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An efficient and robust method to determine the optimal tap coefficients of high speed FIR equalizer

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Abstract The FIR equalizer is a common way to compensate for the effect of ISI in high speed data transmission. One key issue for the FIR equalizer design is to find the optimal tap coefficient (TC) set. In this work, the TC search is formulated as an eye-diagram based global optimization problem and an efficient and robust multi-start point (MSP) optimizer is developed and applied to determine the optimal TC set. To guarantee a robust optimization procedure, an objective function based on the eye diagram is studied. It proves that a 330x speedup is provided by the proposed method as compared with the Monte Carlo method. With the proposed method, the optimal design point of the equalizer can be further estimated.

Keywords tap coefficients, global optimization, multi-start point, eye diagram, Monte Carlo method

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1 Introduction

As CMOS technology is heading to the Gigabits transmission rate, the transmitted data suffers substantially from the intersymbol interferences (ISI) due to the frequency dependent attenuation and multiple reflections [1]. FIR equalizers are the common way to compensate the signal distortion from ISI. As shown in Figure 1, one key issue for a high quality equalizer design is to determine the optimal tap coefficients (TCs) of the FIR filter such that the signal can be recovered.

Many efforts have been made to find the "optimal" TCs for the past decades. Adaptive algorithms are one of the most popular techniques for FIR design[2]. It adaptively adjusts the TCs to accommodate the variations of the transmission channel. A frequency response matching method is proposed in [3] and the TCs can be determined by approximating the inverse of the channel frequency response. Moreover, an eye diagram based optimization method is developed by [4]. A fast eye diagram analysis method is applied to quickly determine the worst case eye pattern based on the step response of the transmission line. Then the optimal TC can be searched using an eye diagram based optimization method.

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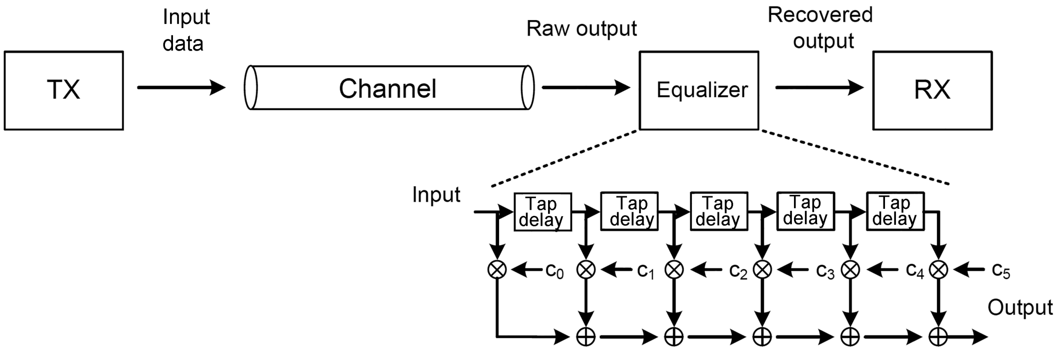

Figure 1 The FIR equalizer. The key to design an FIR equalizer is to determine the TCs $c_0 - c_5$.

Although these existing techniques have largely facilitated the FIR design, most of them focus on either a new formulation [3, 4] or a typical application [2]. The efficiency and robustness of the optimization procedure are not considered carefully and comprehensively. Thus, the obtained “optimal” TC set could correspond to a local optimum point. But in some applications, it is necessary to search for the global or close to global optimal TCs set such that the circuit performance can be maximized without relying on simply increasing the tap number, which in practice will result in more chip area and power dissipation. Meanwhile, with the fast TC performance evaluation, it is possible to estimate the optimal design point of the equalizer by evaluating the best performance of TCs at different design parameters (tap number, tap range and fractional space). However, it is not easy to find such an optimal TC set from a given design parameter range. One straightforward way to search this set of optimal TCs is to run the exhaustive Monte Carlo (MC) simulation. But the computation cost is prohibitively large for traversing all the combinations of TCs, especially as the tap number increases.

This paper introduces a robust and efficient optimal TC design method. The TC search is mapped to an eye diagram based global optimization. A multi-start point (MSP) global optimizer [5, 6] is developed to search the optimal TC set. Although the global optimum is not guaranteed theoretically, the proposed method is proved to be able to significantly enhance the efficiency of the optimal TC search in contrast to the traditional exhaustive search. In order to guarantee a robust optimization procedure, an eye diagram based objective function is carefully studied. A quasi-MC start point generator and a numerical local optimizer are applied to further enhance the efficiency of the MSP global optimizer.

The rest of paper is organized as follows: Section 2 illustrates the proposed method. An extensive study of the robust objective function is carried out. Based on this objective function, an MSP global optimization method is applied to find the optimal TC. In Section 3, an efficiency comparison is performed first. Then, the optimal design point issue of a high speed FIR equalizer is demonstrated by using the proposed method. Finally, a conclusion is drawn in Section 4.

2 Proposed method

An MSP global optimizer is employed to find the optimal TC set in this work. As is presented for a 2-D case in Figure 2, there could exist multiple local regions in the performance surface of the given objective function. The set of TCs provide the best eye opening corresponds to the global optimum. To obtain the global optimum, a local optimizer is used to search the whole performance space from multiple start points. The key of the MSP to achieve a high efficiency is to adopt a “region hit” strategy as compared with the “point hit” of MC method, and it is noted that the probability to hit the region of global optimum is much higher than that to hit a point. Meanwhile, since each local search is independent from each other, the efficiency can be further boosted by current parallel computing technique.

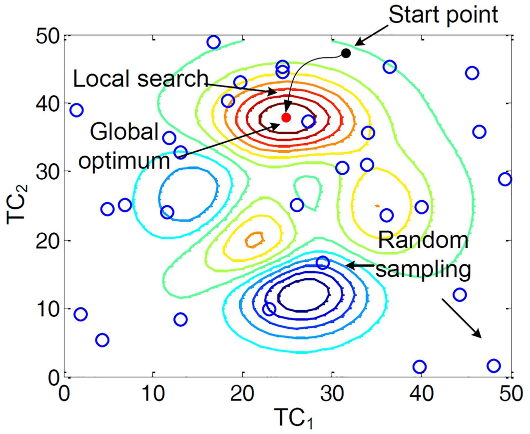

Figure 2 (Color online) Illustration of the MSP method. The global optimum is located if a start point falls into its local region.

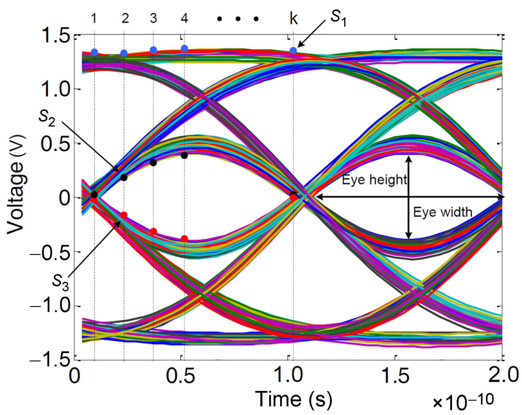

Figure 3 (Color online) Eye diagram with a well selected set of TCs.

2.1 A robust eye-diagram based objective function

An eye diagram based objective function is used in this work. The eye diagram serves as a common metric to judge the quality of the equalizer. A good set of TCs will result in a large eye opening which is shown in Figure 3. On the other hand, the eye collapses to mask if the TCs are not well selected.

To guarantee a robust optimization process, it is critical to define an objective function, which can truthfully represent the eye opening. A poorly-defined objective function will strongly mislead the optimization process in two ways: an unsuccessful optimization or a successful optimization but undesired eye diagram. Both of these situations will largely decrease the number of "effective" start points, thus reducing the efficiency of the MSP.

Three variables $\mathrm{EH}_{\mathrm{ratio}}$, $\mathrm{EH}_{\mathrm{abs}}$ and EW are used simultaneously to characterize the eye opening. As shown in Figure 3, $S_{i|i=1,2,3}$ is defined as the sampled voltage on the eye diagram. Particularly, S_1 are the sampled voltages at the upper bound of eye diagram and marked in blue color. S_2 and S_3 are the sampled voltages at the upper and lower bound of eye opening respectively.

$\mathrm{EH}_{\mathrm{ratio}}$, $\mathrm{EH}_{\mathrm{abs}}$ can be defined as the following forms:

$$\mathrm{EH}_{\mathrm{ratio}} = \sum_{n=1}^{k} S_2(n) \Big/ \sum_{n=1}^{k} S_1(n), \tag{1}$$

$$\mathrm{EH}_{\mathrm{abs}} = \sum_{n=1}^{k} S_2(n) - \sum_{n=1}^{k} S_3(n), \tag{2}$$

$\mathrm{EH}_{\mathrm{ratio}}$ reflects the ratio of the eye opening magnitude over the entire eye diagram. A large value of $\mathrm{EH}_{\mathrm{ratio}}$ indicates the upper bounds of eye diagram and eye opening are close to each other, thus leading to a large ratio. $\mathrm{EH}_{\mathrm{abs}}$ is the sum of the absolute eye opening magnitude at each sampling point. To obtain a large eye opening, it is desired to maximize these two variables.

EW is defined as the eye width. It is the time difference of the eye gaps within the eye opening. Since the magnitude of S_2 tends to zero (a very small positive magnitude) at the gap between two eyes, a threshold voltage is defined and the time difference can be approximated by observing the number of sampled voltage points S_2 whose magnitude is above this voltage. The larger the number, the wider the eye opening will be.

$$\mathrm{EW} : \sum S_2(n)(|V_{S_2(n)}| > V_{\mathrm{threshold}}, n = 1, 2, \ldots). \tag{3}$$

Eventually, the objective function is generalized as

$$\mathrm{obj} = \mathrm{EH}_{\mathrm{ratio}} \times \mathrm{EH}_{\mathrm{abs}} \times \mathrm{EW}. \tag{4}$$

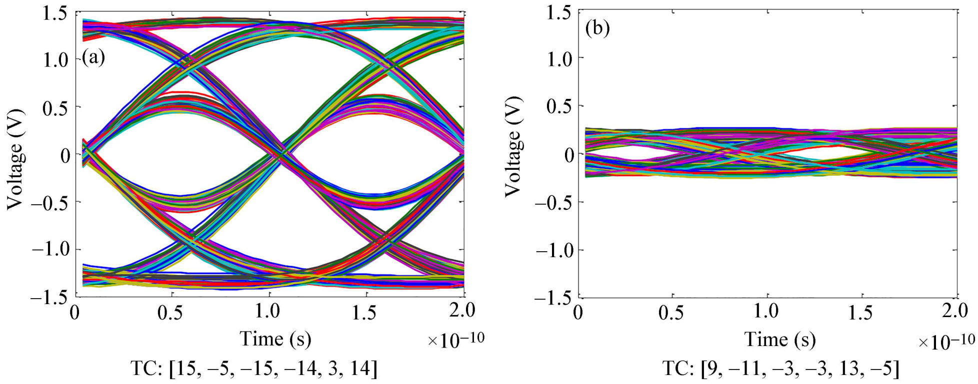

Figure 4 (Color online) Eye diagram obtained from (a) proposed objective function and (b) $\mathrm{EH_{ratio}}$ only.

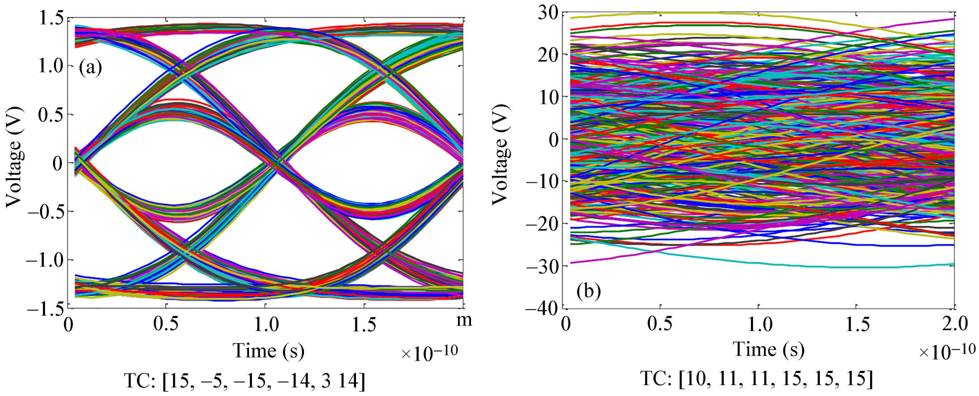

Figure 5 (Color online) Eye diagram obtained from (a) proposed objective function and (b) $\mathrm{EH_{ratio}}$ only.

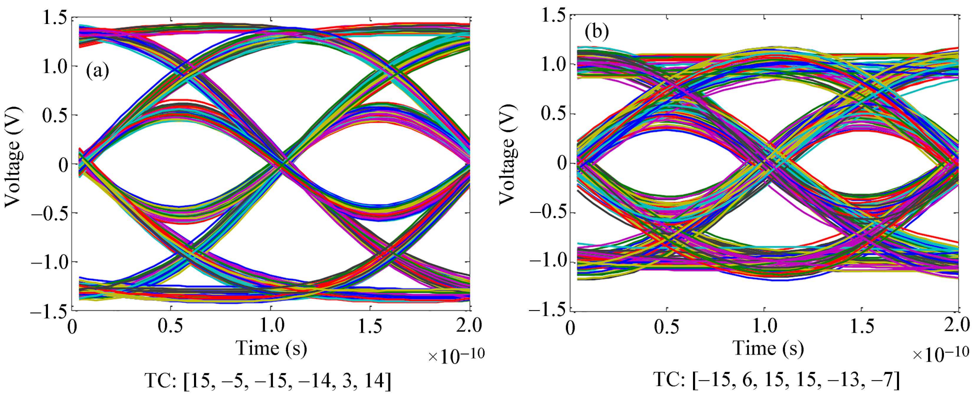

Figure 6 (Color online) Eye diagram obtained from (a) proposed objective function and (b) $\mathrm{EH_{ratio}}\times\mathrm{EH_{abs}}$.

In order to get a big eye opening, the objective function should be maximized. An alternative form can be represented as follows:

$$\mathrm{obj} = w_1 \times \mathrm{EH_{ratio}} + w_2 \times \mathrm{EH_{abs}} + w_3 \times \mathrm{EW}, \tag{5}$$

where $w_i (i = 1, 2, 3)$ is the weight and can be adjusted accordingly for different applications. In this work, Eq. (4) is adopted for the illustration of the proposed method.

To better visualize the robustness of the defined objective function, the comparison of the eye diagrams under different objective functions is performed and illustrated in Figures 4–6. The eye diagrams shown in Figures 4(a)–6(a) are obtained from the proposed objective function. The objective function used for Figure 4(b) is $\mathrm{EH_{ratio}}$ only. It can be observed that the ratio defined by (1) cannot guarantee a desired

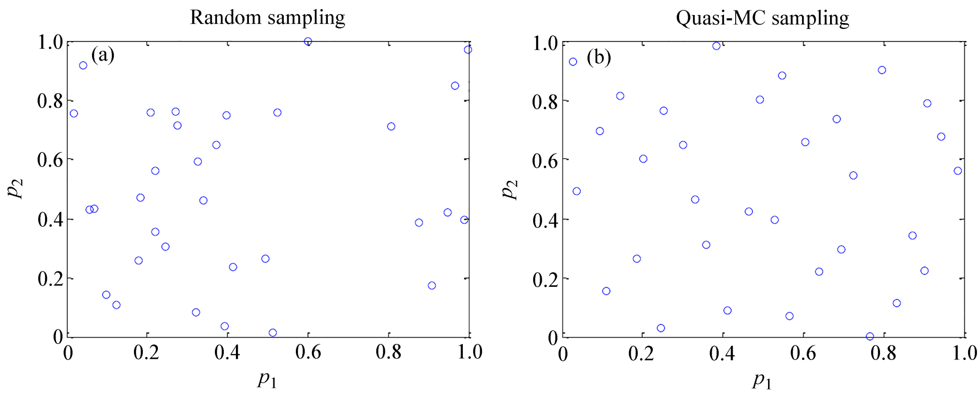

Figure 7 (Color online) Comparison of (a) random sampling and (b) Quasi-MC sampling (p_1, p_2 represent two arbitrary parameters).

eye diagram. Although the computed ratio is large, the absolute value of the eye magnitude could be small. Considering this result, an intuitive idea is to set $\mathrm{EH_{abs}}$ as the objective function. However, using only $\mathrm{EH_{abs}}$ will lead to unsuccessful optimized result which is shown in Figure 5(b). The optimizer tries to widen the eye opening by increasing the overall magnitude of the eye diagram. But the obtained eyes could be completely closed off. Although the robustness of the objective function can be largely improved by combining the first two variables, the eye diagram could still suffer from the timing variation without considering the eye width. This scenario is presented in Figure 6(b). The gap between two adjacent eyes shown in Figure 6(b) is longer than that in Figure 6(a). Thus, a robust objective function should consider all the three variables.

2.2 Optimal TC search

There are two steps to apply the MSP method for optimal TC search. The first step is to generate the start points in the parameter space. To improve the robustness of the global search, a Quasi-MC sequence is utilized to generate the start points:

$$\mathrm{TC}_i = \mathrm{quasimc}(\mathrm{TC}_i^-, \mathrm{TC}_i^+), i = 1, 2, \ldots, n, \tag{6}$$

where $\mathrm{TC}_i^-, \mathrm{TC}_i^+$ represent the lower and upper bound of the defined TC range. As shown for a 2-D parameter space in Figure 7, the start points generated by Quasi-MC present a better uniformity as compared with the ones generated by random sampling. Even though a uniform distribution is adopted for random sampling, some of the start points will still stay too close to each other and the local search from these start points might converge at the same local optimum, thereby, reducing the efficiency of the MSP.

The next step is to search the optimal TCs using a local optimizer. The local search is the core of the MSP since the efficiency of the local optimizer is the determinant factor for the overall performance of the MSP. The local search can be formulated as follows:

$$\begin{aligned} &\max\ \mathrm{obj} = \mathrm{EH_{ratio}} \times \mathrm{EH_{abs}} \times \mathrm{EW}, \\ &\text{s.t.} \quad P = \{\mathrm{TC}_i \in P | \mathrm{TC}_i^- \leqslant \mathrm{TC}_i \leqslant \mathrm{RT}_i^+, i = 1, 2, \ldots, n\}, \end{aligned} \tag{7}$$

where P is the parameter space. Since the objective function is implicit in this case, a numerical optimizer is used in this work to enhance the efficiency of the local search. In order to utilize a numerical optimizer, the key is to map the TC to the value of the defined objective function. This mapping process consists of three steps:

1. Compute the raw output after the channel and convolve it with the given TCs to obtain the recovered data. During this process, a sequence of ideal binary data is used as the input data. FFT is applied to

input data. Then the input data pass through the channel. The channel is modeled in the frequency domain and the model used in this work is presented in (8).

$$c(f) = \mathrm{e}^{-[h_s(1+j)\sqrt{f}+h_d f]l}, \tag{8}$$

where, h_s and h_d are the skin effect factor and dielectric loss factor. f is the frequency and l is the transmission distance. Then, IFFT is applied on the obtained data after the channel, which will be the input data of the equalizer.

2. Construct the eye diagram with the recovered data by cutting the data to pieces in period and overlapping them. In order to run TC for different fractional spaces, a sampling rate of 24 is used in this work. That means there are 24 sampling points in every horizontal line of a single eye. These sampling points are stored during computing the objective function. The sampled voltage $S_{i|i=1,2,3}$ used for objective function can be obtained easily in each vertical slice. For example, $S1$ is the point with the maximum positive magnitude while $S2$ is the one with the minimum positive magnitude in a particular vertical slice.

3. Obtain the variables defined in Subsection 2.1 and evaluate the objective function based on the established eye diagram. After the eye diagram is computed, the objective function can be constructed by using the sample voltage at different positions as is described in Subsection 2.1.

In this way, any change of the TC will be reflected on the obtained objective function value. And this procedure can be treated as a black box during the optimization. The derivatives or conjugate direction needed in the optimization process can be numerically calculated [7]. Thus, more efficient and robust local optimizers such as the Quasi-Newton method [7] or conjugate gradient method [7] can be accessed.

It is worth mentioning that a direct search is adopted in [4] to obtain the optimal TCs. Although the direct search is known as a derivative free optimization method, it in nature has a slower asymptotic convergence speed than many sophisticated gradient-based local optimizers, say, Quasi-Newton since it does not adopt any high order information of the system [8]. Moreover, this property will strongly degrade the performance of the local search, especially when the level set of the function is elongated. The direct search approaches the local minimum by exploring trial step at each dimension of parameter space and moves toward the direction the function reduces. If there is no improvement, the step is halved and the same process is repeated. The process stops until the step is small enough. As is plotted in Figure 8, this strategy will lead to a "zigzagging" track as the contour line of the function is elliptical since the current search direction is not the direction pointing to the local optimum. It will significantly slow down the convergence speed or even leave the optimization process in stagnation when the step reduces to the default tolerance while the local optimum is not reached. In contrast, the number of optimization iteration can be largely reduced by using Quasi-Newton method where the curvature information is considered to control the step and the search direction. Thus, the search efficiency and robustness can be improved.

The flow of the proposed method to find the optimal TC set is generalized as Figure 9. First of all, the objective function is defined, and the initial design parameters (e. g. tap number, fractional space and TCs range) are determined. The main body of the flow is the MSP optimization. The start point is generated and a local search is performed from the given start point. This procedure iterates until a stop criterion is satisfied. An improvement-based stop criterion is used. The simulation is terminated when the improvement of the objective function falls into a predefined threshold. If the obtained eye opening meets the spec, the whole optimization stops. Otherwise, the design parameters are adjusted to start a new optimization procedure.

3 Experimental results

In this section, the efficiency of the proposed method and the TC design issue of a high speed FIR equalizer[9] will be demonstrated. The experiment is performed on the MATLAB in a UNIX system. The channel in the experiment is modeled as (8).

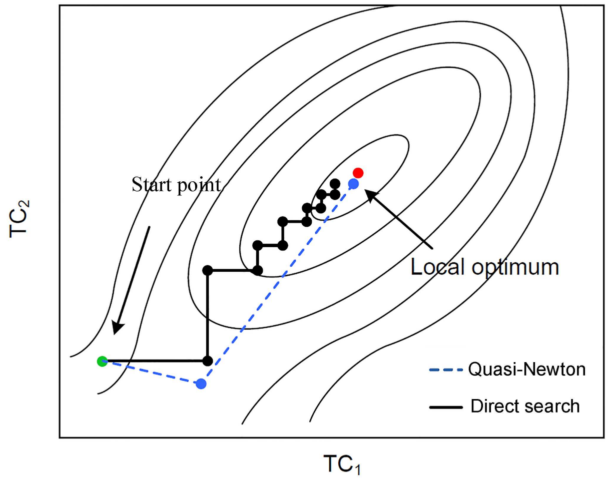

Figure 8 (Color online) Comparison of direct search and Quasi-Newton method.

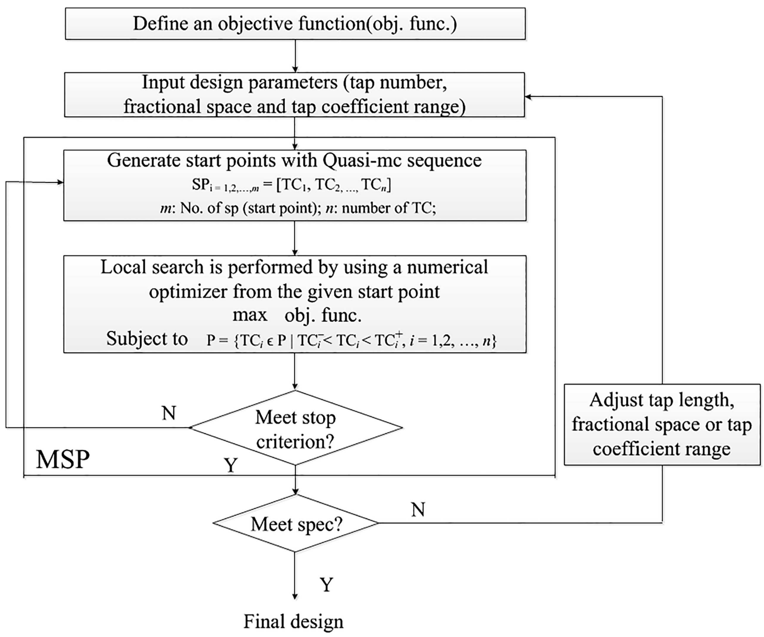

Figure 9 The flow of the optimal TCs search.

The skin effect factor and dielectric loss factor are set to 0.0696 and 0.0073 respectively. The channel is set up at 40 dB attenuation (l = 35 inch) at the half symbol rate frequency of 5 GHz. An 8B/10B encoded data is sent through the channel and equalizer to construct the eye diagram.

The structure of the FIR equalizer is shown in Figure 10. The input or delayed signal is multiplied with TC through the Multiplier DAC (MDAC). Then the current at each stage of MDAC is summed up and converted to output voltage at the summing node. The tap number and fractional space together determine the total tap delay length while MDAC bit range defines the range of a single TC. A fractional space is used to provide a potentially larger bandwidth for each tap delay [9].

3.1 Efficiency comparison

The efficiency of the proposed method is compared with the MC simulation. MC method usually serves as a golden criterion for comparison [10]. Theoretically, the global optimum can be hit if the number of simulation approaches infinity. But it is unacceptable from the point view of running time. In this experiment, a large enough number of simulations are performed. As mentioned before, simulation is

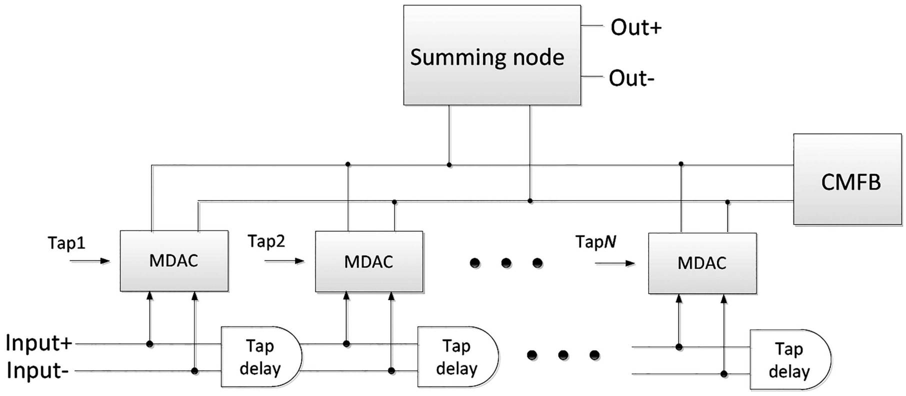

Figure 10 Equalizer architecture.

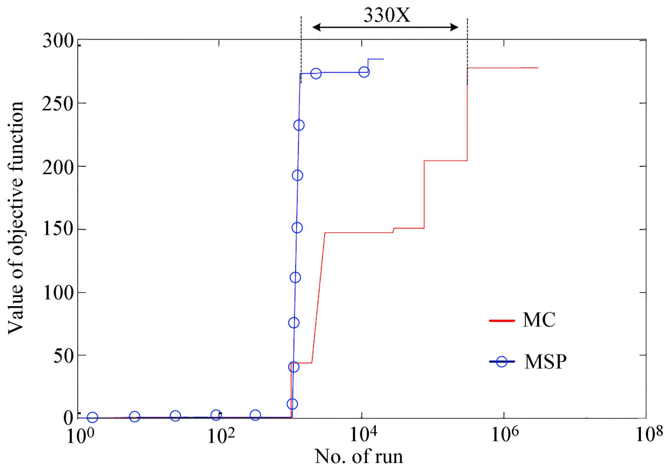

Figure 11 (Color online) Efficiency comparison between MSP and MC. A 330X speedup of simulation time is achieved by the MSP over the MC.

terminated when the performance saturates within a given threshold. Then the efficiency can be compared by the number of simulation runs or running time as the proposed method achieves the same performance.

The experiment is performed for a 6 tap and 1/4 fractional space system. The range for each TC is $[-15, 15]$ (5 bits MDAC). For MC method, each random sample is generated within this range. The eye opening is evaluated based on the previous three mapping steps. The threshold is set to 10 V during the simulation. As shown in Figure 11, 3×10^6 MC simulations are used which takes 3 days on the experimental computer. The objective function eventually saturates after 307000 MC runs, which require 7.2 h. In contrast, MSP only spends 78.4 s to arrive at this level. At this point, a 330X speedup is achieved by MSP.

To further visualize the efficiency of our proposed method, the direct search from the MATLAB toolbox is also performed for the same number of start points. For a total number of 500 start points used by MSP, it takes direct search 213586 runs while Quasi-Newton method only uses 10348 runs. The eye diagrams corresponding to the best TC set obtained by these two methods are presented in Figure 12. It can be observed that the optimal TC set obtained from the Quasi-Newton method outperforms the one obtained by using the direct search while the number of function evaluations with the Quasi-Newton method is only 1/20 as compared with the one using direct search. As discussed in Subsection 2.2, this is due to the different intrinsic mechanisms employed by these two algorithms. Even for the same start point, the direct search may take more steps to reach the local optimum.

Eye diagrams obtained by LMS adaptive algorithm are also shown for comparison in Figure 13. An LMS equalizer object is generated by the "Lineareq" function in the MATLAB for fractional space 4 and 6 taps at the first place. Then the same input (after the channel) is used and the ideal original data is

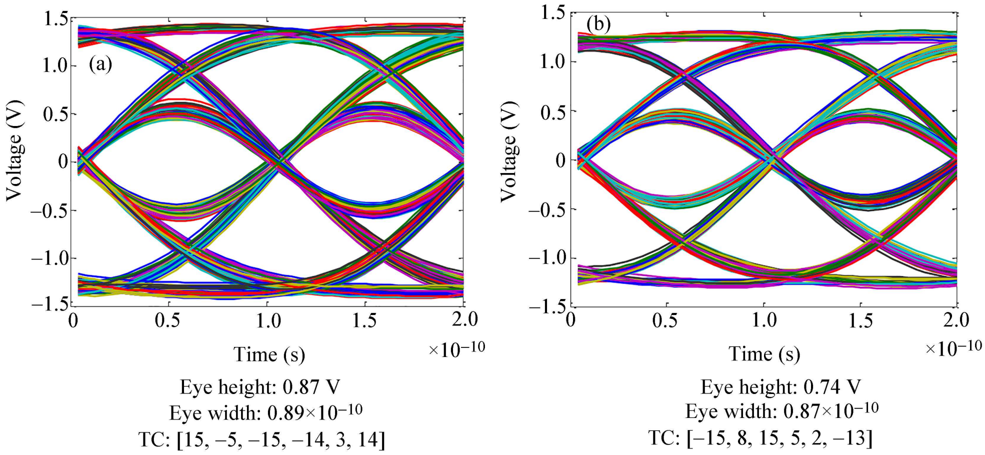

Figure 12 (Color online) Eye diagram comparison. (a) Local search using Quasi-Newton; (b) local search using direct search.

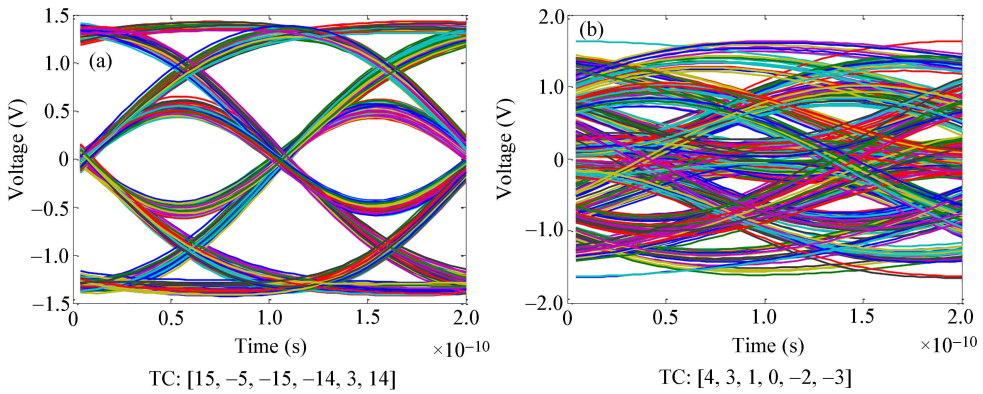

Figure 13 (Color online) Eye diagram comparison. (a) Proposed method; (b) LMS adaptive algorithm.

treated as the reference signal to update the equalizer weight. Although LMS adaptive algorithm has been widely used in the equalizer design, it is in essential a local search. Therefore, it is hard to find a global optimum TC. As shown in the graph, given the default initial value of the equalizer weight (which is all 0), the obtained eye diagram presents a poor eye opening for the provided fractional space and tap length.

3.2 Eye opening comparison for different fractional space, tap length and MDAC bits

Tables 1–3 show the best eye opening obtained by the MSP from 3 to 6 tap numbers and 1/2 to 1/4 fractional spaces. 2000 start points are used for each combination of tap length and fractional space. The eye opening is used

$$\mathrm{FOM} = \mathrm{EH} \times \mathrm{EW}, \tag{9}$$

where EH is the largest absolute eye open height and EW is the eye width as shown in Figure 3.

As mentioned before, a design point can be well estimated by efficiently obtaining the optimal TCs performance at different tap number, fractional space and MDAC bits. Generally, a longer delay line and larger TC range will give a better performance, but at the price of increased parasitic, area and power consumption. The best design point is the TCs which can provide a larger eye opening with the shortest delay length and fewest MDAC bits.

It can be observed from the tables that the FOM presents an overall trend of increase for a longer tap length and larger fractional space as is expected. And there is a diagonal line which is marked at the tables. Below this line, the performance will drop significantly while above it, the increase of the gain

Table 1 Eye opening under TC range [–15, 15][a)]

Fractional space	Tap number			
	3	4	5	6
1/2	2.64	6.47	7.67	8.94
1/3	–	4.62	7.23	8.6
1/4	–	2.66	4.73	7.74

Table 2 Eye opening under TC range [–18, 18][a)]

Fractional space	Tap number			
	3	4	5	6
1/2	2.91	8.59	8.87	10.54
1/3	–	5.08	8.60	9.68
1/4	–	3.60	5.27	9.14

Table 3 Eye opening under TC range [–22, 22][a)]

Fractional space	Tap number			
	3	4	5	6
1/2	4.23	10.12	11.14	13.52
1/3	–	7.2	10.49	12.30
1/4	–	4.59	7.57	11.25

a) The numbers in the tables are given in (9). Since EW is too small, it is rounded up during the multiplication. For example, if EH = 0.7, EW = 8.5e−11. FOM equals to 0.7×8.5.

slows down. The combination of the design parameter in the red line indicates the potential optimal design point. Above this diagonal line, a longer delay and larger area are required for implementation while the performance will not be enhanced significantly.

On the other hand, it should be noted that the increase of the MDAC bits will not result in a significant performance gain. The FOM almost linearly increases with the expansion of the TC range. An appropriate TC range should be set to satisfy the spec requirement of the real application at the price of minimum area.

4 Conclusion

In this work, a robust and efficient method is proposed to determine the optimal TCs of the FIR equalizer. A robust objective function is studied first. Then the TC search is formulated as a global optimization problem and an MSP global optimizer is developed. The experiment results show that a 330x speedup is achieved as compared with MC method. At the same time, the best eye opening at different design parameter such as the tap number, fractional space and MDAC bits are compared and a potential optimal design point is estimated. The proposed methodology will largely shorten the design cycle by providing a fast and accurate TC determination procedure. The future work will focus on the speedup of MSP using GPU based parallel computing.

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Conflict of interest The authors declare that they have no conflict of interest.

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