

# Antenna selection for two-way full duplex massive MIMO networks with amplify-and-forward relay

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**Abstract** In this paper, the antenna selection problem is investigated for the relaying system with the base station having massive multiple antennas, where the relay station works in the full duplex mode and the two-way protocol. The antenna selection scheme is optimized by maximizing the minimum of the two signal to noise ratios (SNRs) with each corresponding to the uplink and the downlink traffics. With the optimized antenna selection scheme, the approximate probability density function (PDF) of the received SNR for each receiver is derived. In what follows, both the overall outage probability and the bit error rate are obtained in the analytical expression for the whole massive multi-input multi-output (MIMO) system. Moreover, the asymptotic overall outage probability is derived in an analytical expression with respect to the growing number of the antennas on the massive MIMO base station. Numerical simulations verify the derived analytical results.

**Keywords** amplify-and-forward, bit error rate, full duplex, massive MIMO, outage probability, performance analysis

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## 1 Introduction

Multi-input multi-out (MIMO) technology provides substantial improvements in terms of spectral efficiency and link reliability [1–5]. Recently, the massive MIMO becomes one of the key technologies in 5G since it can supply much gain of both the spectral efficiency and the energy efficiency [6]. However, this gain is attained at the cost of employing many antennas, which indicates that the computational complexity of the massive MIMO system becomes very huge especially in the regime of the large number of the antennas. Therefore, the antenna selection is very suitable for this case to exploit the advantages of the massive MIMO technology yet without such huge complexity requirement as described in this literature such as [7].

Network coding has been proposed for one decade to improve the spectrum efficiency [8]. Furthermore, the complexity of both the encoder and the decoder is greatly reduced by the developed physical network coding (PNC) [9]. Thus, PNC has attracted a lot of the attentions in both the academia and the industry

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in many references such as [10–12]. This good property of the physical network coding is also applicable to the relaying systems, where the throughput of the relaying system is significantly improved yet the corresponding cost is not that high. The reason is that the physical features are utilized by the physical network coding, which contrasts to the conventional network coding that requires much more complexity, as described in [10]. To exploit both the PNC and the MIMO technologies, many researchers in wireless communications [13,14] try to combine these two technologies by applying the PNC to the MIMO relaying system.

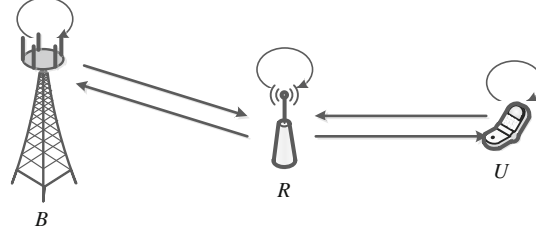
Most existing papers over the antenna selection schemes are limited in the half duplex relaying network [15]. For example, an end-to-end (e2e) single antenna selection scheme is investigated in [16], where the bit error rate (BER) is also derived. The authors in [17] propose an optimal transmit antenna selection scheme with the conclusion that the optimized antenna selection scheme can achieve the full diversity order. Two suboptimal transmit antenna selection schemes are developed in [18] which reduce much complexity yet achieve the full diversity order. In [19], two antenna selection schemes are proposed for the relay station in the MIMO two-way relaying networks. Recently, due to the gain in the doubling the utilization of the spectrum, the full duplex protocol is treated as one of the potential techniques for the 5G communication systems and has been investigated by many researchers in this literature such as [20–22]. In [23–26], the amplify-and-forward and decode-and-forward full duplex relaying protocols have been studied, where several transmission schemes have been proposed by carrying out the power allocation and the relay location. In [27] and [28], both of the joint precoder and the decoder are jointly designed, respectively, for the one-way and the two-way full duplex relaying systems. Furthermore, a couple of the iterative algorithms are proposed to reduce the required computational complexity. Moreover, the authors in [29] examine the joint precoder-and-decoder design with the antenna subset selection in the one way full duplex relaying system to achieve the capacity maximum. In [30], the joint antenna selection scheme is designed for the half-duplex protocol, where all of the antennas on the base station, the two-way relay station and the mobile terminal are jointly selected to maximize the minimum of the two directional channel gain. However, the antenna selection problems as in the above references will become very difficult if the half-duplex protocol at each node becomes the full-duplex protocol that becomes very hot in 5G.

To the author's best knowledge, there is no research reports on the antenna selection scheme design for the two-way full duplex relaying system with the base station having massive MIMO. Under the full-duplex protocol, the results in [30] do not work in the same system model since the self-interference as a result of the full-duplex protocol deteriorates the performance remarkably. Motivated by this observation, an antenna selection scheme is optimized in this paper for the massive MIMO base station in this system. Compared to the transmit beamforming or the precoding scheme from [16–18], the proposed antenna selection scheme substantially reduces the complexity and the power consumption. With the optimized antenna selection scheme in this paper, both the overall outage probability and the bit error rate are analyzed in the closed form expression for the whole system. Simulations demonstrate the effectiveness of the proposed antenna selection scheme with the derived expression of the two performances.

This paper is organized as follows. First, in Section 2, we describe our system model and assumptions and propose our antenna selection scheme. Then we derive the performance of the antenna selection scheme for the two-way full duplex relaying networks with the PNC in Section 3, and in Section 4 simulations are demonstrated to validate our derivations in Section 3. Section 5 is conclusion.

## 2 System model

Consider a full duplex two-way relaying system with the base station (BS) having massive antennas as illustrated in Figure 1. All of the base station, the relay station and the mobile terminal work in the full duplex. Assume that there is no direct link between the base station and the mobile terminal due to the large scale channel fading such as the path loss and the channel shadowing effect. The number of the antennas on BS is  $N_B$  and both the relay station and the mobile terminal are equipped with



**Figure 1** System model of the full-duplex two-way relaying system with the base station having massive antennas.

single antenna.

The channel between each pair of the transmitter and the receiver is modeled by the flat Rayleigh fading. The wireless channel from the base station to the relay station is denoted by  $\mathbf{h} \in \mathbb{C}^{1 \times N_B}$ . The wireless channel from the relay station to the mobile terminal is denoted by  $g \in \mathbb{C}^{1 \times 1}$ .  $\mathbf{h}^{(1)}$  is  $1 \times N_B$  vector, which denotes the residual self-interference channel at the base station, while  $h^{(R)}$  and  $h^{(2)}$  denote the residual self-interference channel at  $R$  and  $U$  respectively. We assume that the wireless channels of all the links are independent and identically distributed (i.i.d.). Based on the known channel state information (CSI), the base station selects one antenna from all the massive antennas at the base station.

In each time slot, both the base station and the mobile terminal transmit, respectively,  $x_1[n]$  and  $x_2[n]$  simultaneously to the relay station in the same frequency, while the relay node receives the two signals and forwards the received signals in the following slot after the relaying amplifying function. In the slot  $n$ , the received signal at the relay station is written below

$$y_R[n] = \sqrt{P_1}h_i x_1[n] + \sqrt{P_2}g x_2[n] + \sqrt{P^{(R)}}h^{(R)}t[n] + n_R[n], \quad (1)$$

where  $i$  means that the  $i$ th antenna is selected at the base station,  $h_i$  is the  $i$ th element in  $\mathbf{h}$ ,  $t[n]$  is the forwarded signal and  $n_R$  is the Additive White Gaussian Noise (AWGN) at  $R$  with the distribution  $n_R \sim \mathcal{CN}(0, \sigma_R^2)$ .

For the full duplex relay, the forwarded signal  $t[n]$  is determined by the received signal  $y_R[n - 1]$  at the relay station. Due to the power constraint at  $R$ , the received signal  $y_R$  is multiplied by the channel gain  $\alpha$  as described below

$$\alpha = \sqrt{\frac{1}{P_1 \|h_i\|^2 + P_2 \|g\|^2 + P^{(R)} \|h^{(R)}\|^2 + \sigma_R^2}}. \quad (2)$$

In the next time slot, the relay station transmits the multiplied signal

$$t[n] = \alpha y_R[n - 1]. \quad (3)$$

With the assumption of the channel reciprocity, the signal is given by

$$y_1[n] = \sqrt{P_R} \alpha h_i^* y_R[n - 1] + \sqrt{P^{(1)}} h_i^{(1)} x_1[n] + n_1[n], \quad (4)$$

$$y_2[n] = \sqrt{P_R} \alpha g^* y_R[n - 1] + \sqrt{P^{(2)}} h^{(2)} x_2[n] + n_2[n], \quad (5)$$

where  $(\cdot)^*$  denotes the complex conjugate,  $n_1$  and  $n_2$  are the AWGN noise received at  $B$  and  $U$ , where each noise is distributed with  $n_1 \sim \mathcal{CN}(0, \sigma_1^2)$  and  $n_2 \sim \mathcal{CN}(0, \sigma_2^2)$ .

After the self-interference cancelation at the base station and the mobile terminal, the equivalent signal is expressed as

$$\tilde{y}_1[n] = \sqrt{P_R P_2} \alpha h_i^* g x_2[n] + \sqrt{P_R P^{(R)}} \alpha h_i^* h^{(R)} t[n] + \sqrt{P_R} \alpha h_i^* n_R[n] + \sqrt{P^{(1)}} h_i^{(1)} x_1[n] + n_1[n], \quad (6)$$

$$\tilde{y}_2[n] = \sqrt{P_R P_1} \alpha g^* h_i x_1[n] + \sqrt{P_R P^{(R)}} \alpha g^* h^{(R)} t[n] + \sqrt{P_R} \alpha g^* n_R[n] + \sqrt{P^{(2)}} h^{(2)} x_2[n] + n_2[n]. \quad (7)$$

Without loss of generality, we assume that  $\sigma_R^2 = \sigma_1^2 = \sigma_2^2 = 1$ . The SNR received at the base station and the mobile terminal can be obtained from (6) and (7)

$$\gamma_1 = \frac{\tilde{P}_2 \tilde{P}_R^{(1)} \|h_i\|^2 \|g\|^2}{(\tilde{P}_R^{(1)} + \tilde{P}_1) \|h_i\|^2 + \tilde{P}_2 \|g\|^2 + 1}, \quad (8)$$

$$\gamma_2 = \frac{\tilde{P}_1 \tilde{P}_R^{(2)} \|h_i\|^2 \|g\|^2}{\tilde{P}_1 \|h_i\|^2 + (\tilde{P}_R^{(2)} + \tilde{P}_2) \|g\|^2 + 1}, \quad (9)$$

where  $\tilde{P}_1 = \frac{P_1}{P^{(R)}(\sigma^{(R)})^2 + 1}$ ,  $\tilde{P}_2 = \frac{P_2}{P^{(R)}(\sigma^{(R)})^2 + 1}$  and  $\tilde{P}_R^{(1)} = \frac{P_R}{P^{(1)}(\sigma_i^{(1)})^2 + 1}$  and  $\tilde{P}_R^{(2)} = \frac{P_R}{P^{(2)}(\sigma_k^{(2)})^2 + 1}$ ,  $(\sigma_j^{(R)})^2 = \mathbb{E}\{\|h^{(R)}\|^2\}$ ,  $(\sigma_i^{(1)})^2 = \mathbb{E}\{\|h_i^{(1)}\|^2\}$  and  $(\sigma_k^{(2)})^2 = \mathbb{E}\{\|h^{(2)}\|^2\}$ ,  $\mathbb{E}(\cdot)$  stands for the expectation over the corresponding variable. Kindly note that the transmit power  $\tilde{P}_1$  in this paper is always smaller than the counterpart  $P_1$  in [30]. This is because one part of the part is partitioned from the total power  $P_1$  to deal with the self-interference. The surplus power  $\tilde{P}_1$  is employed to transmit the signal. In this method, the performance in full-duplex protocol can be improved greatly. Otherwise, the existing schemes for the half-duplex protocol such as [30] cannot deal with the strong self-interferences in full-duplex protocol. Here we assume that the power of the self-interference at each node is relatively smaller than that of the channels.

The proposed antenna selection scheme is carried out at the base station, where single one antenna is chosen from all the massive antennas at this base station to transmit and receive the signal. The objective of the antenna selection scheme is to maximize the minimum of the two SNRs for the two directional traffics as

$$\hat{i} = \arg \max_i \min\{\gamma_1, \gamma_2\}. \quad (10)$$

After some mathematical derivations, the criterion of the antenna selection is further obtained as

$$\hat{i} = \arg \max_i \|h_i\|^2. \quad (11)$$

Considering the fact that there are many antennas at the base station, the SNR corresponding to the selected antenna is usually very large due to the antenna diversity gain, namely, the value of  $\|h_i\|^2$  is very large. Thus, Eqs. (8) and (9) can be approximated as

$$\gamma_1 \approx \frac{\tilde{P}_2 \tilde{P}_R^{(1)} \|h_i\|^2 \|g\|^2}{(\tilde{P}_R^{(1)} + P_1) \|h_i\|^2 + \tilde{P}_2 \|g\|^2}, \quad (12)$$

$$\gamma_2 \approx \frac{\tilde{P}_1 \tilde{P}_R^{(2)} \|h_i\|^2 \|g\|^2}{\tilde{P}_1 \|h_i\|^2 + (\tilde{P}_R^{(2)} + \tilde{P}_2) \|g\|^2}. \quad (13)$$

In a summary, the single antenna selection scheme is designed by maximizing the minimum of the two SNRs corresponding to the two directional traffics. Moreover, the SNR expression of the proposed antenna selection is also derived for the two-way full duplex relaying system. With these SNR expressions, we would like to study the performance such as the bit error rate and the outage probability in the next section.

### 3 Performance analysis of the full duplex two-way relaying system with the proposed antenna selection scheme

In this section, we first derive the exact expression for the probability density function (PDF) of the end-to-end SNR. To facilitate the following performance analysis, we further derive the approximate expression of the obtained PDF. With this result, the bit error rate of the end-to-end link is analyzed with the asymptotic number of the antennas at the base station. In what follows, each wireless channel is distributed with  $h_i \sim \mathcal{CN}(0, \sigma_h^2)$  and  $g \sim \mathcal{CN}(0, \sigma_g^2)$ .

### 3.1 Probability density function of the end-to-end SNR

With the channel distribution  $h_i \sim \mathcal{CN}(0, \sigma_h^2)$  and  $g \sim \mathcal{CN}(0, \sigma_g^2)$ , the PDFs of  $|h_i|^2$  and  $|g|^2$  are given by

$$f_{|h_i|^2}(x) = \frac{1}{\sigma_h^2} e^{-\frac{x}{\sigma_h^2}}, \tag{14}$$

$$f_{|g|^2}(y) = \frac{1}{\sigma_g^2} e^{-\frac{y}{\sigma_g^2}}. \tag{15}$$

After carrying out the proposed antenna selection at the massive MIMO base station, the PDF and the CDF of the equivalent channel  $|h|^2 = \max_i |h_i|^2$  can be obtained as

$$f_{|h|^2}(x) = \frac{N_B}{\sigma_h^2} \left(1 - e^{-\frac{x}{\sigma_h^2}}\right)^{N_B-1} e^{-\frac{x}{\sigma_h^2}}, \tag{16}$$

$$F_{|h|^2}(x) = \left(1 - e^{-\frac{x}{\sigma_h^2}}\right)^{N_B}. \tag{17}$$

Since the wireless channels  $h$  and  $g$  are independently distributed, the joint probability distribution of the two channel coefficients  $h$  and  $g$  is given by

$$p_{|h|^2, |g|^2}(x, y) = \frac{N_B}{\sigma_h^2 \sigma_g^2} \left(1 - e^{-\frac{x}{\sigma_h^2}}\right)^{N_B-1} e^{-\frac{x}{\sigma_h^2}} e^{-\frac{y}{\sigma_g^2}}. \tag{18}$$

It is observed that (12) and (13) are very similar to each other. To facilitate the following derivations, we define two new variables to derive the distribution of these two SNRs. Let  $z = \frac{xy}{ax+by}$  and  $w = \frac{ax^2}{ax+by}$ . The Jacobian determinant is given by

$$\det(J_F(z, w)) = -a - \frac{2abz}{w} - \frac{ab^2z^2}{w^2}. \tag{19}$$

The joint probability distribution of both  $Z$  and  $W$  is written as

$$p_{ZW}(z, w) = |\det(J_F(z, w))| \frac{N_B}{\sigma_h^2 \sigma_g^2} \left(1 - e^{-\frac{w+bz}{\sigma_h^2}}\right)^{N_B-1} e^{-\frac{w+bz}{\sigma_h^2}} e^{-\frac{az + \frac{abz^2}{w}}{\sigma_g^2}}, \tag{20}$$

where  $|\cdot|$  is the absolute value function.

The probability distribution of  $Z$  can be derived as

$$p_Z(z) = \int_0^\infty p_{ZW}(z, w) dw, \tag{21}$$

which can be further expressed by

$$p_Z(z) = \frac{N_B}{\sigma_h^2 \sigma_g^2} \int_0^\infty \det(J_F(z, w)) \left(1 - e^{-\frac{w+bz}{\sigma_h^2}}\right)^{N_B-1} e^{-\frac{w+bz}{\sigma_h^2}} e^{-\frac{az + \frac{abz^2}{w}}{\sigma_g^2}} dw. \tag{22}$$

According to the binomial theorem, we have the following derivations as

$$\left(1 - e^{-\frac{w+bz}{\sigma_h^2}}\right)^{N_B-1} = \sum_{i=0}^{N_B-1} \binom{N_B-1}{i} (-1)^i e^{-i\frac{w+bz}{\sigma_h^2}}, \tag{23}$$

where  $\binom{\cdot}{\cdot}$  is the binomial coefficient. Concerning the integration  $\int_0^\infty x^{a-1} e^{-px-q/x} dx$ , we can further expressed it as

$$\int_0^\infty x^{a-1} e^{-px-q/x} dx = 2 \left(\frac{q}{p}\right)^{a/2} K_a(2\sqrt{pq}), \tag{24}$$

where  $K_a(\cdot)$  is the modified Bessel function of the second kind [31]. Substituting (23) and (24) into (22), the PDF of  $z = \frac{xy}{ax+by}$  is obtained below,

$$p_Z(z) = \frac{N_B}{\sigma_h^2 \sigma_g^2} \sum_{i=0}^{N_B-1} \binom{N_B-1}{i} (-1)^i e^{-(i+1)\frac{bz}{\sigma_h^2} - \frac{az}{\sigma_g^2}} \left[ a \left( \frac{abz^2 \sigma_h^2}{(i+1)\sigma_g^2} \right)^{\frac{1}{2}} K_1 \left( \frac{2z}{\sigma_h^2 \sigma_g^2} \sqrt{(i+1)ab} \right) + \frac{ab^2 z^2}{w^2} \left( \frac{abz^2 \sigma_h^2}{(i+1)\sigma_g^2} \right)^{-\frac{1}{2}} K_{-1} \left( \frac{2z}{\sigma_h^2 \sigma_g^2} \sqrt{(i+1)ab} \right) + \frac{2ab}{w} K_0 \left( \frac{2z}{\sigma_h^2 \sigma_g^2} \sqrt{(i+1)ab} \right) \right]. \quad (25)$$

To this end, the PDF of the two SNRs for both the base station and the mobile terminal is obtained in an analytical expression. With these two PDFs, the CDF for both the base station  $P_{\gamma_1}(z)$  and the mobile terminal  $P_{\gamma_2}(z)$  can be readily attained.

### 3.2 Outage probability analysis for the proposed antenna selection scheme at the base station

As discussed in the previous sections, the outage event occurs as long as any one of the two SNRs is smaller than the predetermined threshold. Thus, the outage probability in our system can be defined as

$$P_{\text{out}} = \Pr \{ \min(\gamma_1, \gamma_2) < \gamma_{\text{th}} \} = 1 - (1 - P_{\gamma_1}(\gamma_{\text{th}}))(1 - P_{\gamma_2}(\gamma_{\text{th}})), \quad (26)$$

which is the function of the CDF of the received SNRs of both the base station and the mobile terminal. The closed form expression of the outage probability can be obtained according to the derivations and the discussions as above. However, the obtained expression is sophisticated. To obtain the insights of this complicated expression, the expression (26) is further approximated by applying the method from [32] below

$$P_{\text{out}} \simeq 1 - \left[ 1 - \left( 1 - e^{-\frac{\gamma_{\text{th}}}{\tilde{\Omega}_1}} \right)^{N_B} \right] e^{-\frac{\gamma_{\text{th}}}{\tilde{\Omega}_2}}, \quad (27)$$

where

$$\tilde{\Omega}_1 = \frac{\tilde{P}_R \tilde{P}_2}{\tilde{P}_R + \tilde{P}_1} \sigma_h^2, \quad (28)$$

$$\tilde{\Omega}_2 = \frac{\tilde{P}_R \tilde{P}_1}{\tilde{P}_R + \tilde{P}_2} \sigma_g^2. \quad (29)$$

It is observed that the number of the antennas on the base station is usually very large due to the massive MIMO base station, which motivates us to derive the concise expression of the outage probability by approximating the exact expression with respect to the growing number of the antennas on the base station. In what follows, we give the following corollary.

**Corollary 1.** When the antenna number of the base station is extremely large, i.e.,  $N_B \gg 1$ , the outage probability can be approximated by

$$P_{\text{out}} \approx 1 - e^{-\frac{\gamma_{\text{th}}}{\tilde{\Omega}_2}}. \quad (30)$$

*Proof.* Since  $N_B \gg 1$ ,  $1 - e^{-\frac{\gamma_{\text{th}}}{\tilde{\Omega}_1}} < 1$ , thus we have

$$\lim_{N_B \rightarrow +\infty} \left( 1 - e^{-\frac{\gamma_{\text{th}}}{\tilde{\Omega}_1}} \right)^{N_B} = 0. \quad (31)$$

The corollary can easily be obtained.

**Remark 1.** From Corollary 1, it is discovered that the asymptotic outage probability of the whole system only depends on the outage probability of the link between the relay station and the mobile terminal. This is because the channel between the base station and the relay station becomes very good after the proposed antenna selection scheme over the massive MIMO base station.

### 3.3 Performance analysis on the overall bit error rate

The bit error rate is defined as the number of error bits divided by the total number of bits during the transmission, while the overall bit error rate is defined as the average of each link's bits error rate in the networks during the transmission period, which can be calculated as

$$P_e \triangleq \frac{1}{2}(P_1 + P_2), \tag{32}$$

where  $P_1$  is the BER of the link  $B \rightarrow R \rightarrow U$ , while  $P_2$  is the BER of the link  $U \rightarrow R \rightarrow B$ . Clearly the overall BER is dominated by the minimum of the two directional links. Thus, we can rewrite (32) as

$$P_e \approx \frac{1}{2} \max\{P_1, P_2\}. \tag{33}$$

**Theorem 1.** The overall BER of the proposed single antenna selection scheme is given by

$$P_e = \frac{\beta_1 \sqrt{\beta_2}}{4\sqrt{\pi}} \left[ \frac{1}{\beta_2^{\frac{1}{2}}} \Gamma\left(\frac{1}{2}\right) - \frac{1}{(\beta_2 + \frac{1}{\Omega_2})^{\frac{1}{2}}} \Gamma\left(\frac{1}{2}\right) + \sum_{i=0}^{N_B} (-1)^i \frac{1}{(\beta_2 + \frac{i}{\Omega_1} + \frac{1}{\Omega_2})^{\frac{1}{2}}} \Gamma\left(\frac{1}{2}\right) \right], \tag{34}$$

where  $\binom{\cdot}{\cdot}$  is the binomial coefficient.

Eq. (34) can be reduced below when the modulation type is BPSK that makes the values of the two coefficients in the above equation as  $\beta_1 = \beta_2 = 1$ ,

$$P_e = \frac{1}{4\sqrt{\pi}} \left[ \Gamma\left(\frac{1}{2}\right) - \frac{1}{(1 + \frac{1}{\Omega_2})^{\frac{1}{2}}} \Gamma\left(\frac{1}{2}\right) + \sum_{i=0}^{N_B} (-1)^i \frac{1}{(1 + \frac{i}{\Omega_1} + \frac{1}{\Omega_2})^{\frac{1}{2}}} \Gamma\left(\frac{1}{2}\right) \right]. \tag{35}$$

*Proof.* As derived in the last subsection, the CDF of the equivalent SNR after the proposed antenna selection scheme is given by

$$P_{\gamma_e}(z) \simeq 1 - \left[ 1 - \left( 1 - e^{-\frac{z}{\Omega_1}} \right)^{N_B} \right] e^{-\frac{z}{\Omega_2}}. \tag{36}$$

Using the binomial theorem, we rewrite the expression as

$$P_{\gamma_e}(z) \simeq 1 - e^{-\frac{z}{\Omega_2}} + \sum_{i=0}^{N_B} (-1)^i e^{-\frac{iz}{\Omega_1} - \frac{z}{\Omega_2}}. \tag{37}$$

According to [33], the BER at the base station and that at the mobile terminal are mathematically formulated as

$$P_b = \mathbb{E}[\beta_1 Q(\sqrt{2\beta_2\gamma})] = \int_0^{+\infty} \beta_1 Q(\sqrt{2\beta_2 z}) p_{\gamma_e}(z) dz, \tag{38}$$

where  $\beta_1$  and  $\beta_2$  are the coefficients corresponding to the modulation order,  $Q(x)$  is the tail probability of the standard normal distribution as given below

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) du. \tag{39}$$

Thus, Eq. (33) can be rewritten as

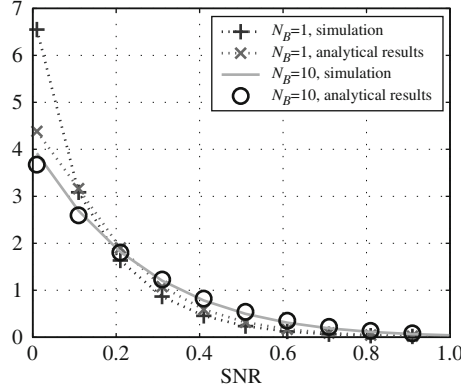
$$P_e \approx \frac{1}{2} \int_0^{+\infty} \beta_1 Q(\sqrt{2\beta_2 z}) p_{\gamma_e}(z) dz = \frac{\beta_1 \sqrt{\beta_2}}{4\sqrt{\pi}} \int_0^{+\infty} \frac{e^{-\beta_2 z}}{\sqrt{z}} P_{\gamma_e}(z) dz. \tag{40}$$

Substituting (37) into (40), where the results from [31] are exploited as

$$\int_0^{\infty} x^{\nu-1} e^{-\mu x} dx = \mu^{-\nu} \Gamma(\nu). \tag{41}$$

We can obtain the result.

In this section, we have derived the probability of the SNR distribution for the full duplex two-way relaying system. Then, the outage probability of each directional traffic is derived in the closed form. Moreover, the asymptotic bit error rate is obtained by exploiting the antenna selection gain at the massive MIMO base station. The derived results will be verified numerically in the next section.



**Figure 2** The PDF of the end-to-end SNR over the transmit SNR.

## 4 Numerical simulations and discussions

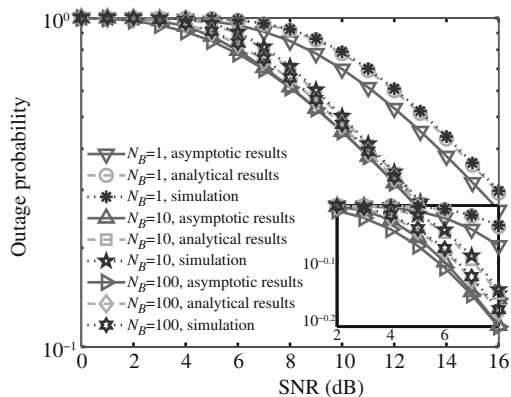
In this section, the numerical results are provided to validate the the obtained performance analysis.

Figure 2 shows the PDF of the end-to-end SNR from the base station to the mobile terminal, when  $\sigma_h^2 = \sigma_g^2 = 2$ ,  $(\sigma^{(R)})^2 = (\sigma^{(1)})^2 = (\sigma^{(2)})^2 = 0.1$ . As shown in this figure, the performance analysis according to (25) by the proposed approximation method is very tight as compared to the exact expression when the number of the antennas is in the high regime. With the increasing number of the antenna at the base station, the gap between the theoretical expression and the numerical simulation results overlaps with each other almost. Especially, when  $N_B = 10$ , the gap tends to be very small.

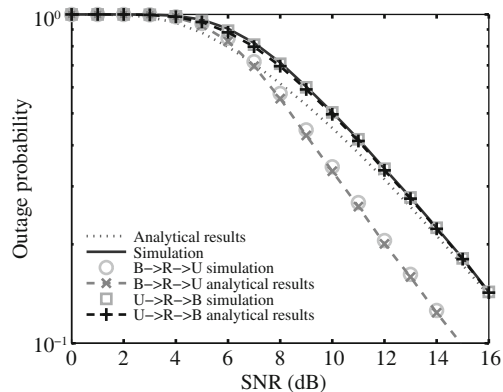
The outage probability based on (27) for the whole system is plotted in Figures 3–5, where  $\sigma_h^2 = \sigma_g^2 = 1.1$ ,  $(\sigma^{(R)})^2 = (\sigma^{(1)})^2 = (\sigma^{(2)})^2 = 0.1$ ,  $P^{(R)} = P^{(1)} = P^{(2)} = 1$  and the threshold of the rate is set to  $R_0 = 1$  bit/s/Hz. In Figures 3 and 4, the outage probability of the whole system is described with the different transmit SNRs. Figure 3 compares the analytical results in (26) with the asymptotic results in (27) as well as the numerical results. From this figure, it is seen that the analytical results almost coincide with the numerical results, which means that the expression in (26) is the closed form expression of the outage probability for the whole system. Meanwhile, it is also seen that the approximation in [32] is suitable in our antenna selection scheme that selects one of the massive antennas on the base station and the approximation in (27) is very tight as compared to the exact expression in (26). Figure 4 depicts the outage probability of the different links versus the transmit SNRs when  $N_B = 10$ , from which we can verify the basic principle for the approximation from (32) to (33). The analytical results for each link is derived from (25). Although the link quality from  $B$  to  $U$  is better than that from  $U$  to  $B$ , the overall outage probability is dominated by the worse one these two links. Thus, in the following derivations of the overall bit error rate, we use the worse link as the substitution of the summation over these two links. In Figure 5, the outage probability of the whole system as a function of the number of the antennas is plotted, which verifies our derivation for the asymptotic outage probability as the number of the antennas at the base tends to the infinity, where the asymptotic results in this figure are plotted according to (30). The figure verifies the correctness of Corollary 1. When the number of the antennas at the base station is extremely large, the outage probability of the worse link tends to the bound in (30), which dominates the overall outage probability.

Figure 6 compares the approximate average BER with the numerical simulations when  $\sigma_h^2 = \sigma_g^2 = 1.1$ ,  $(\sigma^{(R)})^2 = (\sigma^{(1)})^2 = (\sigma^{(2)})^2 = 0.1$  and  $P^{(R)} = P^{(1)} = P^{(2)} = 1$ , which shows that the closed-form expressions derived in the last section provide a good approximation to the simulation results, especially in the high SNR regime. Note that the approximated results are the upper bounds of the exact performances. As the transmit SNR increases, the gap between the exact performance and the approximated results becomes narrower with the fact that the approximation in (33) becomes asymptotically optimal in the high transmit SNR regime.

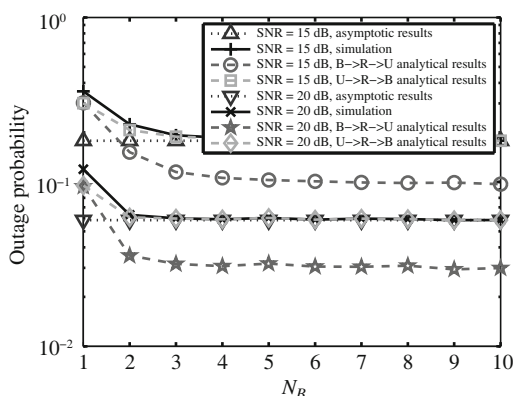




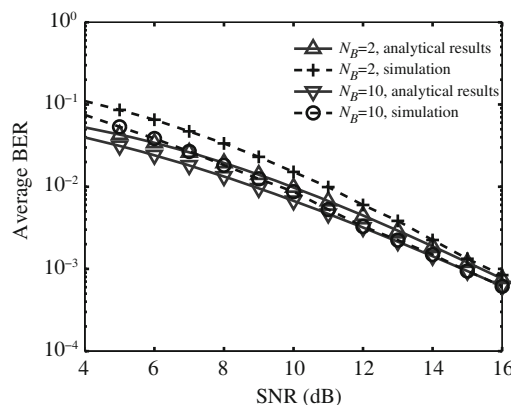
**Figure 3** The overall outage probability over the transmit SNR.



**Figure 4** The outage probability for all links in the system over the transmit SNR.



**Figure 5** The overall outage probability the number of the antennas on the base station.



**Figure 6** The average sum BER over the transmit SNR.

## 5 Conclusion

In this paper, we propose an antenna selection scheme for the massive multi-input multi-output base station in the two-way full duplex relaying system. With the proposed antenna selection scheme, both the outage probability and the bit error rate are derived in the closed form expression, where the probability of the distribution function for the end-to-end SNR is approximated properly. Numerical simulations demonstrate the effectiveness of the derived results. It is discovered that the bottleneck of the whole system comes from the wireless channel between the relay station and the mobile terminal when the number of the antennas on the base station is very large.

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**Conflict of interest** The authors declare that they have no conflict of interest.

## References

- Huang Y M, Yang L X, Bengtsson M, et al. A limited feedback joint precoding for amplify-and-forward relaying. *IEEE Trans Signal Process*, 2010, 58: 1347–1357
- Lin M, An K, Ouyang J, et al. Effect of beamforming on multi-antenna two hop asymmetric fading channels with fixed gain relays. *Prog Electromagn Res*, 2013, 133: 367–390

- 3 Xu W, Dong X D, Huang Y M. Asymptotic achievable rate analysis for selection strategies in amplify-and-forward MIMO two-hop networks with feedback. *IEEE Trans Veh Technol*, 2010, 59: 3662–3668
- 4 Zhao R, Yang L X, Huang Y M. Performance analysis of network coding for multicast relay system over Nakagami-m fading channels. *Sci China Inf Sci*, 2011, 54: 2338–2348
- 5 Huang Y M, He S W, Jin S, et al. Decentralized energy-efficient coordinated beamforming for multicell systems. *IEEE Trans Veh Technol*, 2014, 63: 4302–4314
- 6 Yang A, Xing C W, Fei Z S, et al. Performance analysis for uplink massive MIMO systems with a large and random number of UEs. *Sci China Inf Sci*, 2016, 59: 1–9
- 7 Ni Y, Zhang W C, Chen M. Antenna subset selection in MU large-scale MIMO systems. In: *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC'2013)*, Shanghai, 2013. 1–4
- 8 Ahlswede R, Cai N, Li S Y, et al. Network information flow. *IEEE Trans Inf Theory*, 2000, 46: 1204–1216
- 9 Dai M J, Wang P, Zhang S L, et al. Survey on cooperative strategies for wireless relay channels. *Trans Emerg Telecommun Technol*, 2014, 25: 926–942
- 10 Popovski P, Yomo H. Physical network coding in two-way wireless relay channels. In: *Proceedings of IEEE International Conference on Communications (ICC'2007)*, Glasgow, 2007. 707–712
- 11 Koike-Akino T, Popovski P, Tarokh V. Optimized constellations for two-way wireless relaying with physical network coding. *IEEE J Sel Areas Commun*, 2009, 27: 773–787
- 12 Ji B F, Song K, Huang Y M, et al. A cooperative relay selection for two-way cooperative relay networks in Nakagami channels. *Wirel Pers Commun*, 2013, 71: 2045–2065
- 13 Khina A, Kochman Y, Erez U. Physical-layer MIMO relaying. In: *Proceedings of IEEE International Symposium on Information Theory (ISIT'2011)*, Saint-Petersburg, 2011. 2437–2441
- 14 Zhou X Y, Bai B, Chen W. A low complexity energy efficiency maximization method for multiuser amplify-and-forward MIMO relay systems with a holistic power model. *IEEE Commun Lett*, 2014, 18: 1371–1374
- 15 Dai M J, Wang H, Lin X H, et al. Opportunistic relaying with analogue and digital network coding for two-way parallel relay network. *IET Commun*, 2014, 8: 2200–2206
- 16 Kim J, Kim D. BER analysis of dual-hop amplify-and-forward MIMO relaying with best antenna selection in Rayleigh fading channels. *IEICE Trans Commun*, 2008, 91: 2772–2775
- 17 Peters S, Heath R. Nonregenerative MIMO relaying with optimal transmit antenna selection. *IEEE Signal Process Lett*, 2008, 15: 421–424
- 18 Cao L, Zhang X, Wang Y W, et al. Transmit antenna selection strategy in amplify-and-forward MIMO relaying. In: *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC'2009)*, Budapest, 2009. 1–4
- 19 Yang K, Yang N, Xing C W, et al. Relay antenna selection in MIMO two-way relay networks over Nakagami-m fading channels. *IEEE Trans Veh Technol*, 2013, 63: 2349–2362
- 20 Dai M J, Sung C W, Wang Y. Distributed on-off power control for amplify-and-forward relays with orthogonal space-time block code. *IEEE Trans Wirel Commun*, 2011, 10: 1895–1903
- 21 Zheng G, Krikidis I, Ottersten B. Full-duplex cooperative cognitive radio with transmit imperfections. *IEEE Trans Wirel Commun*, 2013, 12: 2498–2511
- 22 Zhou M X, Cui H Y, Song L Y, et al. Transmit-receive antenna pair selection in full duplex systems. *IEEE Wirel Commun Lett*, 2014, 3: 34–37
- 23 Cheng X L, Yu B, Cheng X, et al. Two-way full-duplex amplify-and-forward relaying. In: *Proceedings of IEEE Military Communications Conference (MILCOM'2013)*, San Diego, 2013. 1–6
- 24 Yu B, Yang L Q, Cheng X, et al. Transmit power optimization for full duplex decode-and-forward relaying. In: *Proceedings of IEEE Global Communications Conference (GLOBECOM'2013)*, Atlanta, 2013. 3347–3352
- 25 Yu B, Yang L Q, Cheng X, et al. Relay location optimization for full-duplex decode-and-forward relaying. In: *Proceedings of IEEE Military Communications Conference (MILCOM'2013)*, San Diego, 2013. 13–18
- 26 Dai M J, Wan S C. Achieving high diversity and multiplexing gains in the asynchronous parallel relay network. *Trans Emerg Telecommun Technol*, 2013, 24: 232–243
- 27 Suraweera H A, Krikidis I, Zheng G, et al. Low-complexity end-to-end performance optimization in MIMO full-duplex relay systems. *IEEE Trans Wirel Commun*, 2014, 13: 913–927
- 28 Zheng G. Joint beamforming optimization and power control for full-duplex MIMO two-way relay channel. *IEEE Trans Signal Process*, 2015, 63: 555–566
- 29 Xia X C, Xu K, Zhang D M, et al. Low-complexity transceiver design and antenna subset selection for cooperative half-and full-duplex relaying systems. In: *Proceedings of IEEE Global Communications Conference (GLOBECOM'2014)*, Austin, 2014. 3314–3319
- 30 Song K, Ji B F, Huang Y M, et al. Performance analysis of antenna selection in two-way relay networks. *IEEE Trans Signal Process*, 2015, 63: 2520–2532
- 31 Gradshteyn I, Ryzhik I, Jeffrey A, et al. *Table of Integrals, Series, and Products*. New York: Academic, 2007. 334–371
- 32 Guo H, Ge J. Performance analysis of two-way opportunistic relaying over Nakagami-m fading channels. *Electron Lett*, 2011, 47: 150–152
- 33 Simon M K, Alouini M S. *Digital Communication over Fading Channels*. New York: Wiley, 2005. 193–218