

# Attribute-based non-interactive key exchange

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## Dear editor,

In this work, we study the notion of attribute-based non-interactive key-exchange (ABNIKE). As a natural extension of the notions of non-interactive key-exchange (NIKE) [1, 2] and identity-based non-interactive key-exchange (IBNIKE) [3, 4], ABNIKE allows users to non-interactively agree on a common shared key. Learning from attribute-based encryption [5], we divide ABNIKE into two forms: key-policy ABNIKE (KP-ABNIKE) and shared-key-policy ABNIKE (SP-ABNIKE). Intuitively, in a KP-ABNIKE scheme, a user who is associated with a policy function  $f$  has a secret key  $sk_f$ . The shared key  $K_x$  will be established according to an attribute set  $x$ . A user can non-interactively generate  $K_x$  if and only if  $x$  satisfies  $f$ , i.e.,  $f(x) = 1$ . On the contrary, in an SP-ABNIKE scheme, a user's secret key is associated with an attribute set  $x$ , while a shared key is associated with a policy function  $f$ . In this work, we first give a formal definition of ABNIKE. We then define the security model for ABNIKE in the dishonest key registration (DKR) setting. Next, by using differing-input obfuscation (diO) [6, 7], we construct an ABNIKE scheme. Finally, we show that the notion of ABNIKE implies IBNIKE and two- or more-party ABNIKE, which have been realized in pre-

vious work.

**Preliminaries.** We now present some definitions that will be used for our construction.

**Differing-input obfuscation.** The definition of diO with auxiliary input follows that of Ananth et al. [6], which is equivalent to that given by Boyle et al. [7]. First, we define the notion of differing-input circuits family.

**Definition 1.** A circuit family  $\mathbb{C}$  with a sampler  $(C_0, C_1, \text{aux}) \leftarrow \text{Samp}(1^\lambda)$  that samples  $C_0, C_1 \in \mathbb{C}$  is said to be a differing-input family, if for all PPT adversaries  $\mathcal{A}$ , we have

$$\Pr[C_0(x) \neq C_1(x) : (C_0, C_1, \text{aux}) \leftarrow \text{Samp}(1^\lambda), x \leftarrow \mathcal{A}(1^\lambda, C_0, C_1, \text{aux})] = \text{negl}(\lambda).$$

We now define the notion of diO for a differing-input circuits family.

**Definition 2.** A uniform PPT machine diO is called a differing-input obfuscator for a differing-input circuits family  $\mathbb{C} = \{\mathbb{C}_\lambda\}$  if it satisfies the following properties:

- **Correctness.** For all security parameters  $\lambda \in \mathbb{N}$ , all  $C \in \mathbb{C}_\lambda$ , and all inputs  $x$ , we have  $\Pr[C'(x) = C(x) : C' \leftarrow \text{diO}(\lambda, C)] = 1$ .
- **Polynomial slowdown.** There exists a universal polynomial  $\text{poly}$  such that for any circuit  $C \in \mathbb{C}_\lambda$  we have  $|C'| \leq \text{poly}(|C|)$ , where  $C' = \text{diO}(C)$ .

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• Differing-inputs. For any (not necessarily uniform) PPT distinguisher  $D$ , all security parameters  $\lambda \in \mathbb{N}$ , and  $(C_0, C_1, \text{aux}) \leftarrow \text{Samp}(1^\lambda)$ , we have

$$\begin{aligned} & |\Pr[D(\text{diO}(\lambda, C_0), \text{aux}) = 1] \\ & - \Pr[D(\text{diO}(\lambda, C_1), \text{aux}) = 1]| = \text{negl}(\lambda). \end{aligned}$$

**Punctured pseudorandom functions.** In punctured PRF [8], one can derive a punctured key  $K_S$  with respect to a subset  $S \subseteq \mathbb{D}$  from the secret key  $K$ . This punctured key enables the evaluation of the PRF in the subset  $\mathbb{D} \setminus S$  of the domain and nowhere else.

**Definition 3.** A punctured PRF consists of a triple of algorithms,  $\mathfrak{F} = (\text{PRF.Key}, \text{PRF.Pun}, F)$ , and a pair of computable functions,  $n(\cdot)$  and  $m(\cdot)$ , satisfying the following conditions:

• **Functionality preserved under puncturing:** for any PPT adversary  $\mathcal{A}$  that outputs a set  $S \subseteq \mathbb{D}$  where  $\mathbb{D} = \{0, 1\}^{n(\lambda)}$  is the domain of the punctured PRF, if  $x \in \mathbb{D} \setminus S$  then

$$\begin{aligned} & \Pr[F(K, x) = F(K_S, x) : K \leftarrow \text{PRF.Key}(1^\lambda), \\ & K_S \leftarrow \text{PRF.Pun}(K, S)] = 1. \end{aligned}$$

• **Pseudorandom at punctured points:** for any PPT adversary  $(\mathcal{A}_1, \mathcal{A}_2)$  such that  $\mathcal{A}_1(1^\lambda)$  outputs a set  $S \subseteq \mathbb{D}$  and a state  $\tau$ , consider an experiment where  $K \leftarrow \text{PRF.Key}(1^\lambda)$  and  $K_S \leftarrow \text{PRF.Pun}(K, S)$ :

$$\begin{aligned} & \Pr[\mathcal{A}_2(\tau, K_S, S, F(K, S)) = 1] \\ & - \Pr[\mathcal{A}_2(\tau, K_S, S, U_{m(\lambda) \cdot |S|}) = 1] = \text{negl}(\lambda), \end{aligned}$$

where  $S = \{x_1, \dots, x_k\}$ ,  $F(K, S)$  is the concatenation of the elements of  $S$  in lexicographic order, i.e.,  $F(K, x_1) \parallel \dots \parallel F(K, x_k)$ , and  $U_\ell$  denotes the uniform distribution over  $\ell$  bits.

**Digital signatures.** A signature scheme  $\mathfrak{S}$  consists of the following three PPT algorithms:  $\text{Sig.Key}$  takes as input a security parameter  $\lambda$  and outputs a signing-verification key pair  $(\text{sk}, \text{vk})$ ;  $\text{Sig.Sign}$  takes as inputs the signing key  $\text{sk}$  and a message  $m$ , and outputs a signature  $\sigma$ ;  $\text{Sig.Vrfy}$  takes as inputs the verification key  $\text{vk}$  and a purported signature  $\sigma$  on a message  $m$ , and outputs 1 if it is valid or 0 otherwise.

For correctness, it is required that for any  $(\text{sk}, \text{vk}) \leftarrow \text{Sig.Key}(1^\lambda)$  and for any message  $m$ ,  $\Pr[\text{Sig.Vrfy}(\text{vk}, m, \text{Sig.Sign}(\text{sk}, m)) = 1] = 1$ .

**Definition 4.** We say that a signature scheme  $\mathfrak{S} = (\text{Sig.Key}, \text{Sig.Sign}, \text{Sig.Vrfy})$  is EU-CMA secure [9] if, for any PPT adversary  $\mathcal{A}$  with oracle access to  $\text{Sig.Sign}$ , the probability that, on input of a uniformly chosen verification key  $\text{vk}$ ,  $\mathcal{A}$  outputs

a pair  $(m^*, \sigma^*)$  such that  $\text{Sig.Vrfy}(\text{vk}, m^*, \sigma^*) = 1$  where  $m^*$  was not queried to  $\text{Sig.Sign}$  oracle, is negligible, where the probability is over  $\text{vk}$  and the randomness of the  $\text{Sig.Sign}$  oracle.

**Definitions.** Let  $\mathbb{A}$  be the universe of possible attributes. A claimed policy over  $\mathbb{A}$  is a Boolean function  $f \in \mathbb{F}$ , where  $\mathbb{F}$  is the space of all possible policy functions. We say that an attribute set  $x \subseteq \mathbb{A}$  satisfies a policy function  $f$  if  $f(x) = 1$ .

Inspired by the classification of attribute-based encryption [5], we divide ABNIKE into two flavors, KP-ABNIKE and SP-ABNIKE. For simplicity, we give the definition of KP-ABNIKE. The notion of SP-ABNIKE can be easily obtained by interchanging the function  $f$  and attribute set  $x$ .

• **AB.Setup( $1^\lambda$ ):** The setup algorithm takes as input the security parameter  $\lambda$ . It outputs the public parameters  $\text{pp}$  and the master key  $\text{msk}$ .

• **AB.KeyGen( $\text{msk}, f$ ):** The key generation algorithm takes as input the master key  $\text{msk}$  and a policy function  $f$ . It outputs a secret key  $\text{sk}_f$ .

• **AB.SharedKey( $\text{pp}, \text{sk}_f, f, x$ ):** Each user can non-interactively generate a common shared key  $K_x \in \text{SHK}$  only if  $f(x) = 1$ , with respect to an attribute set  $x$  using the public parameters  $\text{pp}$ , his secret key  $\text{sk}_f$  and function  $f$ , where  $\text{SHK}$  is the share key space.

For correctness, it is required that for all  $\lambda, f_0, f_1, x$ , and all  $(\text{pp}, \text{msk}) \leftarrow \text{AB.Setup}(1^\lambda)$ ,  $\text{sk}_{f_0} \leftarrow \text{AB.KeyGen}(\text{msk}, f_0)$ , and  $\text{sk}_{f_1} \leftarrow \text{AB.KeyGen}(\text{msk}, f_1)$ , if  $f_0(x) = f_1(x) = 1$ , then we have

$$\begin{aligned} & \text{AB.SharedKey}(\text{pp}, \text{sk}_{f_0}, f_0, x) = \\ & \text{AB.SharedKey}(\text{pp}, \text{sk}_{f_1}, f_1, x) = K_x. \end{aligned}$$

**Construction.** We now construct an ABNIKE scheme. Our ABNIKE scheme follows the punctured program technique (with  $\text{diO}$  instead of  $\text{iO}$ ) devised by Sahai and Waters [8]. To generate the secret key for  $f$ , we choose a signature scheme  $\mathfrak{S} = (\text{Sig.Key}, \text{Sig.Sign}, \text{Sig.Vrfy})$  to sign on  $f$  and set the resulting signature as the secret key. In addition, we also choose a punctured PRF  $\mathfrak{F} = (\text{PRF.Key}, \text{PRF.Pun}, F)$ . For simplicity, we assume that the  $\text{Sig.Sign}$  and  $F$  algorithms will take inputs with appropriate length.

The following is a KP-ABNIKE scheme. By using universal circuits, we can easily construct an SP-ABNIKE scheme.

• **AB.Setup( $1^\lambda$ ):** The setup algorithm takes as input a security parameter  $\lambda$  and does the following. It first runs the  $\text{Sig.Key}(1^\lambda)$  and  $\text{PRF.Key}(1^\lambda)$  algorithms to produce a signing-verification key pair  $(\text{sk}, \text{vk})$  and a PRF key  $K$ , respectively. It then builds an obfuscated program

$\text{diO}(\mathcal{P})$ , where the program  $\mathcal{P}$  contains two constant values,  $\text{vk}$  and  $K$ . Then, any user can run this program on inputs an attribute set  $x$ , his policy function  $f$  and secret key  $\text{sk}_f$ . Formally, the program  $\mathcal{P}$  is defined below.

(1) Given inputs  $(x, f, \text{sk}_f)$ , the program first checks that  $f(x) \stackrel{?}{=} 1$  and  $\text{Sig.Vrfy}(\text{vk}, f, \text{sk}_f) \stackrel{?}{=} 1$  holds or not.

(2) If any checks fail, then it outputs  $\perp$ ; else, it outputs  $K_x \leftarrow F(K, x)$ .

The public parameters  $\text{pp}$  consist of the descriptions of the attribute universe  $\mathbb{A}$ , the space of policy functions  $\mathbb{F}$ , the shared key space  $\text{SHK}$ , and the obfuscated program  $\text{diO}(\mathcal{P})$ . The master key is  $\text{msk} = \text{sk}$ .

- $\text{AB.KeyGen}(\text{msk}, f)$ : To generate the secret key for a function  $f \in \mathbb{F}$ , the key generation algorithm runs the signing algorithm of the signature scheme  $\text{sk}_f \leftarrow \text{Sig.Sign}(\text{sk}, f)$  and outputs  $\text{sk}_f$ .

- $\text{AB.SharedKey}(\text{pp}, \text{sk}_f, f, x)$ : Each user runs the obfuscated program  $\text{diO}(\mathcal{P})$  on the inputs  $(x, f, \text{sk}_f)$  and outputs the result.

*Conclusion.* In this work, we define the notion of attribute-based non-interactive key-exchange. In addition, by using differing-input obfuscation, we give a concrete construction of such cryptographic primitive. Due to space limitations, the security of our construction and further analysis are available in the supporting information.

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**Supporting information** The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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