## MOO Paper

# Regional Path Moving Horizon Tracking Controller Design for Autonomous Ground Vehicles

**Hongyan GUO[1,2], Feng LIU[2] , Ru YU[2], Zhenping SUN[3], Hong CHEN[1,2]***

1. State Key Laboratory of Automotive Simulation and Control, Jilin University (Campus NanLing), Changchun 130025, PR China;

2. College of Communication Engineering, Jilin University (Campus NanLing),

Changchun 130025, PR China;

3. School of Mechatronic Engineering and Automation, National University of Defense Technology, Changsha 410073, PR China

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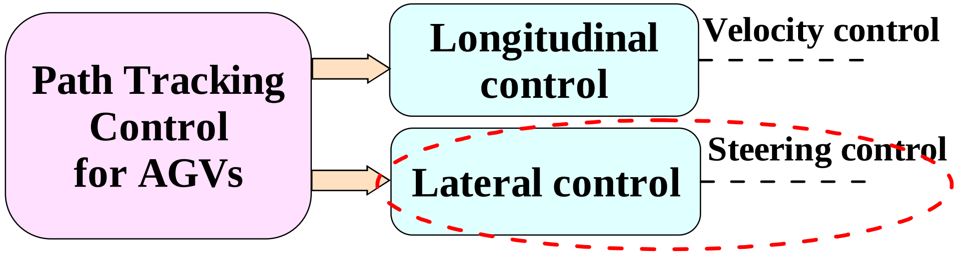
# Outline

- Introduction
- Regional path tracking problem
- Regional path tracking control
- Implementation and experiments
- Conclusion

# Introduction

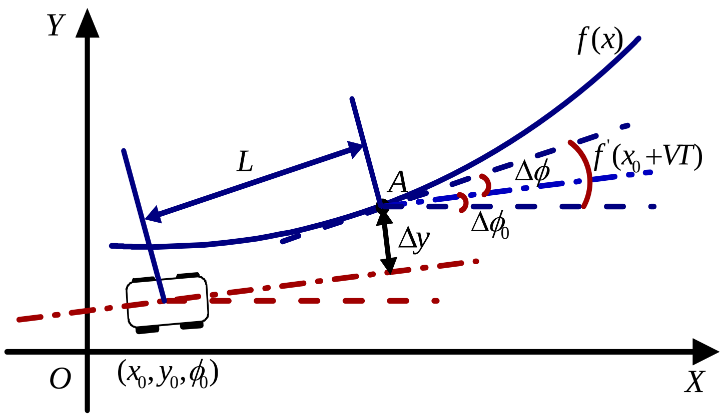
## ➢ Path tracking control for AGVs




**Steering control:**

- tracks the desired lateral path
- related to lateral stability

## ➢ Problems in the existing steering control

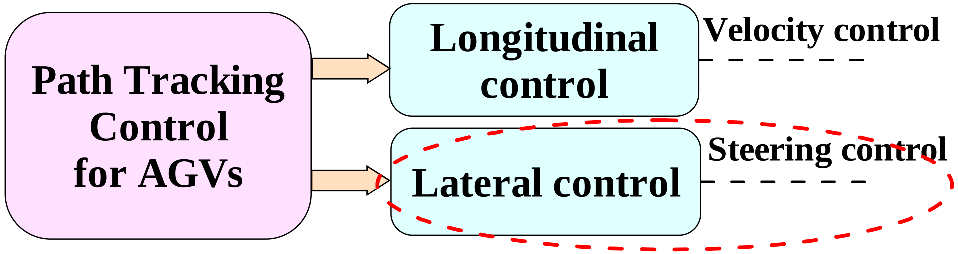



- road width is ignored
- vehicle shape is ignored

crashing road boundary

# Introduction

## ➢ Path tracking control for AGVs




**Steering control:**

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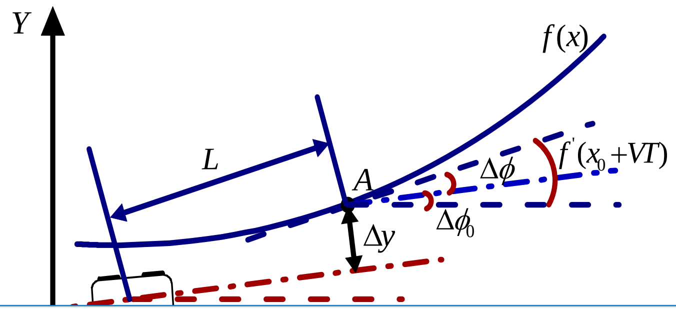



- ➢ road width is ignored
- ➢ vehicle shape is ignored

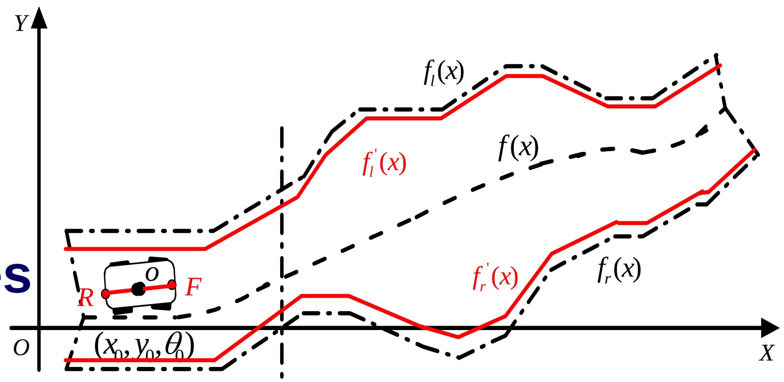
### Control objective:

make control decision repeatedly for AGVs according to the previewed traffic environment and road information, considering the road width and vehicle shape, and keeping AGVs run in the feasible region

# Regional path tracking problem

## Regional path tracking problem description

- road boundaries are $f_l(x)$ and $f_r(x)$
- vehicle is considered as a rectangle with width $w$ and length $l$
- keeps vehicle run in the road boundaries
- tracks the centerline $f(x)$



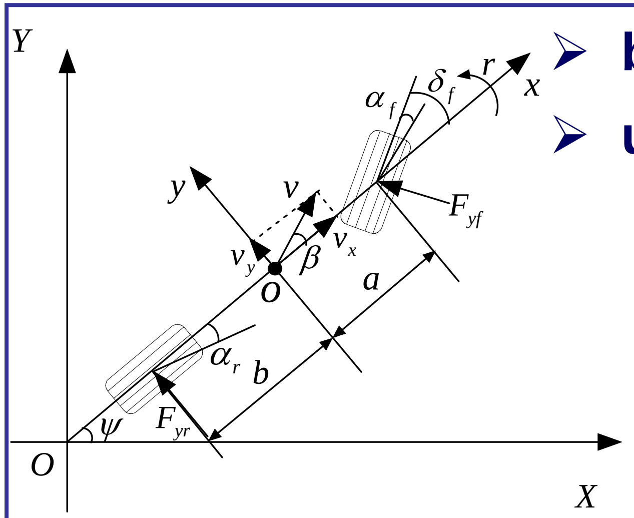

road boundary shrinks $\frac{w}{2}$ ➡ vehicle equals to a bar with length $l$

## Control objective of regional path tracking

- keep *F* and *R* in the determined region
- control AGVs travel along the road centerline
- make driving route as short as possible
- consider mechanical characteristics of actuator

# Regional path tracking problem

## ➢ Vehicle model

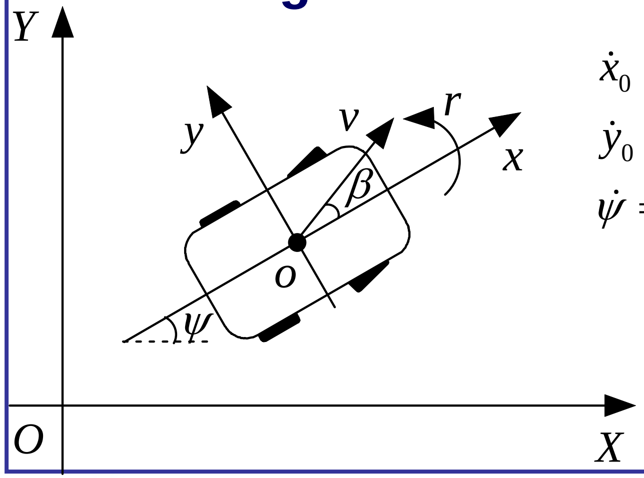



- based on the small angle assumption of $\beta$ and $\psi$
- using linear tire model $F_{yf} = C_f \alpha_f, \; F_{yr} = C_r \alpha_r$

$$\dot{\beta} = \frac{C_f + C_r}{mv}\beta + \left(\frac{aC_f - bC_r}{mv^2} - 1\right) r - \frac{C_f}{mv}\delta_f$$

$$\dot{r} = \frac{aC_f - bC_r}{I_z}\beta + \frac{a^2 C_f + b^2 C_r}{I_z v} r - \frac{aC_f}{I_z}\delta_f$$

**considering vehicle as a rigid body**




$$\dot{x}_0 = v$$

$$\dot{y}_0 = v(\beta + \psi)$$

$$\dot{\psi} = r$$

**vehicle could be described as**

$$\dot{y}_o = v(\psi + \beta)$$

$$\dot{\psi} = r$$

$$\dot{\beta} = \frac{C_f + C_r}{mv}\beta + \left(\frac{aC_f - bC_r}{mv^2} - 1\right) r - \frac{C_f}{mv}\delta_f$$

$$\dot{r} = \frac{aC_f - bC_r}{I_z}\beta + \frac{a^2 C_f + b^2 C_r}{I_z v} r - \frac{2aC_f}{I_z}\delta_f$$

$$y_{out} = y_o$$

# Regional path tracking problem

## ➢ Vehicle model

**longitudinal velocity is assumed as constant**

$m$ **and** $I_z$ **varied so slowly that assume as constant**

**CoG is considered as constant** ➡ $a$ **and** $b$ **is invariant**

$C_f$ **and** $C_r$ **varied so slowly that assume as constant**

**The vehicle model could be treated as linear-time-invariant system**

$$\dot{x} = Ax + Bu$$

$$y_{out} = Cx$$

$$A = \begin{bmatrix} 0 & v & v & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{C_f + C_r}{mv} & \frac{aC_f - bC_r}{mv^2} - 1 \\ 0 & 0 & \frac{aC_f - bC_r}{I_z} & \frac{a^2 C_f + b^2 C_r}{I_z v} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ -\frac{C_f}{mv} \\ -\frac{aC_f}{I_z} \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

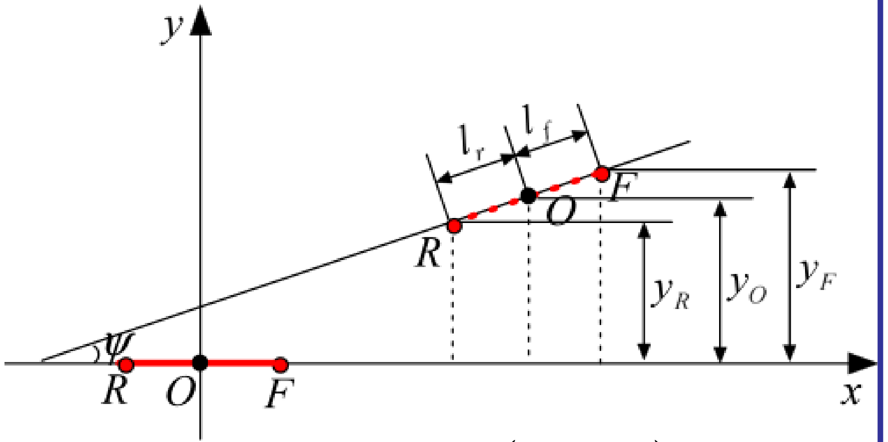
$$u = \delta_f$$

$$y_{out} = y$$

$$x = [y \quad \phi \quad \beta \quad r]$$

# Regional path tracking problem

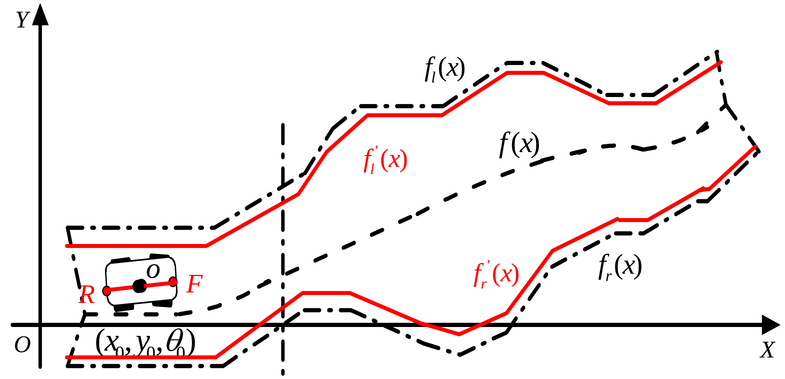
**According to the relationship between $y_o$ , $y_F$ and $y_R$**




$$y_F = y_o + l_f\left(\psi + \beta\right)$$

$$y_R = y_o - l_r\left(\psi + \beta\right)$$

**lateral positions of front and rear end keeps in the region**




$$f_l^{'}(x) \le y_F \le f_r^{'}(x)$$

$$f_l^{'}(x) \le y_R \le f_r^{'}(x)$$

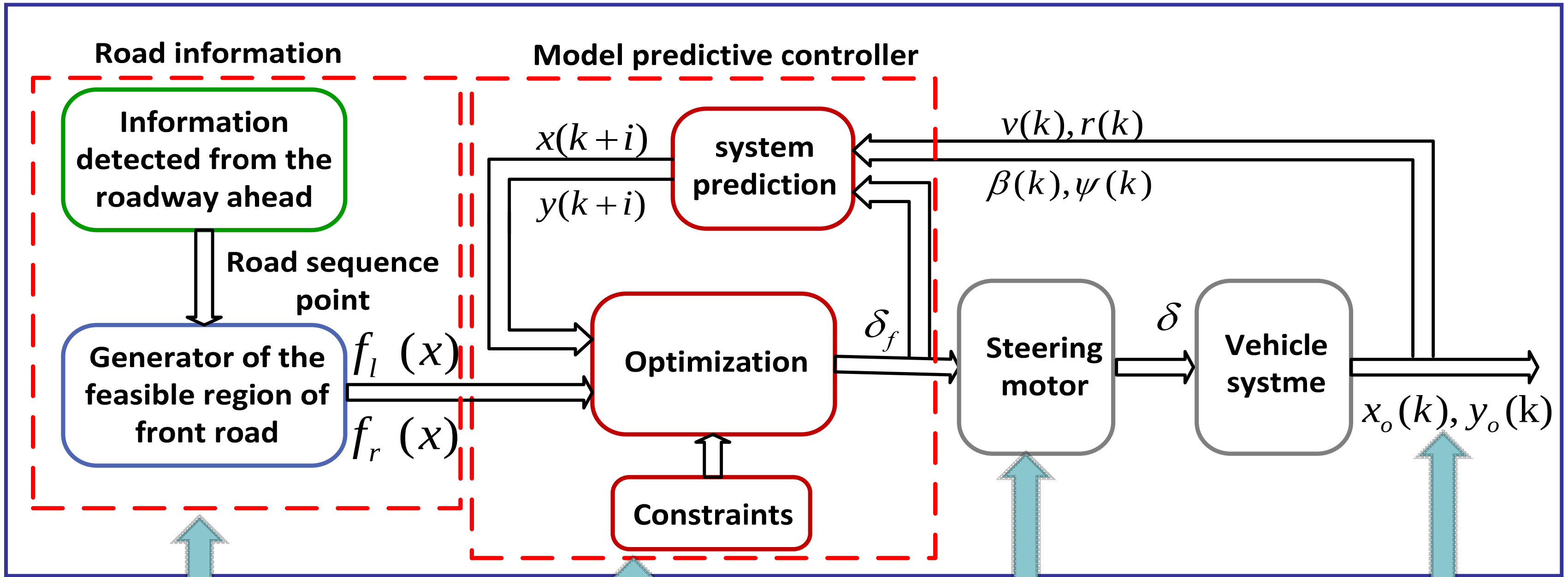
**The front and rear end in the region could be transformed into the constraints of lateral position**

$$f_r^{'}(x) - l_f(\beta + \psi) \le y \le f_l^{'}(x) - l_f(\beta + \psi)$$

$$f_r^{'}(x) + l_r(\beta + \psi) \le y \le f_l^{'}(x) + l_r(\beta + \psi)$$

# Regional path tracking control

## ➢ Regional path tracking control scheme




Road information detected from the roadway ahead is a series of position points

Model predictive controller includes system prediction, constraints description and optimization

Steering motor is actuator, it's control is not discussed

Measured states are

$v, r, \beta, \psi$

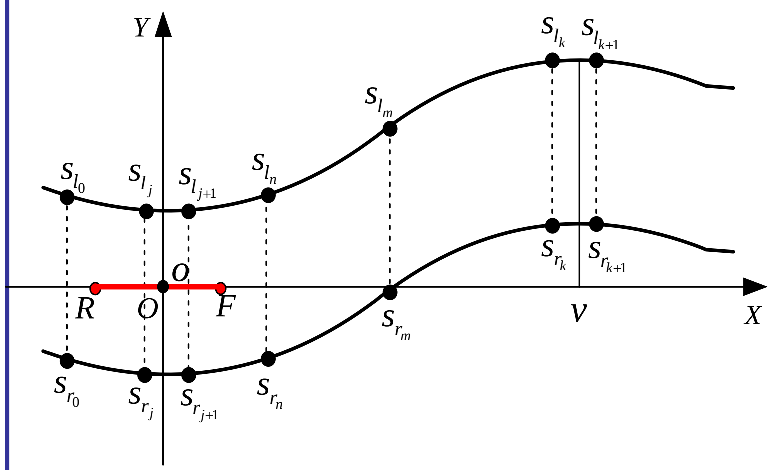
# Regional path tracking control

## Road information processing

$$
\begin{cases}
f_r(x) = \sum \prod\limits_{i \neq p} \dfrac{(x - x_r(i))}{(x_r(p) - x_r(i))} y_r(p) \\
f_l(x) = \sum \prod\limits_{i \neq p} \dfrac{(x - x_l(i))}{(x_l(p) - x_l(i))} y_l(p)
\end{cases}
\quad
\begin{aligned}
p &= j, n, m, k; \\
i &= j, n, m, k.
\end{aligned}
$$

- road point sequence $(x_r, y_r, x_l, y_l)$ is obtained from perception system
- $j, n, m, k$ : four sets of interpolation points position chosen from given road points
- four sets of interpolation points equally spaced
- interpolation points are selected based on quadratic search algorithm

$$
n = \left\lfloor \frac{k-j}{3} \right\rfloor + j, \qquad m = \left\lfloor \frac{k-j}{3} \right\rfloor + n
$$




- ✓ first time: find start point $(s_{r_j}, s_{l_j})$

$$
\begin{cases}
x_r(j) \cdot x_r(j+1) \le 0 \\
x_l(j) \cdot x_l(j+1) \le 0
\end{cases}
$$

- ✓ second time: find final point $(s_{r_k}, s_{l_k})$

$$
\begin{cases}
(x_r(k) - v) \cdot (x_r(k+1) - v) \le 0 \\
(x_l(k) - v) \cdot (x_l(k+1) - v) \le 0
\end{cases}
$$

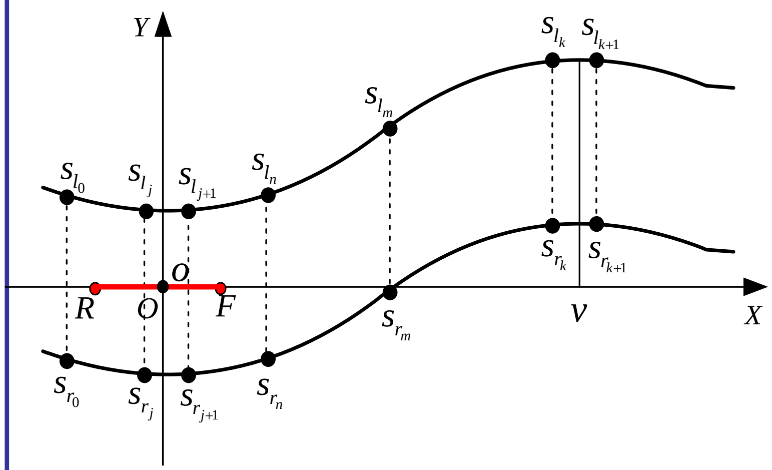
# Regional path tracking control

## ➢ Road information processing

$$\begin{cases} f_r(x) = \sum \prod\limits_{i \neq p} \dfrac{(x - x_r(i))}{(x_r(p) - x_r(i))} y_r(p) \\ f_l(x) = \sum \prod\limits_{i \neq p} \dfrac{(x - x_l(i))}{(x_l(p) - x_l(i))} y_l(p) \end{cases} \quad \begin{matrix} p = j, n, m, k; \\ i = j, n, m, k. \end{matrix}$$

- road point sequence $(x_r, y_r, x_l, y_l)$ is obtained from perception system
- $j, n, m, k$ : four sets of interpolation points position chosen from given road points
- four sets of interpolation points equally spaced

$\lfloor \; 3 \; \rfloor \qquad \lfloor \; 3 \; \rfloor$




✓ first time: find start point $(s_{r_j}, s_{l_j})$

$$\begin{cases} x_r(j) \cdot x_r(j+1) \leq 0 \\ \ldots \\ (x_l(k) - v) \cdot (x_l(k+1) - v) \leq 0 \end{cases}$$

**When the road boundary is determined, the road centerline could be obtained**

$$f(x) = \frac{1}{2}\left( f_l(x) + f_r(x) \right)$$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# Regional path tracking control

## Moving horizon controller design

### Vehicle system dynamic prediction

By discretizing the continuous-time model, the system is described as

$$x(k+1) = A_c x(k) + B_c u(k)$$
$$y_c(k) = C_c x(k)$$

Define road centerline as reference input

$$R(k) = \left[ f(k) \quad f(k+1) \quad \cdots \quad f(k+P-1) \right]$$

Define the predicted input and output sequence

$$U(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+N-1) \end{bmatrix}$$

$$Y(k+1|k) = \begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+P) \end{bmatrix}$$

### Control objective description

objective 1 : runs along the centerline

$$J_1 = \left\| Y(k+1|k) - R(k) \right\|^2$$

objective 2 : minimize driving route

$$J_2 = \sum_{i=1}^{P} \left( \left\| \Delta x_d(k+i) \right\|^2 + \left\| \Delta y_d(k+i) \right\|^2 \right)$$

objective 3 : control action is limited

$$J_3 = \left\| U(k) \right\|^2$$

objective 4 : vehicle runs in the region

$$\begin{cases} f_r^{'}(k+i) - l_f(\psi(k+i)+\beta(k+i)) \le y_o(k+i) \le f_l^{'}(k+i) - l_f(\psi(k+i)+\beta(k+i)) \\ f_r^{'}(k+i) + l_r(\psi(k+i)+\beta(k+i)) \le y_o(k+i) \le f_l^{'}(k+i) + l_r(\psi(k+i)+\beta(k+i)) \end{cases}, i = 1, \cdots, P$$

objective 5 : steering actuator saturation

$$\begin{cases} \left| \delta_f(k+i) \right| \le \delta_{fsat} \\ \left| \Delta\delta_f(k+i) \right| \le \Delta\dot{\delta}_{fsat} \cdot T_s \end{cases}, i = 0, \cdots, N-1$$

# Regional path tracking control

## ➢ Moving horizon controller design

### ➢ Vehicle system dynamic prediction

**By discretizing the continuous-time model, the system is described as**

$$x(k+1)=A_c x(k)+B_c u(k)$$

$$y_c(k)=C_c x(k)$$

**Define road centerline as reference input**

$$R(k)=\left[f(k) \quad f(k+1) \quad \cdots \quad f(k+P-1)\right]$$

**Define the predicted input and output sequence**

$$Y(k+1|k)=\begin{bmatrix} \vdots \\ y(k+2) \\ \vdots \\ y(k+P)\end{bmatrix}$$

### ➢ Control objective description

objective 1 : runs along the centerline

$$J_1=\left\|Y(k+1|k)-R(k)\right\|^2$$

objective 2 : minimize driving route

$$J_2=\sum_{i=1}^{P}\left(\left\|\Delta x_d(k+i)\right\|^2+\left\|\Delta y_d(k+i)\right\|^2\right)$$

objective 3 : control action is limited

$$J_3=\left\|U(k)\right\|^2$$

**Minimize these three objectives simultaneously is contradictive, weighting factors are introduced. The multi-objective function could be obtained as**

$$J=\left\|\Gamma_y(Y(k+1|k)-R(k))\right\|^2+\left\|\Gamma_u U(k)\right\|^2+\sum_{i=1}^{P}\Gamma_{d,i}\left(\left\|\Delta x_d(k+i)\right\|^2+\left\|\Delta y_d(k+i)\right\|^2\right)$$

$$\begin{cases}\left|\delta_f(k+i)\right|\le\delta_{fsat} \\ \left|\Delta\delta_f(k+i)\right|\le\Delta\dot{\delta}_{fsat}\cdot T_s\end{cases}, i=0,\cdots,N-1$$

# Regional path tracking control

## ➢ Optimization problem description

$$\min_{U(k)} \quad J = \left\|\Gamma_y (Y(k+1|k) - R(k))\right\|^2 + \left\|\Gamma_u U(k)\right\|^2 + \sum_{i=1}^{P} \Gamma_{d,i} \left( \left\|\Delta x_d (k+i)\right\|^2 + \left\|\Delta y_d (k+i)\right\|^2 \right)$$

**Subject to:**

$$x(k+i+1) = A_c x(k+i) + B_c \delta_f (k+i) + B_{dc} d(k+i)$$

$$y_o = C_c x(k+i)$$

$$f_r^{'}(k+i) - l_f(\psi(k+i) + \beta(k+i)) \le y_o(k+i) \le f_l^{'}(k+i) - l_f(\psi(k+i) + \beta(k+i))$$

$$f_r^{'}(k+i) + l_r(\psi(k+i) + \beta(k+i)) \le y_o(k+i) \le f_l^{'}(k+i) + l_r(\psi(k+i) + \beta(k+i))$$

$$\left|\delta_f (k+i)\right| \le \delta_{fsat}$$

$$\left|\Delta\delta_f (k+i)\right| \le \Delta\dot{\delta}_{fsat} \cdot T_s$$

**The related matrices and variables could be computed as**

$$A_c = e^{AT_s}, \quad B_c = \int_0^{T_s} e^{A\tau} d\tau \cdot B, \quad B_{dc} = B_d \quad C_c = C$$

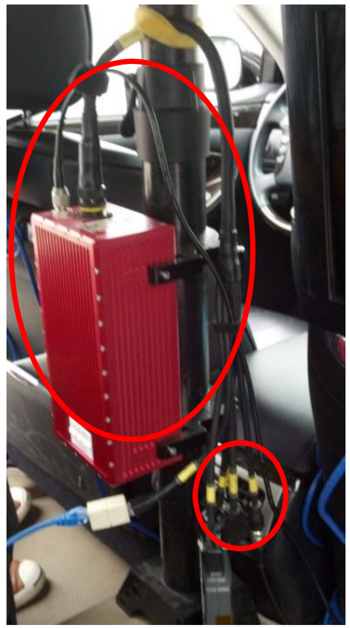
$$\Delta x_d (k+i) = v(k) \cdot T_s, \quad \Delta y_d (k+i) = y_o (k+i) - y_o (k+i)$$

$$\Delta\delta_f (k+i) = \delta_f (k+i) - \delta_f (k+i-1)$$

**Only the first element $U(k)$ is applied to the vehicle**

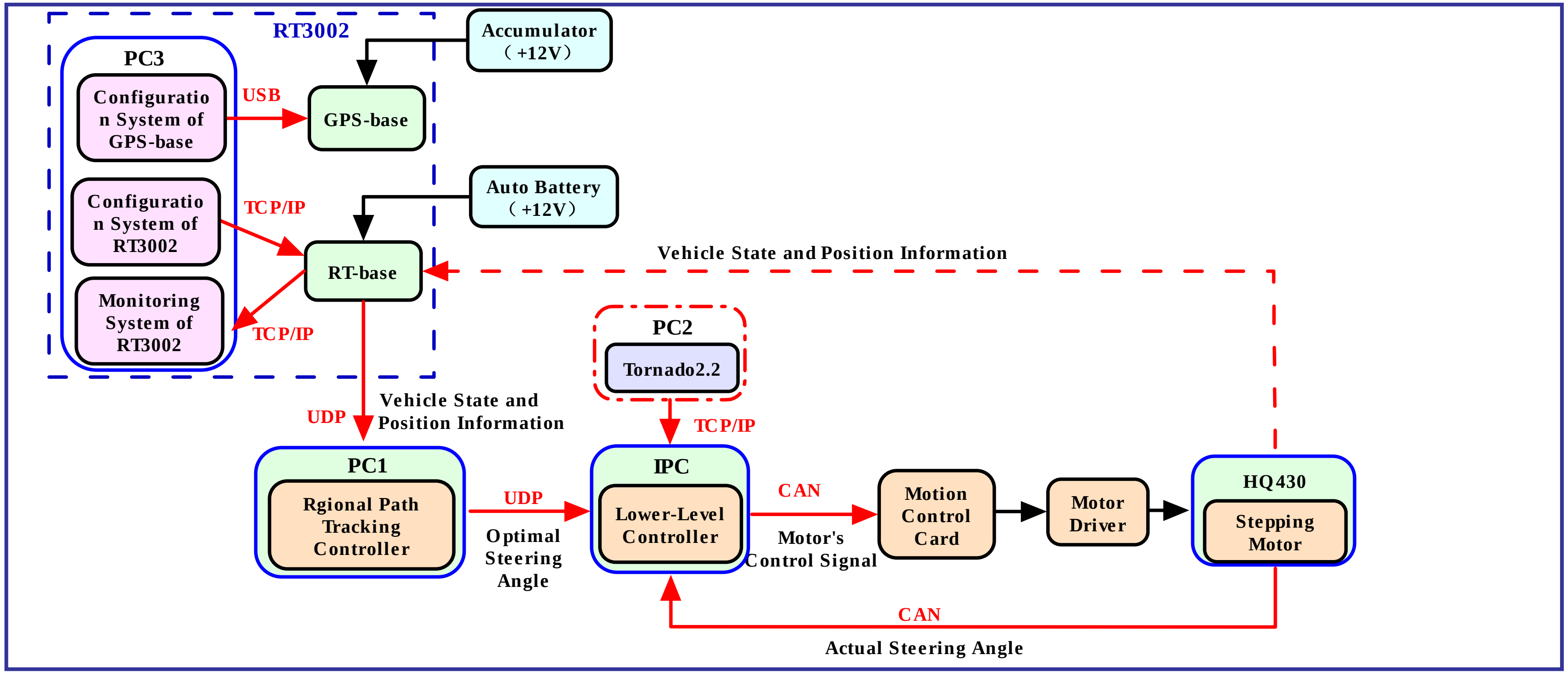
# Implementation and experiments

- **Implementation**
  - **MPC algorithm is implemented based on C language**
  - **differential evolution is employed to solve the optimization**
- **Experiment setup**
  - **vehicle： Hongqi vehicle HQ430**
  - **sensor： OTXS RT3002 and 2 cameras**
  - **experimental system: VxWorks and Windows**
  - **tools: Visual Studio 2010 and Tornado2.2**
  - **computers: 2 Thinkpad T420**

# Implementation and experiments

## ➢ communication system used in experiments




RT3002 ➡ regional path tracking controller : UDP

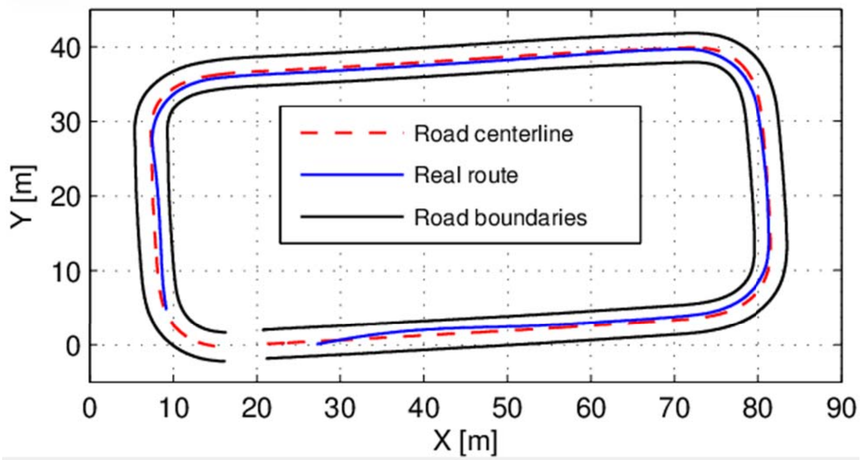
Regional path tracking controller ➡ driving control system: UDP

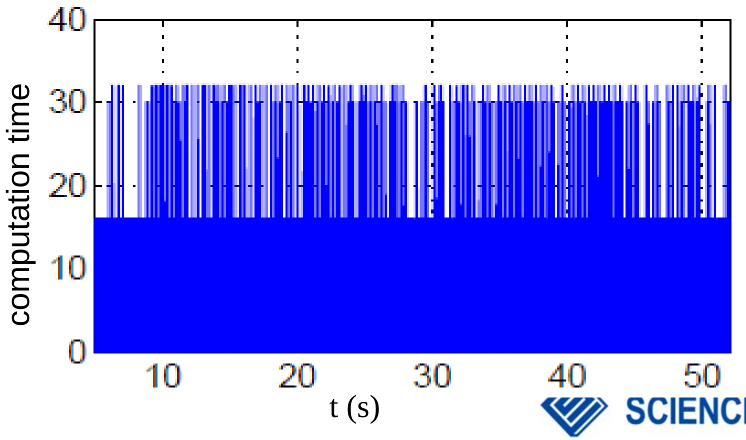
Driving control system ➡ steering motor: CAN

Hongqi AGV ➡ driving control system and RT3002: CAN

# Implementation and experiments

➤ **experiments result**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# Conclusions

- A novel description of AGVs' path tracking issue is proposed
- Regional path moving horizon control is presented
- Road perception information is processed
- Experiments are carried out based on Hongqi AGV HQ430
- Effectiveness of the proposed method is verified

# Thank you