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Model-based diagnosis of incomplete discrete-event system with rough set theory

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Abstract Fault diagnosis of discrete-event system (DES) is important in the preventing of harmful events in the system. In an ideal situation, the system to be diagnosed is assumed to be complete; however, this assumption is rather restrictive. In this paper, a novel approach, which uses rough set theory as a knowledge extraction tool to deal with diagnosis problems of an incomplete model, is investigated. DESs are presented as information tables and decision tables. Based on the incomplete model and observations, an algorithm called Optimizing Incomplete Model is proposed in this paper in order to obtain the repaired model. Furthermore, a necessary and sufficient condition for a system to be diagnosable is given. In ensuring the diagnosability of a system, we also propose an algorithm to minimize the observable events and reduce the cost of sensor selection.

Keywords model-based diagnosis, diagnosability, discrete-event system, finite state machine, rough set theory

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1 Introduction

In recent decades, fault diagnosis has received considerable attention because it ensures the safety of a system. Fault diagnosis of dynamic systems is crucial to their efficient operation. Continuous dynamic systems can be considered as discrete-event systems (DESs) with a higher level of abstraction. The main approach to solve diagnosis problems in DES is by employing model-based diagnosis (MBD). Many modeling formalisms have been proposed to model DES. The most frequently used modeling formalism is finite state machines (FSMs). A seminal work of fault diagnosis using FSM in DESs was proposed in the 1990s [1, 2]. In recent years, researchers focused mainly on relevant yet different aspects. Some researchers have studied algorithms to analyze different kinds of DESs, including stochastic DESs [3] and fuzzy DESs [4–6]. In [7, 8], researchers used different algorithms to verify the diagnosability of DESs. Another work [9] addressed how sensors are selected. To reduce the cost of sensor selection, the problem of minimizing observable events has been presented in [10]. The papers mentioned earlier achieved excellent results. The algorithms presented above assume that a system to be diagnosed is complete. However,

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an absolute complete model, including all behaviors and states of the system, does not exist because the restrictive assumption is unpractical. Thus, studying the incomplete model is necessary.

Rough set theory (RST) was first introduced by Zdzislaw Pawlak in 1982 [11]. RST can analyze and handle uncertain, inconsistent and incomplete problems; hence it has been widely used in many fields of artificial intelligence, such as machine learning, data mining, and so on. Fault diagnosis with RST is a novel approach developed in recent years. In this paper, employing MBD with RST is proposed. The information table and decision table are used to describe the incomplete model and the observations monitored by the sensors. Then, based on these observations, we propose an algorithm called Optimizing Incomplete Model (OPTINM) to obtain a repaired model. Employing the repaired model helps guarantee the diagnosability of a system. Moreover, an algorithm called Minimizing Observable Events (MINOE) is presented to minimize observable events in ensuring the diagnosability of the system.

The contributions of this paper are three-fold. First, we present a methodology change to MBD that uses a combination of RST. An algorithm for obtaining the repaired model by optimizing the incomplete model is presented. Second, we describe a necessary and sufficient condition for the diagnosability of DES based on RST. Third, we propose an algorithm that minimizes observable events to fall in the category of sensor selection problems.

2 Preliminaries

In this section, some definitions and frequently used terms in RST and diagnosis are presented.

2.1 Rough set theory

In the succeeding section, we review the concepts of rough sets, information systems, decision systems, indiscernibility, and reduct [10].

Definition 1 (Information system). An information system (IS) is a pair (U, A), where U refers to a non-empty finite set of objects and A is a non-empty finite set of attributes.

Each attribute $a \in A$ is associated to a function $f_a : U \to V_a$, where V_a is the set of values of a, which is called the domain of a. Objects in U are described by the values of attributes in A.

Decision system, which is an extended version of information system, is defined as follows.

Definition 2 (Decision system). A decision system (DS) is a pair $(U, C \cup \{d\})$, where $d \notin C$ is called the decision attribute.

The elements in C are called condition attributes. A decision attribute can be interpreted as a classification of objects in the domain given by an expert. Given a decision table, the value of the decision attribute determines a partition on U. Note that the condition attributes in DS are equivalent to attributes in IS.

Definition 3 (Indiscernibility). Let $\text{IS} = (U, A)$ be an information system, then for any $B \subseteq A$, there is an associated equivalence relation

$$\text{IND}_{\text{IS}}(B) = \{(x, x') \in U^2 \mid \forall a \in B, a(x) = a(x')\}, \tag{1}$$

where $\text{IND}_{\text{IS}}(B)$ is called the B-indiscernibility relation.

When $(x, x') \in \text{IND}_{\text{IS}}(B)$, objects x and x' are indiscernible from each other because of the attributes of B. Generally, objects with the same information are indiscernible. The equivalence classes of the B-indiscernibility relation are denoted by $[x]_B$.

Given $S = (U, A)$, let $B \subseteq A$ be a set of attributes, and $X \subseteq U$ be a set of objects. We can approximate X using only the information contained in B by constructing the *B-lower* and *B-upper* approximations of X, denoted $\overline{B}X$ and $\underline{B}X$, respectively, where

$$\overline{B}X = \{x | [x]_B \subseteq X\}, \tag{2}$$

$$\underline{B}X = \{x | [x]_B \cap X \neq \emptyset\}. \tag{3}$$

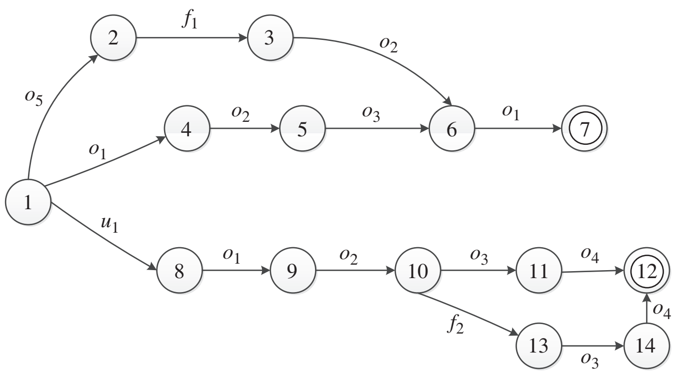

Figure 1 An FSM G.

Suppose C is the set of condition attributes in a DS, and D is the decision attribute of DS. Let $c \in C$, where attribute c is *dispensable* if $\mathrm{POS}_C(D) = \mathrm{POS}_{(C-\{c\})}(D)$, otherwise attribute c is indispensable. $\mathrm{POS}_C(D)$ is the *C-positive* region of D:

$$\mathrm{POS}_C(D) = \bigcup_{X \in U/D} \underline{C}X. \tag{4}$$

$T = (U, R, D)$ is *independent* if all $c \in C$ are indispensable in T, where U is the objects, R is a set of condition attributes, and D is the decision attribute.

Definition 4 (Reduct)**.** The set of condition attributes $R \subseteq C$ is called a *reduct* of C if $S = (U, R, D)$ is independent and $\mathrm{POS}_R(D) = \mathrm{POS}_C(D)$.

The reducts of a DS have two characteristics:

(1) Reducts only keep condition attributes that preserve the indiscernibility relation and, consequently, set an approximation.

(2) Typically, there are several subsets of attributes that are minimal.

2.2 Diagnosis

We review the definitions of MBD in DES in this subsection. The FSM proposed in our paper is different from the traditional automata in [1]. Here, we add a set of final states into the automata.

Definition 5 (Finite state machine)**.** A finite state machine is defined as a tuple

$$G = (X, E, T, x_0, F), \tag{5}$$

where X is the state space, E is the set of events, $T \subseteq X \times E \times X$ is the partial transition function, x_0 is the initial state of the system, and F is the set of final states of G.

The event set E is partitioned as $E = E_o \cup E_{\mathrm{uo}}$, where E_o and E_{uo} denote the sets of observable and unobservable events, respectively. Note that $E_f \subseteq E_{\mathrm{uo}} \subseteq E$ denotes the set of failure events to be diagnosed. Observable events can be directly detected by the sensors, hence failure events are usually supposed to be unobservable. A transition between x_1 and x_2 is represented as $\mathrm{tran}(x_1, \sigma) = x_2$, where $x_1,\ x_2 \in X$, $\sigma \in E$. Thus x_2 is reachable from x_1 driven by event σ.

Figure 1 shows an FSM G, where the set of observable events is given by $E_o = \{o_1,\ o_2,\ o_3,\ o_4,\ o_5\}$ and the set of unobservable events is given by $E_{\mathrm{uo}} = \{u_1,\ f_1,\ f_2\}$. In addition, $E_f = \{f_1,\ f_2\}$ is the set of failure events to be diagnosed. $\mathrm{tran}(3,\ o_2) = 6$ is a transition of G. State 1 is the initial state of G, and $\{7,\ 12\}$ is the set of final states of G. Let E^* denote the set of all finite-length sequences formed by events in E. The behavior of the system is described by the prefix-closed language L. L is a subset of E^*. A *path* denotes an arbitrary element of E^*. A *trace* is a special path, which begins from the initial state of G and ends up with the final state of G. In Figure 1, four traces in G exist, i.e., $o_5 f_1 o_2 o_1$, $o_1 o_2 o_3 o_1$, $u_1 o_1 o_2 o_3 o_4$, and $u_1 o_1 o_2 f_2 o_3 o_4$, respectively. Suppose s is a path of G, *projection* $\mathrm{Pj}(s)$ removes the unobservable events from s. The reverse operation of projection is $\mathrm{Pj}_L^{-1}(s_0) = \{s \in L : \mathrm{Pj}(s) = s_0\}$. L/s denotes the set of possible continuations of a path s.

In the real world, most models are incomplete, the definition of incomplete model is described as follows.

Definition 6 (Incomplete model)**.** An incomplete model is an FSM which only includes part of the behaviors and states of the system.

Assessing the diagnosability of a system is crucial in MBD. Diagnosability property refers to the ability to detect the occurrence of failure events, based on the observations and using model-based inferencing. Specifically, a failure event is diagnosable if every event occurrence can be detected after a bounded number of events by a diagnostic engine driven by the observable events of the FSM; this property remains over the entire language generated by the FSM. The formal description of diagnosability of a system is presented in [1].

Definition 7 (Diagnosability)**.** A failure event f is diagnosable with respect to a projection Pj if

$$(\exists n_i \in \mathbb{N})(\forall s_f = f)(\forall t)[||t|| \geqslant n_i \Rightarrow D], \tag{6}$$

where s_f denotes the final event of the path s, t is a continuation of the path s, and the diagnosability condition function $D : E^* \to \{0, 1\}$ is given by

$$D(st) = \begin{cases} 1, & \text{if } \omega \in \mathrm{Pj}_L^{-1}[\mathrm{Pj}(st)] \Rightarrow f \in \omega, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 7 can be explained as follows: s is a path of G, and ends up with a failure event f. In addition, t is an arbitrary sufficiently long continuation of s. f is diagnosable *if and only if* every trace, which produces the same projection of st, contains f. Moreover, if all the failure events in the system are diagnosable, the system is diagnosable [1]. Let us take the DES in Figure 1 as an example. Trace $o_5 f_1 o_2 o_1$ contains failure event f_1, and its projection is unique in the model. Therefore, f_1 is diagnosable. Traces $u_1 o_1 o_2 o_3 o_4$ and $u_1 o_1 o_2 f_2 o_3 o_4$ produce the same projection $o_1 o_2 o_3 o_4$, thus we cannot confirm whether failure event f_2 occurred after observation $o_1 o_2 o_3 o_4$. Thus, f_2 is not diagnosable. Moreover, system G is not diagnosable.

3 Optimizing the incomplete model

Incomplete models only include part of the behaviors and states of the system. According to the observations, repairing the incomplete model is necessary. In this section, we translate the observations and incomplete model into IS. An algorithm, called OPTINM, is proposed to optimize the incomplete model according to the ISs of the observations and incomplete model. Finally, we can obtain the repaired model by using OPTINM. The repaired model is more complete than the incomplete model. Note that the repaired model obtained through OPTINM is also incomplete. In fact, an absolute complete model cannot be computed. Based on the repaired model, the diagnosability of the system can be determined.

3.1 Related definitions

In this subsection, we combine FSM with RST. We explain how IS is used to represent the incomplete model and the observations.

Definition 8 (FSM-IS)**.** An FSM-IS is an information system, where the objects are the traces of the FSM and the attributes are the observable events of FSM.

An attribute value that is equal to n (where n is a natural number) indicates that the order of the observable event occurs in the projection of the trace, and 0 means that the observable event does not occur in the trace. We take the incomplete model of G, i.e., Img, in Figure 2 as an example. The FSM-IS of Img is shown in Table 1. Similarly, all the projections can be represented as the objects in IS. Suppose $o_5 o_2 o_1$, $o_1 o_2 o_3 o_1$, and $o_1 o_2 o_3 o_4$, are the observations, Table 2 shows the IS of these observations.

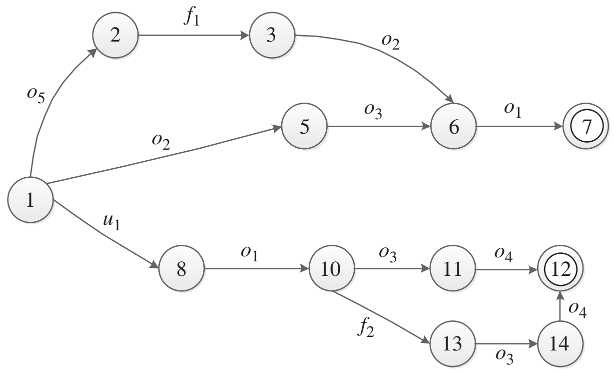

Figure 2 An incomplete system Img.

Table 1 FSM-IS of Img

U	o_1	o_2	o_3	o_4	o_5
1	3	2	0	0	1
2	3	1	2	0	0
3	1	0	2	3	0
4	1	0	2	3	0

Table 2 Observations of G

U	o_1	o_2	o_3	o_4	o_5
1	3	2	0	0	1
2	1,4	2	3	0	0
3	1	2	3	4	0

Table 3 Trim-FSM-IS of Img

U	o_1	o_2	o_3	o_4	o_5
1	1	1	0	0	1
2	1	1	1	0	0
3	1	0	1	1	0
4	1	0	1	1	0

Table 4 Trim-FSM-IS of observations

U	o_1	o_2	o_3	o_4	o_5
1	1	1	0	0	1
2	1	1	1	0	0
3	1	1	1	1	0

Particularly, the value of an attribute of an object is not unique, because an event may occur more than once in a trace. In order to describe the optimized algorithm, we define a trim-FSM-IS below.

Definition 9 (Trim-FSM-IS)**.** A trim-FSM-IS is a pair (U, A), where the attribute value is equal to 1 means that the observable event occurs in the trace, and 0 means that the observable event does not occur in the trace.

Consider the incomplete model Img in Figure 2. The trim-FSM-IS of Img is shown in Table 3. The trim-FSM-IS of observations is shown in Table 4.

In Definition 2, two kinds of attributes in DS were introduced, i.e., consider attribute and decision attribute. In MBD with RST, the value of the decision attribute is used to describe the failure events

Table 5 FSM-DS of G

U	o_1	o_2	o_3	o_4	o_5	dia
1	3	2	0	0	1	f_1
2	1,4	2	3	0	0	N
3	1	2	3	4	0	N
4	1	2	3	4	0	f_2

that occurred in the traces. We formally define the FSM-DS bellow.

Definition 10 (FSM-DS)**.** An FSM-DS is a pair $(U, C \cup \{d\})$ where the objects are the traces of the FSM, the condition attributes are the observable events of the FSM, and the decision attribute is the result of diagnosis.

In FSM-DS, the decision attribute is denoted as dia. If there does not exist any failure event in the object, the value of dia is N (N denotes normal). Otherwise, the value of dia is the failure event that occurred in each object. Consider the FSM G in Figure 1, the FSM-DS of G is shown in Table 5.

Definition 11 (Similar degree)**.** A similar degree between two objects a and b is denoted by λ_{ab}, and defined as follows: $\lambda_{ab} = |\mathrm{CD}_{\mathrm{same}}|/|\mathrm{CD}|$, where $|\mathrm{CD}_{\mathrm{same}}|$ is the number of condition attributes whose values are equal in a and b, and $|\mathrm{CD}|$ is equal to the number of condition attributes.

Considering objects 1 and 2 in Table 3 as an example, five condition attributes exist in the objects, and condition attributes o_1, o_2 and o_4 have the same values; thus, λ_{12} between objects 1 and 2 is equal to 3/5.

Definition 12 (Inconsistent)**.** An inconsistent decision system is a DS with objects whose condition values are the same. However, the decision values of these objects are different.

Table 5 is an inconsistent decision system. Objects 3 and 4 have the same condition values (1 2 3 4 0); however, their decision values are different. Object 3 has decision value N, whereas object 4 has decision value f_2.

Definition 13 (Repaired model)**.** An repaired model is an FSA after optimizing the incomplete model by the observations.

In Subsection 3.2, we introduce the algorithm to obtain the repaired model.

3.2 Optimized algorithm

In this subsection, we present an algorithm called OPTINM to compute the repaired model of the incomplete model. OPTINM is shown in Algorithm 1, with the following inputs: IM and OBS denote the

Algorithm 1 OPTINM: an algorithm for optimizing the incomplete model

Input: $\mathrm{IM}, \mathrm{OBS}, \mathrm{TIM}, \mathrm{TOBS}, \lambda$;
Output: RM;
$\mathrm{RM} \leftarrow \mathrm{IM}$;
for $i = 1; i \leqslant |\mathrm{TIM}|; i++$ **do**
  for $j = 1; i \leqslant |\mathrm{TOBS}|; j++$ **do**
    if $\max(\lambda_{ij}) \geqslant \lambda$ **then**
      $\mathrm{ini}(u_i[n]), \mathrm{ini}(u_j[n])$;
      while $u_i[n] = u_j[n]$ **do**
        if $\mathrm{find}(x)$ **then**
          $u_i[x] = u_j[x]$;
          for each $u_i[z] > u_i[x]$ **do**
            $u_i[z] = u_i[z] + 1$;
          end for
        end if
      end while
      refresh(RM);
    end if
  end for
end for

FSM-ISs of the incomplete DES and observations, respectively; TIM and TOBS are the trim-FSM-ISs of IM and OBS, respectively; and λ signifies the restrictive value of the similar degree. Finally, the repaired model RM can be obtained.

We initialize the repaired model RM by duplicating the incomplete DES IM (line 1). In line 2, |TIM| is equal to the number of the objects in TIM. Similarly, |TOBS| is equal to the number of objects in TOBS (line 3). In line 5, the operation of function $\mathrm{ini}(u_i[n])$ (or $\mathrm{ini}(u_j[n])$) means that an array $u_i[n]$ (or $u_j[n]$) is initialized by the values of the condition attributes of object i (or j) in IM (or OBS). In line 7, function $\mathrm{find}(x)$ denotes that condition attribute x that satisfies the following requirements is searched:

(1) $u_i[x] \neq u_j[x]$;

(2) $(\forall y, u_i[y] = u_j[y])$ s.t. $(u_i[y] < u_i[x])$.

After finding x, let $u_i[x] = u_j[x]$ and every $u_i[z]$ $(u_i[z] > u_i[x])$ in IM pluses one. Condition attribute x refers to the observable event that can be detected in the observation by the sensor. However, the observable event cannot be determined in the trace of the incomplete model. Thus, we should add x in the trace of the incomplete model. The loop in lines 6–12 are continued until $u_i[n] = u_j[n]$. Finally, we renew the ith object in RM by the condition value in array $u_i[n]$ (line 13). Note that if more than one similar degree exists between object i from TIM and object j from TOBS, which is larger than λ, then the larger one is chosen for the computation, because the similarity between the observation and trace increases with λ. In line 4, $\max(\lambda_{ij})$ denotes the largest similar degree. If the value of a condition attribute is not unique, each value should be computed separately.

We conclude this subsection with an example that illustrates the application of OPTINM. Algorithm OPTINM is applied to the incomplete model Img shown in Figure 2. First, we suppose $\lambda = 3/5$. The influence of the value of λ is discussed after the example.

Step 1. The similar degree λ_{11} between (1 1 0 0 1) (the first object in TIM) and (1 1 0 0 1) (the first object in TOBS) is 1, which is larger than 3/5. We initialize arrays and begin the loop. Then, $u_{\mathrm{IM1}}[n] =$ (3 2 0 0 1) is equal to $u_{\mathrm{OBS1}}[n] =$ (3 2 0 0 1), which satisfies the condition in line 6; thus, the first object in RM is not changed. Meanwhile, λ_{11} is the largest one among λ_{11}, λ_{12} and λ_{13}; hence, the next object is considered in TIM.

Step 2. A similar degree λ_{21} between (1 1 1 0 0) and (1 1 0 0 1) is 3/5. λ_{22} between (1 1 1 0 0) and (1 1 1 0 0) is equal to 1, which is larger than 3/5, and λ_{22} is larger than λ_{21} and λ_{23}. After the loop operation, the second object in RM is refreshed by (1,4 2 3 0 0).

Step 3. A similar degree λ_{33} between (1 0 1 1 0) and (1 1 1 1 0) is 4/5, which is larger than 3/5 (the first two comparisons, λ_{31} and λ_{32}, are omitted). We initialize arrays and begin the loop. Finally, the third object in RM is refreshed by (1 2 3 4 0).

Step 4. The condition values of the fourth object and third object are equal; hence, step 4 is the same as step 3.

Finally, the repaired model RM is obtained and the model is the same as Img shown in Table 5. In line 4, a restriction value on the similar degree is introduced. According to the example, λ is crucial in the efficiency of Algorithm 1. The number of loops is based on the value of λ. Setting a better value of the restrictive value on the similar degree still depends on many aspects, such as the scale of the system, the completeness of the system, and so on. The greater the restriction on the similar degree, the more complete the repaired model is; however, the lower the efficiency is.

3.3 Testing the diagnosability

Verifying the diagnosability of the system is an important task in MBD. The diagnosability property identifies the occurrence of failure events from observations. In Subsection 3.2, the repaired model of the incomplete DES is achieved. In this subsection, when we describe a DES as an FSM-DS, a necessary and sufficient condition for a DES to be diagnosable is proposed.

Theorem 1. An FSM is considered diagnosable *if and only if* there are no inconsistent objects in the corresponding table FSM-DS.

Table 6 Trim-FSM-DS of G

U	o_1	o_2	o_3	o_4	o_5	*dia*
1	1	1	0	0	1	f_1
2	1	1	1	0	0	N
3	1	1	1	1	0	N
4	1	1	1	1	0	f_2

Proof. Necessity: We first prove that if an FSM is diagnosable, then there are no inconsistent objects in the FSM-DS. By contrast, assume objects a and b exist, such that these objects have the same condition values and different decision values. In FSM-DS, the condition values of a and b present the projections of traces s_1 and s_2 in FSM. The condition values of a and b are equal; hence, the projections of s_1 and s_2 are the same. Given that the FSM is diagnosable, then the failure events that occurred in s_1 and s_2 are the same. However, the decision values of a and b are different. Therefore, the assumption violates the conclusion. Hence, no inconsistent objects exist in a diagnosable FSM-DS.

Sufficiency: Assume that the FSM-DS does not have inconsistent objects. Pick objects a and b, such that a and b have the same condition values. Here, a and b represent the projections of traces s_1 and s_2 in FSM, respectively, and both produce the same projection. The decision values of a and b are the same; hence, the occurrence of the same failure event can be detected after the observations of s_1 and s_2. Based on Definition 7, FSM is diagnosable.

The DES in Table 5 is illustrated to test the necessary and sufficient condition. Table 5 is an inconsistent DS because objects 3 and 4 have the same condition values (1 2 3 4 0). However, the decision values of these objects are different. Hence, according to Theorem 1, the DES is not diagnosable. In FSM G shown in Figure 1, traces $u_1o_1o_2o_3o_4$, and $u_1o_1o_2f_2o_3o_4$ have the same projection $o_1o_2o_3o_4$. Thus, the occurrence of failure event f_2 cannot be confirmed. The diagnosability is the same as the result concluded by Theorem 1.

4 Minimizing the observable events

Sensor selection is a complicated work in MBD [9], such that the larger the number of observable events, the more difficult the work becomes. Hence, in ensuring the diagnosability of the system, minimizing the observable events is important. This falls in the category of sensor selection problems. In this section, an algorithm to minimize observable events is presented.

The FSM-DS also has a trim formalism. A more formal definition of the trim-FSM-DS is given below.

Definition 14 (Trim-FSM-DS)**.** A trim-FSM-DS is a pair $(U, C \cup \{d\})$ where the condition attributes are the observable events in the FSM and the decision attribute is the result of diagnosis.

Consider FSM G in Figure 1, the trim-FSM-DS of G is shown in Table 6. We introduce an algorithm called MINOE to minimize the observable events of the system. MINOE is shown in Algorithm 2. The input of MINOE is TRS, which signifies the trim-FSM-DS of the system.

Algorithm 2 MINOE: an algorithm for minimizing the observable events

Input: TRS;
Output: OBSet;
if inconsistent objects exist in TRS **then**
    merge(TRS);
end if
$C \leftarrow$ condition attribute of TRS;
$D \leftarrow$ decision attribute of TRS;
OBSet $\leftarrow$ reduct$_D(C)$;

Note that minimizing the observable events should not affect the diagnosability of a system that is deemed not diagnosable. Thus, if inconsistent objects exist in TRS, the inconsistent objects should be merged first (line 2). The scheme of merging two inconsistent objects a and b is described as follows:

Case 1. If $dia(a) = dia(b)$, delete object b from the trim-FSM-DS;

Case 2. If $dia(a) \neq dia(b)$, delete objects a and b from the trim-FSM-DS.

Here, a and b denote the ambiguous traces in the DES. In case 1, when $dia(a)$ is equal to $dia(b)$, the value of $dia(a)$ or $dia(b)$ is the diagnosable failure event in the trace represented by objects a and b. The diagnosable failure events should be reserved after merging. In case 2, the ambiguous traces are deleted. In Algorithm 2, sets C and D are initialized by the condition attributes and decision attribute of TRS, respectively (lines 3 and 4). Definition 4 introduced the concept of *reduct*. Generally, given a set of decision attributes D, a *reduct* of C relative to D is a minimal set MS (MS $\subseteq C$), where the elements in MS produce the same decision value of D as that produced by C. Verification is provided in the paper of RST [1]. In line 5, function $\text{reduct}_D(C)$ computes the *reduct* of C based on D. The result is the minimal set of observable events that ensures the diagnosability of the DES.

We take the trim-FSM-DS in Table 6 as an example. Inconsistent objects exist in Table 6, and the inconsistent objects satisfy case 2. Thus, objects 3 and 4 are deleted from the trim-FSM-DS. Set C is $\{o_1, o_2, o_3, o_4, o_5\}$, and set D is $\{dia\}$. Finally, the *reduct* of C $\{o_4, o_5\}$ is obtained. Hence, $\{o_4, o_5\}$ is the minimal set of observable events of G. The following information from the *reduct* $\{o_4, o_5\}$ is determined:

(1) If the value of $\{o_4, o_5\}$ is $\{0, 1\}$, failure event f_1 can be detected;

(2) If the value of $\{o_4, o_5\}$ is $\{0, 0\}$, the system is normal.

The reduct of C is not unique, and $\{o_3, o_4\}$ is also a *reduct* of C. Therefore, the minimal set of observable events of an FSM may not be unique. When the *reduct* is $\{o_3, o_4\}$, the result is as follows:

(1) If the value of $\{o_3, o_4\}$ is $\{0, 0\}$, failure event f_1 can be detected;

(2) If the value of $\{o_3, o_4\}$ is $\{1, 0\}$, the system is normal.

5 Comparisons

In [12], the MBD of DES with an incomplete model is proposed, and DES is modeled as an automaton, which is the same as FSM. The verification of the diagnosability of DES has been introduced in [1,2,7,8]. These papers model the DESs as FSMs. In [10], an approach for computing the minimal set of observable events in ensuring diagnosability of DES has been described. FSM is basically a tree structure, which is complicated and unstable in terms of synchronization and realization. The complexity of verifying diagnosability and optimizing the model in previous papers is a result of the exponential in the number of states of the FSM. Hence, solving diagnosis problems with FSM is complex.

In our paper, we transform FSM into RST, and solve the MBD of DES with RST. RST has been successfully used in knowledge acquisition, such as intelligent data analysis. After combining DES and RST, the diagnosis problems, including incomplete problems, diagnosability, and minimization, can be easily solved based on the properties of RST. Given an FSM G, $|t|$ is the number of traces of G and $|\text{Obs}|$ is the number of observable sequences. Based on OPTINM, the complexity of optimizing the incomplete model is expressed as $O(|t| \times |\text{Obs}|)$. The basic operation of verifying the diagnosability of DES involves determining the inconsistency of the FSM-DS by comparing the objects. Hence, the complexity of verifying diagnosability is $O(|t|^2)$, and the complexity of the proposed algorithm is significantly improved.

The experiments were conducted on a cluster of Intel Core i5-3470 64-bit processors running at 3.2 GHz with 8 GB of RAM. We compare our algorithms by using the twin-plant approach proposed in [7]. The complexity of the twin-plant approach, which is better than other previous algorithms, is $|X|^4 \times |\text{Obs}| \times |F|$, where $|X|$ is the number of states and $|F|$ is the number of failure events. A benchmark does not exist in the diagnosis of DES, thus we randomly generated different scales of FSMs to prove the efficiency and accuracy of our algorithms. Figures 3–6 present the performance on the random FSMs, while Figure 3 shows the runtimes for some random FSMs, whose number of events is 3 and numbers of states range from 4 to 8. We performed the process ten times for each number of states and the number of transitions is unequivocal every time. Figures 4–6 present the runtimes for the FSMs whose numbers of states are 5, 10 and 15, respectively. We find that the number of the failure events does not affect the efficiency in our

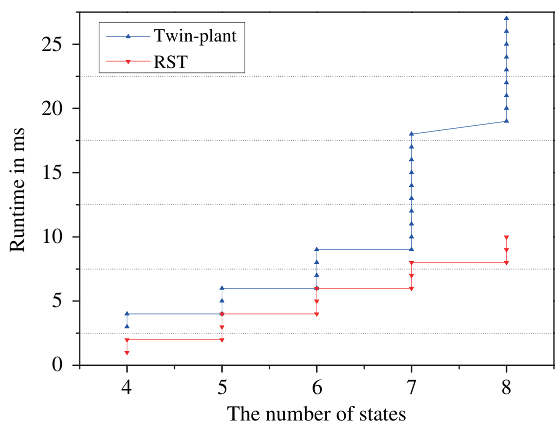

Figure 3 FSMs include 3 events.

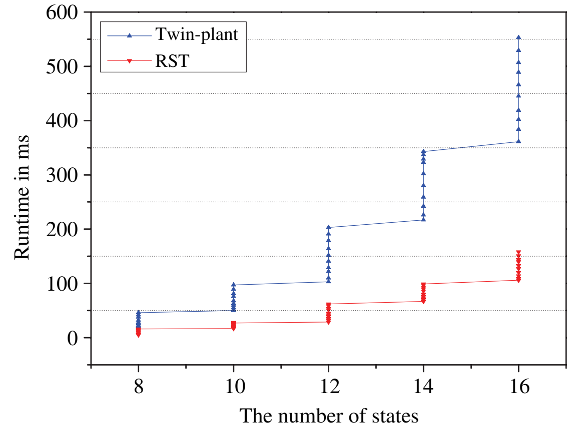

Figure 4 FSMs include 5 events.

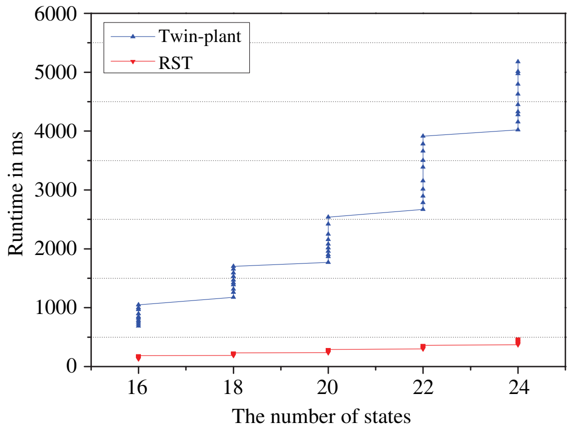

Figure 5 FSMs include 10 events.

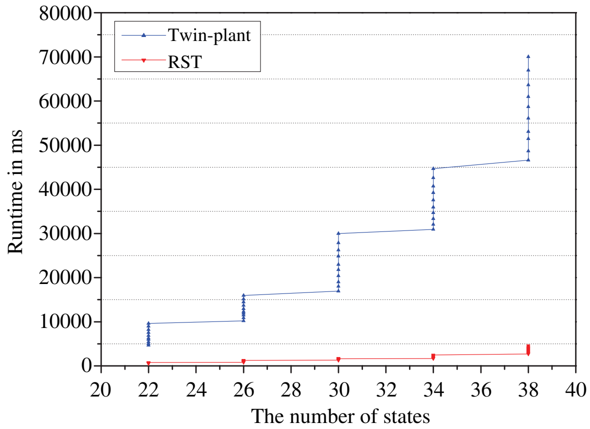

Figure 6 FSMs include 15 events.

algorithms. However, the runtime of the twin-plant approach increases with the number of failure events, suggesting that the twin-plant approach spends less time when there is only one failure event in the system. Therefore, we suppose that there only exists one failure event in the systems in our experiments. Results show that the performance of our algorithms is much better than that of the twin-plant approach.

6 Related work

In 1989, Luca Console proposed an incomplete model for a static system [13]. Motivated by this work, some researchers focused on studying the incomplete model for DES in recent years. A novel concept called P-synchronization product diagnosed DES with an incomplete model was proposed in [12]. The algorithm is primarily used to complete the incomplete model. However, the main task of diagnosis is to ascertain whether a failure behavior has occurred in the system, which verifies the diagnosability of the system. Most algorithms verify the diagnosability of a system through a twin-plant or a diagnoser. Both configurations are tree structures. The complexity of testing diagnosability using a tree structure is exponential in the number of states of the DES and doubly exponential in the number of failure types. Thus, the previous algorithms are complex. Efficiency is a key issue in diagnosis; hence, determining a new formalism to diagnose the DES is necessary.

Other studies have proven that using RST can efficiently solve the fault diagnosis problems [14–17]. These papers applied RST to different kinds of systems. RST has been used to diagnose power transformers in [14] and [15]. A study examined the fault diagnosis problem in diesel engines [16], while another study investigated the diagnosis of a multi-sensor information fusion [17]. Different methods have been proposed for various kinds of failure systems in these studies. MBD is a diagnosis method that can be applied to many kinds of systems because systems can be translated into the same formalism [18, 19]. To diagnose more kinds of systems with RST, an approach employing MBD with RST is proposed in the current paper. We handle the diagnosis problem using RST and compare our algorithms with the previous methods.

7 Conclusion

FSM is one of the most popular formalism in MBD. Based on FSM, various diagnosis problems can be solved. In this paper, we combine FSM and RST, which is then used to diagnose all the dynamic systems that can be modeled as FSM. We propose OPTINM to optimize the incomplete DES according to the observations. A necessary and sufficient condition for the diagnosability of DES is presented. To avoid sensor selection problems, we also introduce MINOE as a tool to minimize the observable events of the FSM in ensuring the diagnosability of the system.

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Conflict of interest The authors declare that they have no conflict of interest.

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