SCIENCE CHINA
Information Sciences

• **LETTER** •

December 2016, Vol. 59 129301:1–129301:3
doi: 10.1007/s11432-016-5591-z

Bi-conjugate gradient based computation of weight vector in space-time adaptive processing

Gatai BAI[1,3], Ran TAO[1,2,3*] & Yue WANG[1,2,3]

[1]*School of Mathematics and Statistics, Beijing Institute of Technology, Beijing* 100081*, China;*
[2]*School of Information and Electronics, Beijing Institute of Technology, Beijing* 100081*, China;*
[3]*Beijing Key Laboratory of Fractional Signals and Systems, Beijing* 100081*, China*

Received December 11, 2015; accepted January 25, 2016; published online June 22, 2016

Citation Bai G T, Tao R, Wang Y. Bi-conjugate gradient based computation of weight vector in space-time adaptive processing. Sci China Inf Sci, 2016, 59(12): 129301, doi: 10.1007/s11432-016-5591-z

Dear editor,

Space-time adaptive processing (STAP) techniques can effectively suppress strong ground clutter and detect moving target for airborne phased array radar by combing spatial signals and the temporal pulses simultaneously [1]. It is known that, when the clutter satisfies the independent and identically-distributed (i.i.d.) condition, the sample matrix inversion (SMI)-based STAP [1] requires twice the number of degree of freedom in the system of secondary samples to estimate the clutter plus noise covariance matrix. The inversion of this covariance matrix is used to compute the weight vector, i.e., the STAP filter, which is used to detect target in the test cell. However, because of shortage of the i.i.d. secondary data and large computational burden of the direct inversion of the clutter plus noise covariance matrix, the SMI-STAP method is difficult to be applied in the practical system because its computational complexity of the inversion of the covariance matrix is $O((NM)^3)$, where N and M are the number of pluses in the CPI and the number of antenna elements, respectively.

In the past three decades, many reduced-dimension and reduced-rank STAP algorithms have been studied for reducing computational burden of the clutter plus noise covariance matrix. However, the direct inversion of the covariance matrix in these STAP methods is still required, and thus the STAP is still difficult to be applied in practical system. Several recursive methods have been developed to solve this problem in [2–4]. However, each of these methods can only be applied to some special forms of covariance matrix. A recursive algorithm [2] and a modified method [3] were proposed based on inverse QR decomposition, where the obtained adaptive weight vector was only an approximate to that of the SMI-based STAP method, because it was difficult to set the initial inverse matrix for calculation. In addition, a range-recursive algorithm [4] was proposed to reject the range dependent clutter. However, the direct inversion of the covariance matrix is still performed in each range cell, while the computational burden is still high in the STAP.

In order to avoid the direct inversion of the covariance matrix, the pulse-order recursive (POR) method was proposed in [5] by using the block Hermitian matrix property of the covariance matrix, which is an effective way to reduce the computational burden of the matrix inversion. However, this method is still restricted in practical system because its computational complexity is $O((NM)^3)$.

In this letter, we propose the BICG-STAP

* Corresponding author (email: rantao@bit.edu.cn)
The authors declare that they have no conflict of interest.

method by using the bi-conjugate gradient (BICG) method [6, 7], which obtains directly the weight vector from a linear equation without computing the inversion of matrix. Since the BICG can converge to the true solution only in several iterations and its computational complexity at each iteration is $O((NM)^2)$, it can obtain the weight vector faster and accurately.

Computation of the weight vector by BICG. Many researches have shown in the literature [1] that the signal model of the STAP is given by the two hypotheses as

$$\begin{aligned} &\mathrm{H}_0 : \boldsymbol{x} = \boldsymbol{x}_{c+n}, \\ &\mathrm{H}_1 : \boldsymbol{x} = \alpha \boldsymbol{v}_t + \boldsymbol{x}_{c+n}, \end{aligned} \tag{1}$$

where $\boldsymbol{x}_{c+n}$, α, and $\boldsymbol{v}_t$ are clutter-plus-noise steering vector, an unknown signal amplitude, and the target steering vector, respectively. The clutter plus noise covariance matrix $\boldsymbol{R} \in \mathbb{C}^{NM\times NM}$ is given as

$$\boldsymbol{R} = E[\boldsymbol{x}_{c+n}\boldsymbol{x}_{c+n}^{\mathrm{H}}] = \boldsymbol{R}_c + \sigma^2 \boldsymbol{I}, \tag{2}$$

where $E[\cdot]$ is the expectation operator, the superscript H is the conjugate transpose operator, $\boldsymbol{R}_c$ is the clutter covariance matrix, σ^2 denotes the noise variance, and $\boldsymbol{I}$ is the identity matrix. In practice, $\boldsymbol{R}$ has a good estimation, which uses K ($K \geqslant 2MN$) number of i.i.d. training samples around the range under test, given as [1]

$$\tilde{\boldsymbol{R}} = \frac{1}{K}\sum_{k=1}^{K} \boldsymbol{x}_k \boldsymbol{x}_k^{\mathrm{H}}, \tag{3}$$

where $\boldsymbol{x}_k$ is the sample in the kth test cell. Then $\tilde{\boldsymbol{R}}$ is a positive-definite Hermitian matrix. By using (3), the weight vector $\boldsymbol{w}$ is given by [1]

$$\boldsymbol{w} = \tilde{\boldsymbol{R}}^{-1}\boldsymbol{s}_t, \tag{4}$$

where $\boldsymbol{s}_t$ is a target steering vector. Because of the large computational burden in calculating $\tilde{\boldsymbol{R}}^{-1}$, the direct method is difficult to be applied in practical system. In order to reduce this computational burden in obtaining $\boldsymbol{w}$ from (4), many researches focus on reducing the computational burden of the inversion $\tilde{\boldsymbol{R}}^{-1}$. In this letter, our method is to compute $\boldsymbol{w}$ directly from a linear equation without computing the inversion.

It is known from (4) that the weight vector $\boldsymbol{w}$ is the solution of the complex linear equation as

$$\tilde{\boldsymbol{R}}\boldsymbol{w} = \boldsymbol{s}_t. \tag{5}$$

We use the iterative method of linear equation to solve $\boldsymbol{w}$. Assume $\mathrm{rank}(\boldsymbol{R}_c) = r$. Then we know from [1] that r is relatively less than NM. Assume that the nonzero eigenvalues of $\boldsymbol{R}_c$ are $\lambda_1, \ldots, \lambda_r$. Then $\lambda_i > 0$ ($1 \leqslant i \leqslant r$). Thus, the eigenvalues of can be regarded as $\lambda_1 + \sigma^2, \ldots, \lambda_r + \sigma^2, \sigma^2, \ldots, \sigma^2$. Thus, we can regard that the distribution of the eigenvalues of $\tilde{\boldsymbol{R}}$ is relatively concentrated. It is known that, the more concentrated the distribution of the eigenvalues of coefficient matrix is, the faster the iterative method converges. Then the iterative method can relatively faster converge to the solution of (5).

The conjugate gradient (CG) method is an efficient iterative method to solve the linear equation with the positive-definite symmetrical coefficient matrix. For the general real linear equation, Fletcher [6] proposes a generalized CG method, i.e., BICG method, which uses both the bi-orthogonality condition and the bi-conjugacy condition to construct the sequence of iterations. By iterations, the BICG converges to the solution by minimizing the residual. The detailed procedures of the BICG are illustrated in [6]. Since (5) is a complex linear equation, we use the improved BICG iterative method, i.e., the complex-version BICG [7], which can deal with the linear equation with the complex coefficient matrix. We use the procedures of the improved BICG to solve (5). The detailed steps of the BICG are illustrated in the following as:

Step 1: Input $\tilde{\boldsymbol{R}} \in \mathbb{C}^{NM\times NM}$ and $\boldsymbol{s}_t \in \mathbb{C}^{NM\times 1}$, and set a tolerance ε. Take initial $\boldsymbol{w}_0 \in \mathbb{C}^{NM\times 1}$. Let $\boldsymbol{r}_0 = \boldsymbol{s}_t - \tilde{\boldsymbol{R}}\boldsymbol{w}_0$, $\tilde{\boldsymbol{r}}_0 = \boldsymbol{r}_0^*$, $\boldsymbol{h}_0 = \boldsymbol{r}_0$, and $\tilde{\boldsymbol{h}}_0 = \tilde{\boldsymbol{r}}_0$. Set $k = 0$.

Step 2: Calculate

$$\begin{aligned} &\alpha_k = \mathrm{Re}(\langle \boldsymbol{r}_k, \tilde{\boldsymbol{r}}_k\rangle)/\mathrm{Re}(\langle \tilde{\boldsymbol{R}}\boldsymbol{h}_k, \tilde{\boldsymbol{h}}_k\rangle), \\ &\boldsymbol{w}_{k+1} = \boldsymbol{w}_k + \alpha_k \boldsymbol{h}_k, \\ &\boldsymbol{r}_{k+1} = \boldsymbol{r}_k - \alpha_k \tilde{\boldsymbol{R}}\boldsymbol{h}_k, \\ &\tilde{\boldsymbol{r}}_{k+1} = \tilde{\boldsymbol{r}}_k - \alpha_k \tilde{\boldsymbol{R}}^{\mathrm{H}}\tilde{\boldsymbol{h}}_k, \\ &\beta_k = -\mathrm{Re}(\langle \boldsymbol{r}_{k+1}, \tilde{\boldsymbol{r}}_{k+1}\rangle)/\mathrm{Re}(\langle \boldsymbol{r}_k, \tilde{\boldsymbol{r}}_k\rangle), \\ &\boldsymbol{h}_{k+1} = \boldsymbol{r}_k - \beta_k \boldsymbol{h}_k, \\ &\tilde{\boldsymbol{h}}_{k+1} = \tilde{\boldsymbol{r}}_k - \beta_k \tilde{\boldsymbol{h}}_k \end{aligned}$$

Step 3: If $\|\boldsymbol{r}_{k+1}\|_2/\|\boldsymbol{s}_t\|_2 < \varepsilon$, break and output $\boldsymbol{w} = \boldsymbol{w}_{k+1}$, else let $k = k+1$ and turn to Step 2.

In these steps, the symbols $(\cdot)^*$, $\mathrm{Re}(\cdot)$, $\langle\cdot,\cdot\rangle$, and $\|\cdot\|_2$ represent the conjugate operator, the real part of $(\cdot)$, the inner product in the unitary space, and the 2-norm, respectively.

It can be seen in above steps that the BICG converges to the solution of (5) by minimizing the norm of the residual $\boldsymbol{r}_{k+1}$ by iterations. In addition, it is noted that, since $\tilde{\boldsymbol{R}}$ is a Hermitian matrix, the calculation of the conjugate transpose operator H in Step 2 can be avoided.

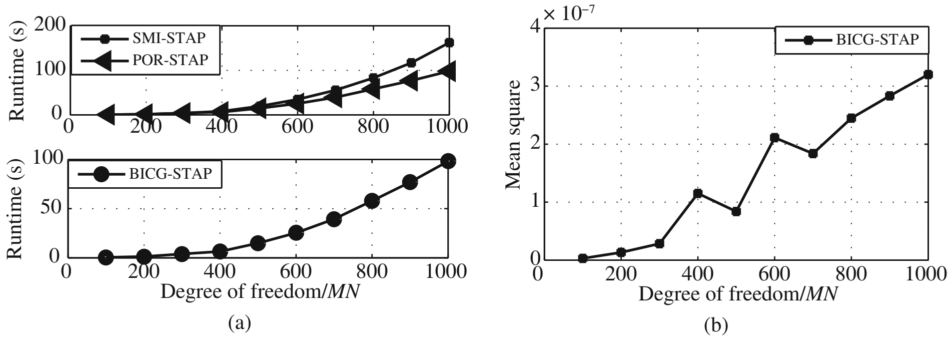

Figure 1 (a) The runtime comparisons of the three methods; (b) the accuracy of the BICG-STAP.

Complexity analysis. It can be known from the steps of the BICG that the computational burden of the BICG at each iteration is contributed by the multiplication between the matrix and the vector. Then the computational complexity of the BICG-STAP is $O((NM)^2)$. It is known that the computational complexity of the SMI-STAP is $O((NM)^3)$. In [6, 7], the POR-based STAP is proposed to compute $\tilde{\boldsymbol{R}}^{-1}$ and the analysis shows that it outperforms other methods, and its computational complexity is $O((NM)^3)$. Therefore, the computational complexity of the BICG-STAP is lower than that of the SMI-STAP and the POR-based STAP.

Simulations. In this simulation, we generate an $N \times M \times K$ space-time clutter data cube according to the form of the steering vector in STAP to show the computational complexities and the accuracies of the BICG-STAP, the SMI-STAP, and the POR-based STAP in computing the weight vector. The noise level is 30 dB. Let $K = 2MN$. Initial $\boldsymbol{w}_0$ and tolerance ε are the zero vector and 0.0001, respectively. Figure 1(a) shows the runtimes of the SMI-STAP, the POR-based STAP, and the BICG-STAP with the increase of the degree of freedom (MN) on the computer. It can be seen that the BICG-STAP takes very little time, while the SMI-STAP and the POR-based STAP take much time when MN becomes large. We regard the weight vector obtained by the SMI-STAP as the true weight vector. Since the inverse method in POR-based STAP is completely equal with the direct inversion, we show only the accuracy of the BICG-STAP. This accuracy is measured by the mean square of the error between the SMI-STAP and the BICG-STAP. Figure 1(b) shows the Mean Square of the BICG-STAP with the increase of the degree of freedom. It can be seen that the error level of the BICG-STAP is 10^{-7}. Therefore, the BICG-STAP is a significant method to obtain the weight vector fast and accurately.

Conclusion. In this letter, we propose the BICG-STAP to compute the weight vector in the STAP. In this method, the weight vector is obtained from a complex linear equation by the BICG iterative method. Analysis shows that the BICG-STAP is a low complexity method, and can obtain the weight vector with a high convergent rate. Simulations show that the BICG-STAP can obtain the weight vector faster and accurately.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant Nos. 61421001, 61331021).

References

1 Melvin W L. A STAP overview. IEEE Aerosp Electron Syst Mag, 2004, 19: 19–35
2 Ma L, Dickson K, McAllister J, et al. QR decomposition-based matrix inversion for high performance embedded MIMO receiver. IEEE Trans Signal Process, 2011, 59: 1859–1867
3 Cao J, Wang X. New recursive algorithm for matrix inversion. J Syst Eng Electron, 2008, 19: 381–384
4 Beau S, Marcos S. Range dependent clutter rejection using range-recursive space-time adaptive processing algorithms. Signal Process, 2010, 90: 57–68
5 Yang X P, Liu Y X, Long T. A pulse-order recursive method for inverse matrix computation applied to space-time adaptive processing. Sci China Inf Sci, 2013, 56: 042312
6 Fletcher R. Conjugate gradient methods for indefinite systems. In: Watson G A, ed. Numerical Analysis. Berlin/Heidelberg: Springer-Verlag, 1976. 73–89
7 Joly P, Meurant G. Complex conjugate gradient methods. Numer Math, 1993, 4: 379–406