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Stability of second-order stochastic neutral partial functional differential equations driven by impulsive noises

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Abstract This paper investigates the stability of second-order stochastic neutral partial functional differential equations driven by impulsive noises. Some sufficient conditions ensuring pth moment exponential stability of the second-order stochastic neutral partial functional differential equations driven by impulsive noises are obtained by establishing a new impulsive-integral inequality. These existing results are generalized and improved by the present study. Finally, an example is given to show the effectiveness of our results.

Keywords second-order stochastic neutral partial functional differential equations, exponential stability, impulsive noises, sine family, cosine family

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1 Introduction

The study of existence, uniqueness, and stability of solutions of stochastic partial differential equations (SPDEs) in Hilbert or Banach space has attracted great interest due to their applications in many fields such as biological systems, physical phenomena, electricity and engineering [1–5].

Unlike the strong solutions of SPDEs in infinite dimensions, the Itô formula of infinite dimensions [5] does not directly apply to the mild solutions of SPDEs. Therefore new techniques and methods need to be developed for the mild solutions. Nevertheless, the study of the mild solutions of stochastic partial differential equations has been developed by the fixed point theory and some integral inequalities. For example, Luo [6, 7] discussed the stability of stochastic partial differential equations with delays based on fixed point theory. Taniguchi [8] discussed exponential stability for stochastic delay partial differential equations. Sakthivel et al. [9] studied the existence of solutions to nonlinear fractional stochastic differential equations. Chen [10] discussed exponential stability for neutral stochastic partial differential equations with delays by establishing integral inequality.

In addition, impulsive effects often exist in real world [11–13], which can lead to mutations of dynamic systems and affect the system stability. For example, exponential stability of mild solutions to impulsive

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stochastic neutral partial differential equations was investigated by establishing new impulsive-integral inequalities [14–16]. Arthi et al. [17] studied the existence and exponential stability of neutral impulsive stochastic integrodifferential equations driven by a fractional Brownian motion. Zhang et al. [18] studied the stability of neutral impulsive stochastic evolution equations with time-varying delays. The asymptotic stability of impulsive SPDEs has been studied by fixed point theory [19,20]. Besides, impulsive stochastic partial functional differential equations (SPFDEs) have been also examined extensively [21–24].

On the other hand, second-order partial differential equations [25–27] have recently gained popularity because of their applications in modeling the charge on a capacitor and mechanical vibrations. The interest of studying second-order systems is to directly deal with them, not to convert them to first-order systems. Recently, the existence, stability, and controllability of mild solutions of second-order SPDEs have been discussed extensively. For example, there are reports on the existence of mild solutions of non-autonomous stochastic second-order differential equations [28, 29] and the dynamic behavior of second-order stochastic evolution equations [30–32]. Furthermore, Arora and Sukavanam [33] discussed the approximate controllability of second order semilinear stochastic system with nonlocal conditions. Chen [34] discussed the stability of second-order neutral SPDEs with infinite delay by using integral-inequality. Arthi et al. [35] discussed exponential stability of second-order neutral stochastic differential equations with impulses. Motivated by the above studies, in this paper, we investigate second-order stochastic neutral partial functional differential equations driven by impulsive noises by establishing a new integral inequality.

The paper is organized as follows. In Section 2, some preliminaries are introduced. In Section 3, we first establish a new integral inequality, which improves the inequality presented in [35] and then we give the stability of second-order stochastic neutral partial functional differential equations driven by impulsive noises by using the new integral inequality. In Section 4, an example is offered to illustrate the effectiveness of our results. Section 5 concludes the paper.

2 Preliminaries

Throughout this paper let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, P)$ be a complete probability space with the filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., the filtration is right continuous while $\mathcal{F}_0$ contains all P-null sets). Let Υ and Γ be the two real separable Hilbert space with norm $\|\cdot\|$. $L(\Gamma, \Upsilon)$ is the space of all bounded linear operators from Υ to Γ with the norm $\|\cdot\|$. $\{w(t) : t \geqslant 0\}$ is a Γ-valued Wiener process defined on the probability space with covariance operator Q, which is a positive, self-adjoint and trace class operator on Γ. Let $\Gamma_0 = Q^{1/2}\Gamma$ be the Hilbert space with respect to the Q-Wiener process $w(t)$ and $L_2^0 = L_2(\Gamma_0, \Upsilon)$ be the space of all Q-Hilbert-Schmidt operators from Γ_0 to Υ, which is a separable Hilbert space with the norm $\|\phi\|^2_{L_2^0} = \mathrm{tr}(\phi Q^{1/2}(\phi Q^{1/2})^*)$ for any $\phi \in L_2^0$. For any bounded operator $\phi \in L(\Gamma, \Upsilon)$, $\|\phi\|^2_{L_2^0} = \mathrm{tr}(\phi Q \phi^*)$. For details, see [2] and the references therein.

Let $\overline{R}_+ = [0, +\infty)$, $\tau > 0$ and $C = C([-\tau, 0], \Upsilon)$ be the family of all right continuous function η defined on $[-\tau, 0]$ with the norm $\|\eta\|_\tau = \sup_{-\tau\leqslant\vartheta\leqslant 0} \|\eta(\vartheta)\|$. For all $\ell = 1, 2, \ldots$, $\mathfrak{C}([-\tau, 0], \Upsilon)$ is the space of the functions $\zeta : [-\tau, 0] \to \Upsilon$ such that ζ is continuous at $t \neq t_\ell$, $\zeta(t_\ell^-) = \zeta(t_\ell)$ and $\zeta(t_\ell^+)$ exist. Let $\mathfrak{C}^b_{\mathcal{F}_0}([-\tau, 0], \Upsilon)$ be the family of all bounded $\mathcal{F}_0$-measurable, $\mathfrak{C}([-\tau, 0], \Upsilon)$ -valued random variables ς with the norm $\|\varsigma\|^p_{\mathfrak{C}} = \sup_{-\tau\leqslant\vartheta\leqslant 0} E\|\varsigma(\vartheta)\|^p_\Upsilon$.

Consider the following second-order stochastic neutral partial functional differential equations driven by impulsive noises:

$$\begin{cases} \mathrm{d}[x'(t) - D(t, x_t)] = [Ax(t) + f(t, x_t)]\mathrm{d}t + g(t, x_t)\mathrm{d}w(t),\ t \geqslant 0,\ t \neq t_\ell,\ \ell = 1, 2, \ldots, \\ \Delta x(t_\ell) = \hat{I}_\ell(x(t_\ell^-)),\ \ell = 1, 2, \ldots, \\ \Delta x'(t_\ell) = \check{I}_\ell(x(t_\ell^-)),\ \ell = 1, 2, \ldots, \\ x(s) = \varphi(s) \in \mathfrak{C}^b_{\mathcal{F}_0},\ s \in [-\tau, 0],\ x'(0) = \psi, \end{cases} \tag{1}$$

where $A : D(A) \subset \Upsilon \to \Upsilon$ is the infinitesimal generator of a strongly continuous cosine family of bounded linear operators $(C(t))_{t\in R}$ defined on Υ. $x_t : [-\tau, 0] \to \Upsilon$, $x_t(\vartheta) = x(t+\vartheta)$ for $t \geqslant 0$. ψ is a $\mathcal{F}_0$-measurable Υ-valued random variable independent of $w(t)$. $f, D : \overline{R}_+ \times \mathfrak{C} \to \mathfrak{C}$, $g : \overline{R}_+ \times \mathfrak{C} \to L_2^0(\Gamma, \Upsilon)$, $0 < t_1 < t_2 < \cdots < t_\ell < \cdots$ and $\lim_{\ell\to+\infty} t_\ell = +\infty$. $\hat{I}_\ell, \check{I}_\ell : \mathfrak{C} \to \Upsilon$, $\Delta\xi(t) = \xi(t^+) - \xi(t^-)$, where $\xi(t^+)$ and $\xi(t^-)$ are the right and left limits of ξ at t, respectively.

Definition 1 ([34])**.** A one-parameter family of bounded linear operators $(C(t))_{t\in R} \in L(\Upsilon, \Upsilon)$ is called a strongly continuous cosine family if
(i) for all $v, t \in R$, $C(v+t) + C(v-t) = 2C(v)C(t)$;
(ii) $C(0) = I$;
(iii) $C(t)x$ is continuous in t on R, for each fixed $x \in \Upsilon$.

Assume $A : \Upsilon \to \Upsilon$ is the infinitesimal generator of $(C(t))_{t\in R}$, which is defined by $Ax = \frac{\mathrm{d}^2}{\mathrm{d}t^2} C(t)|_{t=0}, x \in D(A)$, where $D(A) = \{x \in \Upsilon : C(t)x$ is twice continuously differential in $t\}$. The corresponding strongly continuous sine family $(S(t))_{t\in R}$ is defined by $S(t)x = \int_0^t C(s)x\mathrm{d}s$, for $x \in \Upsilon$.

Lemma 1 ([25])**.** Let A be the infinitesimal generator of a cosine family of operators $(C(t))_{t\in R}$. Then, the following properties hold.
(i) There exists $M^* \geqslant 1$ and $\delta \geqslant 0$ such that $\|C(t)\| \leqslant M^*\mathrm{e}^{\delta t}$ and therefore $\|S(t)\| \leqslant M^*\mathrm{e}^{\delta t}$.
(ii) For all $0 \leqslant s \leqslant t < +\infty$, $A\int_s^t S(v)x\mathrm{d}v = (C(t) - C(s))x$.
(iii) For all $0 \leqslant s \leqslant t < +\infty$, there exists $M^{**} \geqslant 1$ such that $\|S(s) - S(t)\| \leqslant M^{**}|\int_s^t \mathrm{e}^{\delta s}\mathrm{d}s|$.

Definition 2. A stochastic process $\{x(t), t \in \overline{R}_+\}$, is called a mild solution of (1) if
(i) $x(t)$ is an $\mathcal{F}_t(t \geqslant 0)$-adapted process;
(ii) $x(t) \in \Upsilon$ has a càdlàg path on $t \geqslant 0$ almost surely and for any $t \in \overline{R}_+$, $x(t)$ satisfies

$$\begin{aligned} x(t) = {} & C(t)\varphi(0) + S(t)(\psi - D(0,\varphi)) + \int_0^t C(t-v)D(v,x_t)\mathrm{d}v + \int_0^t S(t-v)f(v,x_t)\mathrm{d}v \\ & + \int_0^t S(t-v)g(v,x_t)\mathrm{d}w(v) + \sum_{0<t_\ell<t} C(t-t_\ell)\hat{I}_\ell(x(t_\ell^-)) + \sum_{0<t_\ell<t} S(t-t_\ell)\check{I}_\ell(x(t_\ell^-)). \end{aligned} \tag{2}$$

Definition 3. The mild solution (2) of (1), or simply (1), is said to be pth moment exponentially stable for $p \geqslant 2$, if there exist two constants $\mu > 0$ and $\overline{M} \geqslant 1$ such that

$$E\|x(t)\|^p \leqslant \overline{M}\mathrm{e}^{-\mu t}, \quad t \geqslant 0, \quad p \geqslant 2.$$

Especially, if $p = 2$, Eq. (1) is said to be mean square exponentially stable.

In order to gain stability of (1), we impose the following hypotheses:
(H1) There exist constants $M \geqslant 1$ and $\alpha > 0$, $\beta > 0$ such that for $t \geqslant 0$, $\|C(t)\| \leqslant M\mathrm{e}^{-\alpha t}$, and $\|S(t)\| \leqslant M\mathrm{e}^{-\beta t}$.
(H2) There exist constants $\kappa \geqslant 0$, $K_1 > 0$, $K_2 > 0$, and $K_3 > 0$ such that

$$\|D(t,x) - D(t,y)\| \leqslant K_1\|x - y\|,$$

$$\|f(t,x) - f(t,y)\| \leqslant K_2\|x - y\|,$$

$$\|g(t,x) - g(t,y)\|_{L_2^0} \leqslant K_3\|x - y\|,$$

and $\|D(t,0)\| = \|f(t,0)\| = \|g(t,0)\|_{L_2^0} \leqslant \kappa$.
(H3) For $x, y \in \Upsilon$ and $\sum_{\ell=1}^{+\infty} a_\ell < +\infty$, $\sum_{\ell=1}^{+\infty} b_\ell < +\infty$, there exist positive numbers a_ℓ, and b_ℓ ($\ell = 1, 2, \dots$) such that

$$\|\hat{I}_\ell(x) - \hat{I}_\ell(y)\| \leqslant a_\ell\|x - y\|,$$

$$\|\check{I}_\ell(x) - \check{I}_\ell(y)\| \leqslant b_\ell\|x - y\|,$$

and $\hat{I}_\ell(0) = \check{I}_\ell(0) = 0$.

3 Main results

In this section, we will establish the stability criterion of the mild solutions of (1). First, we establish the following impulsive integral inequality to overcome the interference of impulses.

Lemma 2. Let $\tau > 0$, $\Psi(t) : [-\tau, +\infty) \to [0, \infty)$ be Borel measurable and assume that there exist nonnegative constants ϱ_i $(i = 1, 2, 3, 4)$, ζ_ℓ, ς_ℓ $(\ell = 1, 2, 3, \dots)$ and σ such that

$$\Psi(t) \leqslant \varrho_1 e^{-\eta_1 t} + \varrho_2 e^{-\eta_2 t}, \quad t \in [-\tau, 0], \tag{3}$$

and

$$\begin{aligned}\Psi(t) \leqslant\ & \varrho_1 e^{-\eta_1 t} + \varrho_2 e^{-\eta_2 t} + \varrho_3 \int_0^t e^{-\eta_1 (t-s)} \sup_{\vartheta \in [-\tau, 0]} \Psi(s+\vartheta) ds + \varrho_4 \int_0^t e^{-\eta_2 (t-s)} \sup_{\vartheta \in [-\tau, 0]} \Psi(s+\vartheta) ds \\ & + \sum_{t_\ell < t} \zeta_\ell e^{-\eta_1 (t-t_\ell)} \Psi(t_\ell^-) + \sum_{t_\ell < t} \varsigma_\ell e^{-\eta_2 (t-t_\ell)} \Psi(t_\ell^-) + \sigma, \quad t \geqslant 0, \end{aligned} \tag{4}$$

for $\eta_1, \eta_2 > 0$. If

$$\varrho \triangleq \frac{\varrho_3}{\eta_1} + \frac{\varrho_4}{\eta_2} + \sum_{\ell=1}^{+\infty} \zeta_\ell + \sum_{\ell=1}^{+\infty} \varsigma_\ell < 1. \tag{5}$$

Then there exist constants $\lambda \in (0, \eta_1 \wedge \eta_2)$ and $\aleph \geqslant \varrho_1 + \varrho_2$ such that

$$\Psi(t) \leqslant \aleph e^{-\lambda t} + \frac{\sigma}{1-\varrho}, \quad \text{for } t \geqslant -\tau, \tag{6}$$

where λ and $\aleph$ satisfy that

$$\varrho_\lambda \triangleq \frac{\varrho_3 e^{\lambda\tau}}{\eta_1 - \lambda} + \frac{\varrho_4 e^{\lambda\tau}}{\eta_2 - \lambda} + \sum_{\ell=1}^{+\infty} \zeta_\ell + \sum_{\ell=1}^{+\infty} \varsigma_\ell < 1 \quad \text{and} \quad \aleph \geqslant \frac{\varrho_1 + \varrho_2}{1 - \varrho_\lambda}; \tag{7}$$

or

$$\varrho_\lambda \leqslant 1 \quad \text{and} \quad \aleph \geqslant \frac{(\eta_1 - \lambda)(\eta_2 - \lambda)[(\varrho_1 + \varrho_2) - (\frac{\varrho_3}{\eta_1} + \frac{\varrho_4}{\eta_2})\frac{\sigma}{1-\varrho}]}{e^{\lambda\tau}(\varrho_3(\eta_2 - \lambda) - \varrho_4(\eta_1 - \lambda))}, \tag{8}$$

where $\varrho_3(\eta_2 - \lambda) \neq \varrho_4(\eta_1 - \lambda)$.

Proof. From (5), there exists a constant $\lambda \in (0, \eta_1 \wedge \eta_2)$ such that $\varrho_\lambda < 1$ or $\varrho_\lambda \leqslant 1$. Then there exists a constant $\aleph \geqslant \varrho_1 + \varrho_2$ such that $\aleph$ in (7) or (8) can be defined well for given ϱ_1 and ϱ_2. For any $\widetilde{\aleph} > \aleph$, we firstly claim that

$$\Psi(t) < \widetilde{\aleph} e^{-\lambda t} + \frac{\sigma}{1-\varrho} = z(t), \quad \text{for } t \geqslant -\tau. \tag{9}$$

Obviously, Eq. (9) holds for any $t \in [-\tau, 0]$. By reductio ad absurdum, if Eq. (9) is not true, then we affirm that there exists a $\widetilde{t} > 0$ such that

$$\Psi(\widetilde{t}) \geqslant z(\widetilde{t}), \quad \Psi(t) < z(t), \quad \text{for } t \in [-\tau, \widetilde{t}). \tag{10}$$

In what follows, we will obtain a contradiction to the above conditions.

From (4) and (9), we have

$$\begin{aligned}\Psi(\widetilde{t}) \leqslant\ & \varrho_1 e^{-\eta_1 \widetilde{t}} + \varrho_2 e^{-\eta_2 \widetilde{t}} + \varrho_3 \int_0^{\widetilde{t}} e^{-\eta_1 (\widetilde{t}-s)} \left[\widetilde{\aleph} e^{-\lambda s} e^{\lambda\tau} + \frac{\sigma}{1-\varrho}\right] ds \\ & + \varrho_4 \int_0^{\widetilde{t}} e^{-\eta_2 (\widetilde{t}-s)} \left[\widetilde{\aleph} e^{-\lambda s} e^{\lambda\tau} + \frac{\sigma}{1-\varrho}\right] ds + \sum_{t_\ell < \widetilde{t}} \zeta_\ell e^{-\eta_1 (\widetilde{t}-t_\ell)} \left[\widetilde{\aleph} e^{-\lambda t_\ell} + \frac{\sigma}{1-\varrho}\right]\end{aligned}$$

$$
\begin{aligned}
&+\sum_{t_\ell<\widetilde{t}}\varsigma_\ell \mathrm{e}^{-\eta_2(\widetilde{t}-t_\ell)}\left[\widetilde{\aleph}\mathrm{e}^{-\lambda t_\ell}+\frac{\sigma}{1-\varrho}\right]+\sigma\\
&\leqslant \varrho_1\mathrm{e}^{-\eta_1\widetilde{t}}+\varrho_2\mathrm{e}^{-\eta_2\widetilde{t}}+\left(\frac{\varrho_3}{\eta_1}+\frac{\varrho_4}{\eta_2}+\sum_{\ell=1}^{+\infty}\zeta_\ell+\sum_{\ell=1}^{+\infty}\varsigma_\ell\right)\frac{\sigma}{1-\varrho}\\
&+\left(\frac{\varrho_3\mathrm{e}^{\lambda\tau}}{\eta_1-\lambda}+\frac{\varrho_4\mathrm{e}^{\lambda\tau}}{\eta_2-\lambda}+\sum_{\ell=1}^{+\infty}\zeta_\ell+\sum_{\ell=1}^{+\infty}\varsigma_\ell\right)\widetilde{\aleph}\mathrm{e}^{-\lambda\widetilde{t}}-\frac{\varrho_3\sigma}{(1-\varrho)\eta_1}\mathrm{e}^{-\eta_1\widetilde{t}}-\frac{\varrho_4\sigma}{(1-\varrho)\eta_2}\mathrm{e}^{-\eta_2\widetilde{t}}+\sigma. \qquad (11)
\end{aligned}
$$

If Eq. (7) holds, i.e. $\varrho_\lambda < 1$, then Eq. (11) together with (5) and (7) yields

$$
\Psi(\widetilde{t})\leqslant(\varrho_1+\varrho_2)\mathrm{e}^{-\lambda\widetilde{t}}+\varrho_\lambda\widetilde{\aleph}\mathrm{e}^{-\lambda\widetilde{t}}+\frac{\sigma}{1-\varrho}\leqslant\widetilde{\aleph}\mathrm{e}^{-\lambda\widetilde{t}}+\frac{\sigma}{1-\varrho}, \qquad (12)
$$

which contradicts to (10).

If Eq. (8) holds, i.e. $\varrho_\lambda \leqslant 1$, then Eq. (11) together with (5) and (8) yields

$$
\begin{aligned}
\Psi(\widetilde{t})&\leqslant\left(\varrho_1-\frac{\sigma\varrho_3}{(1-\varrho)\eta_1}-\frac{\varrho_3\widetilde{\aleph}\mathrm{e}^{\lambda\tau}}{\eta_1-\lambda}\right)\mathrm{e}^{-\eta_1\widetilde{t}}+\left(\varrho_2-\frac{\sigma\varrho_4}{(1-\varrho)\eta_2}-\frac{\varrho_4\widetilde{\aleph}\mathrm{e}^{\lambda\tau}}{\eta_2-\lambda}\right)\mathrm{e}^{-\eta_2\widetilde{t}}+\varrho_\lambda\widetilde{\aleph}\mathrm{e}^{-\lambda\widetilde{t}}+\frac{\sigma}{1-\varrho}\\
&\leqslant\left(\varrho_1+\varrho_2-\frac{\sigma\varrho_3}{(1-\varrho)\eta_1}-\frac{\varrho_3\widetilde{\aleph}\mathrm{e}^{\lambda\tau}}{\eta_1-\lambda}-\frac{\sigma\varrho_4}{(1-\varrho)\eta_2}-\frac{\varrho_4\widetilde{\aleph}\mathrm{e}^{\lambda\tau}}{\eta_2-\lambda}\right)\mathrm{e}^{-\lambda\widetilde{t}}+\varrho_\lambda\widetilde{\aleph}\mathrm{e}^{-\lambda\widetilde{t}}+\frac{\sigma}{1-\varrho}\\
&\leqslant\varrho_\lambda\widetilde{\aleph}\mathrm{e}^{-\lambda\widetilde{t}}+\frac{\sigma}{1-\varrho}\leqslant\widetilde{\aleph}\mathrm{e}^{-\lambda\widetilde{t}}+\frac{\sigma}{1-\varrho}, \qquad (13)
\end{aligned}
$$

which contradicts to (10). Therefore, Eq. (9) is true. As $\widetilde{\aleph}\to\aleph$, (6) holds. The proof is completed.

Remark 1. Let $\varrho_i>0$ $(i=1,2,3,4)$, $\sigma=0$ and $\varrho_3(\eta_2-\lambda)\neq\varrho_4(\eta_1-\lambda)$ in Lemma 2, then we can obtain Lemma 2.4 in [35]. In Lemma 2, we only need to find λ from $\varrho_\lambda\leqslant 1$ or $\varrho_\lambda<1$. In [35] the conditions $\varrho_\lambda=1$ and $\varrho_i>0$ $(i=1,2,3,4)$ are too harsh, which are relaxed in Lemma 2. In this sense, Ref. [35] is generalized and improved.

Remark 2. Lemma 2, which is different from the lemmas of [16,17], plays an important role here. The main lemmas in [16,17] cannot stand in the present study, because their conditions are stronger than those of Lemma 2. That is, the conditions in the lemmas of [16,17,35] need to satisfy equalities, whereas our lemma aims to meet inequalities (7) or (8), which are more effective and calculated easily. Thus, the present study improves the existing results.

Theorem 1. Assume that (H1)–(H3) hold. Then Eq. (1) is the pth moment exponentially stable for $p\geqslant 2$ if the following inequality holds:

$$
\begin{aligned}
&4^{p-1}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_1^p\alpha^{-p}+4^{p-1}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_2^p\beta^{-p}\\
&+4^{p-1}c_p2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_3^p\beta^{-\frac{p}{2}}\left(\frac{2(p-1)}{p-2}\right)^{1-\frac{p}{2}}+16^{p-1}M^p\left(\sum_{\ell=1}^{+\infty}a_\ell\right)^p+16^{p-1}M^p\left(\sum_{\ell=1}^{+\infty}b_\ell\right)^p<1, \qquad (14)
\end{aligned}
$$

where $c_p=(p(p-1)/2)^{p/2}$.

Proof. From (2), we have

$$
\begin{aligned}
E\|x(t)\|^p\leqslant\ &4^{p-1}E\|C(t)\varphi(0)+S(t)(\psi-D(0,\varphi))\\
&+\sum_{0<t_\ell<t}C(t-t_\ell)\hat{I}_\ell(x(t_\ell^-))+\sum_{0<t_\ell<t}S(t-t_\ell)\check{I}_\ell(x(t_\ell^-))\|^p\\
&+4^{p-1}E\left\|\int_0^tC(t-v)D(v,x_t)\mathrm{d}v\right\|^p+4^{p-1}E\left\|\int_0^tS(t-v)f(v,x_t)\mathrm{d}v\right\|^p\\
&+4^{p-1}E\left\|\int_0^tS(t-v)g(v,x_t)\mathrm{d}w(v)\right\|^p\\
\leqslant\ &16^{p-1}E\|C(t)\varphi(0)\|^p+16^{p-1}E\|S(t)(\psi-D(0,\varphi))\|^p
\end{aligned}
$$

$$
\begin{aligned}
&+16^{p-1}E\Big\|\sum_{0<t_\ell<t}C(t-t_\ell)\hat{I}_\ell(x(t_\ell^-))\Big\|^p+16^{p-1}E\Big\|\sum_{0<t_\ell<t}S(t-t_\ell)\check{I}_\ell(x(t_\ell^-))\Big\|^p\\
&+4^{p-1}E\left\|\int_0^t C(t-v)D(v,x_t)\mathrm{d}v\right\|^p+4^{p-1}E\left\|\int_0^t S(t-v)f(v,x_t)\mathrm{d}v\right\|^p\\
&+4^{p-1}E\left\|\int_0^t S(t-v)g(v,x_t)\mathrm{d}w(v)\right\|^p\\
:=\;&16^{p-1}\sum_{i=1}^{4}F_i+4^{p-1}\sum_{i=5}^{7}F_i. \qquad (15)
\end{aligned}
$$

By (H1) and (H2), we have $F_1 \leqslant M^pE\|\varphi\|^p\mathrm{e}^{-p\alpha t} \leqslant M^pE\|\varphi\|^p\mathrm{e}^{-\alpha t}$ and $F_2 \leqslant M^p2^{p-1}(E\|\psi\|^p + K_1^pE\|\varphi\|^p)\mathrm{e}^{-\beta t}$. By (H1) and (H3), we have

$$
\begin{aligned}
F_3 &\leqslant M^pE\left(\sum_{0<t_\ell<t}\mathrm{e}^{-\alpha(t-t_\ell)}a_\ell\|x(t_\ell^-)\|\right)^p\\
&\leqslant M^pE\left(\sum_{\ell=1}^{+\infty}a_\ell^{\frac{p-1}{p}}a_\ell^{\frac{1}{p}}\mathrm{e}^{-\alpha(t-t_\ell)}\|x(t_\ell^-)\|\right)^p\\
&\leqslant M^p\left(\sum_{\ell=1}^{+\infty}a_\ell\right)^{p-1}\sum_{0<t_\ell<t}q_\ell\mathrm{e}^{-p\alpha(t-t_\ell)}E\|x(t_\ell^-)\|^p\\
&\leqslant M^p\left(\sum_{\ell=1}^{+\infty}a_\ell\right)^{p-1}\sum_{0<t_\ell<t}a_\ell\mathrm{e}^{-\alpha(t-t_\ell)}E\|x(t_\ell^-)\|^p,
\end{aligned}
$$

and

$$
F_4 \leqslant M^p\left(\sum_{\ell=1}^{+\infty}b_\ell\right)^{p-1}\sum_{0<t_\ell<t}b_\ell\mathrm{e}^{-\beta(t-t_\ell)}E\|x(t_\ell^-)\|^p.
$$

By (H1), (H2) and the Hölder inequality, we have

$$
\begin{aligned}
F_5 &\leqslant E\left(\int_0^t M\mathrm{e}^{-\alpha(t-v)}\|D(v,x_v)\|\mathrm{d}vf\right)^p\\
&\leqslant M^p\left(\int_0^t\left(\mathrm{e}^{-\alpha(t-v)+\frac{-\alpha(t-v)}{p}}\right)^{\frac{p}{p-1}}\mathrm{d}v\right)^{p-1}\int_0^t\mathrm{e}^{-\alpha(t-v)}E\|D(v,x_v)\|^p\mathrm{d}v\\
&\leqslant M^p\alpha^{1-p}\int_0^t\mathrm{e}^{-\alpha(t-v)}E\|D(v,x_v)-D(v,0)+D(v,0)\|^p\mathrm{d}v.
\end{aligned}
$$

When $\kappa > 0$,

$$
F_5 \leqslant 2^{p-1}M^pK_1^p\alpha^{1-p}\int_0^t\mathrm{e}^{-\alpha(t-v)}\sup_{-\tau\leqslant\vartheta\leqslant0}E\|x(v+\vartheta)\|^p\mathrm{d}v+2^{p-1}M^p\kappa^p\alpha^{-p}.
$$

When $\kappa = 0$,

$$
F_5 \leqslant M^pK_1^p\alpha^{1-p}\int_0^t\mathrm{e}^{-\alpha(t-v)}\sup_{-\tau\leqslant\vartheta\leqslant0}E\|x(v+\vartheta)\|^p\mathrm{d}v.
$$

That is,

$$
F_5 \leqslant 2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_1^p\alpha^{1-p}\int_0^t\mathrm{e}^{-\alpha(t-v)}\sup_{-\tau\leqslant\vartheta\leqslant0}E\|x(v+\vartheta)\|^p\mathrm{d}v+2^{p-1}M^p\kappa^p\alpha^{-p},
$$

where $\mathrm{sgn}(\cdot)$ is a sign function. Similarly, by (H1), (H2) and the Hölder inequality, we have

$$
F_6 \leqslant M^p\beta^{1-p}\left(\int_0^t\mathrm{e}^{-\beta(t-v)}E\|f(v,x_v)-f(v,0)+f(v,0)\|^p\mathrm{d}v\right)
$$

$$\leqslant 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_2^p \beta^{1-p} \int_0^t \mathrm{e}^{-\beta(t-v)} \sup_{-\tau\leqslant\vartheta\leqslant 0} E\|x(v+\vartheta)\|^p \mathrm{d}v + 2^{p-1} M^p \kappa^p \beta^{-p}.$$

According to Prato and Zabczyk [2, Lemma 7.7, p. 194], (H1) and (H2), we have

$$\begin{aligned} F_7 &\leqslant c_p M^p \left(\int_0^t \left(\mathrm{e}^{-\beta p(t-v)} E\|g(v,x_v)-g(v,0)+g(v,0)\|_{L_2^0}^p \right)^{\frac{2}{p}} \mathrm{d}v \right)^{\frac{p}{2}} \\ &\leqslant c_p 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_3^p \left(\frac{2\beta(p-1)}{p-2} \right)^{1-\frac{p}{2}} \int_0^t \mathrm{e}^{-\beta(t-v)} \sup_{\vartheta\in[-\tau,0]} E|x(v+\vartheta)|^p \mathrm{d}v \\ &\quad + c_p 2^{p-1} \beta^{-1} M^p \kappa^p \left(\frac{2\beta(p-1)}{p-2} \right)^{1-\frac{p}{2}}, \end{aligned}$$

where $c_p = (p(p-1)/2)^{p/2}$. These, together with (15) yield

$$\begin{aligned} E\|x(t)\|^p &\leqslant 16^{p-1} M^p E\|\varphi\|^p \mathrm{e}^{-\alpha t} + 16^{p-1} M^p 2^{p-1} (E\|\psi\|^p + K_1^p E\|\varphi\|^p) \mathrm{e}^{-\beta t} \\ &\quad + 4^{p-1} 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_1^p \alpha^{1-p} \int_0^t \mathrm{e}^{-\alpha(t-v)} \sup_{-\tau\leqslant\vartheta\leqslant 0} E\|x(v+\vartheta)\|^p \mathrm{d}v \\ &\quad + 4^{p-1} 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_2^p \beta^{1-p} \int_0^t \mathrm{e}^{-\beta(t-v)} \sup_{-\tau\leqslant\vartheta\leqslant 0} E\|x(v+\vartheta)\|^p \mathrm{d}v \\ &\quad + 4^{p-1} c_p 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_3^p \left(\frac{2\beta(p-1)}{p-2} \right)^{1-\frac{p}{2}} \int_0^t \mathrm{e}^{-\beta(t-v)} \sup_{\vartheta\in[-\tau,0]} E\|x(v+\vartheta)\|^p \mathrm{d}v \\ &\quad + 16^{p-1} M^p \left(\sum_{\ell=1}^{+\infty} a_\ell \right)^{p-1} \sum_{0<t_\ell<t} a_\ell \mathrm{e}^{-\alpha(t-t_\ell)} E\|x(t_\ell^-)\|^p \\ &\quad + 16^{p-1} M^p \left(\sum_{\ell=1}^{+\infty} b_\ell \right)^{p-1} \sum_{0<t_\ell<t} b_\ell \mathrm{e}^{-\beta(t-t_\ell)} E\|x(t_\ell^-)\|^p \\ &\quad + 8^{p-1} M^p \kappa^p \alpha^{-p} + 8^{p-1} M^p \kappa^p \beta^{-p} + 8^{p-1} c_p \beta^{-1} M^p \kappa^p \left(\frac{2\beta(p-1)}{p-2} \right)^{1-\frac{p}{2}} \\ &\leqslant 16^{p-1} M^p E\|\varphi\|^p \mathrm{e}^{-\alpha t} + 16^{p-1} M^p 2^{p-1} (E\|\psi\|^p + K_1^p E\|\varphi\|^p) \mathrm{e}^{-\beta t} \\ &\quad + 4^{p-1} 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_1^p \alpha^{1-p} \int_0^t \mathrm{e}^{-\alpha(t-v)} \sup_{-\tau\leqslant\vartheta\leqslant 0} E\|x(v+\vartheta)\|^p \mathrm{d}v \\ &\quad + \left[4^{p-1} 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_2^p \beta^{1-p} + 4^{p-1} c_p 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_3^p \left(\frac{2\beta(p-1)}{p-2} \right)^{1-\frac{p}{2}} \right] \\ &\quad \times \int_0^t \mathrm{e}^{-\beta(t-v)} \sup_{\vartheta\in[-\tau,0]} E\|x(v+\vartheta)\|^p \mathrm{d}v \\ &\quad + 16^{p-1} M^p \left(\sum_{\ell=1}^{+\infty} a_\ell \right)^{p-1} \sum_{0<t_\ell<t} a_\ell \mathrm{e}^{-\alpha(t-t_\ell)} E\|x(t_\ell^-)\|^p \\ &\quad + 16^{p-1} M^p \left(\sum_{\ell=1}^{+\infty} b_\ell \right)^{p-1} \sum_{0<t_\ell<t} b_\ell \mathrm{e}^{-\beta(t-t_\ell)} E\|x(t_\ell^-)\|^p \\ &\quad + 8^{p-1} M^p \kappa^p \alpha^{-p} + 8^{p-1} M^p \kappa^p \beta^{-p} + 8^{p-1} c_p \beta^{-1} M^p \kappa^p \left(\frac{2\beta(p-1)}{p-2} \right)^{1-\frac{p}{2}}. \end{aligned}$$

Let $\widetilde{\varrho}_1 := M^p E\|\varphi\|^p$, $\widetilde{\varrho}_2 := 16^{p-1} M^p 2^{p-1}(E\|\psi\|^p + K_1^p E\|\varphi\|^p)$, $\widetilde{\varrho}_3 := 4^{p-1} 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_1^p \alpha^{1-p}$, $\widetilde{\varrho}_4 := 4^{p-1} 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_2^p \beta^{1-p} + 4^{p-1} c_p 2^{(p-1)\mathrm{sgn}(\kappa)} M^p K_3^p (\frac{2\beta(p-1)}{p-2})^{1-\frac{p}{2}}$, $\hat{\delta}_\ell = 16^{p-1} M^p (\sum_{\ell=1}^{+\infty} a_\ell)^{p-1} a_\ell$, $\check{\delta}_\ell = 16^{p-1} M^p (\sum_{\ell=1}^{+\infty} b_\ell)^{p-1} b_\ell$. If Eq. (14) holds, i.e., $\frac{\widetilde{\varrho}_3}{\alpha} + \frac{\widetilde{\varrho}_4}{\beta} + \sum_{\ell=1}^{+\infty} \hat{\delta}_\ell + \sum_{\ell=1}^{+\infty} \check{\delta}_\ell < 1$, then by (16) and Lemma 2 there exist $\lambda \in (0, \alpha \wedge \beta)$ and $\widehat{M}$ such that $E\|x(t)\|^p \leqslant \widehat{M} \mathrm{e}^{-\lambda t}$. The proof is completed.

Corollary 1. Assume that (H1)–(H3) hold. Then Eq. (1) is mean square exponentially stable for $p=2$, provided

$$\begin{aligned} &4\times 2^{\mathrm{sgn}(\kappa)}M^2K_1^2\alpha^{-2}+4\times 2^{\mathrm{sgn}(\kappa)}M^2K_2^2\beta^{-2}+4\times 2^{\mathrm{sgn}(\kappa)}M^2K_3^2\beta^{-1}\\ &+16M^2\left(\sum_{\ell=1}^{+\infty}a_\ell\right)^2+16M^2\left(\sum_{\ell=1}^{+\infty}b_\ell\right)^2<1. \end{aligned} \tag{16}$$

Moreover, if $\kappa=0$, then Eq. (1) is mean square exponentially stable for $p=2$, provided

$$4M^2K_1^2\alpha^{-2}+4M^2K_2^2\beta^{-2}+4M^2K_3^2\beta^{-1}+16M^2\left(\sum_{\ell=1}^{+\infty}a_\ell\right)^2+16M^2\left(\sum_{\ell=1}^{+\infty}b_\ell\right)^2<1.$$

If Eq. (1) has no impulsive effects, Eq. (1) is transformed into the following second-order neutral stochastic partial functional differential equations:

$$\begin{cases} \mathrm{d}[x'(t)-D(t,x_t)]=[Ax(t)+f(t,x_t)]\mathrm{d}t+g(t,x_t)\mathrm{d}w(t), & t\geqslant 0,\\ x(s)=\varphi(s)\in\mathfrak{C}^b_{\mathcal{F}_0}, \quad s\in[-\tau,0], \quad x'(0)=\psi. \end{cases} \tag{17}$$

By using the technique of Theorem 1, we can obtain the following result.

Corollary 2. Assume that (H1) and (H2) hold. Then Eq. (17) is the pth moment exponentially stable for $p\geqslant 2$, provided

$$\begin{aligned} &4^{p-1}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_1^p\alpha^{-p}+4^{p-1}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_2^p\beta^{-p}\\ &+4^{p-1}(p(p-1)/2)^{p/2}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_3^p\beta^{-\frac{p}{2}}\left(\frac{2(p-1)}{p-2}\right)^{1-\frac{p}{2}}<1. \end{aligned}$$

If $D(t,x_t)=0$, Eq. (1) becomes the following second-order impulsive stochastic partial functional differential equations:

$$\begin{cases} \mathrm{d}x'(t)=[Ax(t)+f(t,x_t)]\mathrm{d}t+g(t,x_t)\mathrm{d}w(t),\ t\geqslant 0,\ t\neq t_\ell,\ \ell=1,2,\ldots,\\ \Delta x(t_\ell)=\hat{I}_\ell(t_\ell^-),\ \ell=1,2,\ldots,\\ \Delta x'(t_\ell)=\check{I}_\ell(t_\ell^-),\ \ell=1,2,\ldots,\\ x(s)=\varphi(s)\in\mathfrak{C}^b_{\mathcal{F}_0},\ s\in[-\tau,0],\ x'(0)=\psi. \end{cases} \tag{18}$$

Corollary 3. Assume that (H1)–(H3) hold. Then Eq. (18) is pth moment exponentially stable for $p\geqslant 2$, provided

$$\begin{aligned} &3^{p-1}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_2^p\beta^{-p}+3^{p-1}(p(p-1)/2)^{p/2}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_3^p\beta^{-\frac{p}{2}}\left(\frac{2(p-1)}{p-2}\right)^{1-\frac{p}{2}}\\ &+12^{p-1}M^p\left(\sum_{\ell=1}^{+\infty}a_\ell\right)^p+12^{p-1}M^p\left(\sum_{\ell=1}^{+\infty}b_\ell\right)^p<1. \end{aligned}$$

If Eq. (1) has no impulsive effects and neutral term, Eq. (1) becomes the following second-order stochastic partial functional differential equations:

$$\begin{cases} \mathrm{d}x'(t)=[Ax(t)+f(t,x_t)]\mathrm{d}t+g(t,x_t)\mathrm{d}w(t), & t\geqslant 0,\\ x(s)=\varphi(s)\in\mathfrak{C}^b_{\mathcal{F}_0}, \quad s\in[-\tau,0], \quad x'(0)=\psi. \end{cases} \tag{19}$$

By using the technique in Theorem 1, we can obtain the following result.

Corollary 4. Assume that (H1) and (H2) hold. Then Eq. (19) is pth moment exponentially stable for $p\geqslant 2$, provided

$$\begin{aligned} &3^{p-1}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_1^p\alpha^{-p}+3^{p-1}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_2^p\beta^{-p}\\ &+3^{p-1}(p(p-1)/2)^{p/2}2^{(p-1)\mathrm{sgn}(\kappa)}M^pK_3^p\beta^{-\frac{p}{2}}\left(\frac{2(p-1)}{p-2}\right)^{1-\frac{p}{2}}<1. \end{aligned}$$

Remark 3. It should be pointed out that Theorem 3.4 of [34] could be given by Corollary 2. It is important that if $\kappa > 0$, Theorem 3.1 in [35] cannot be obtained by their lemma, which is improved in our paper. When $\kappa = 0$ and the delays of system (1) are constant, Theorem 3.1 of [35] can be directly obtained as a special case of our results. The conditions of our results are weaker. In this sense, our results generalize and improve the existing results.

Remark 4. In [14–17], first-order stochastic partial differential equations have been discussed by establishing integral inequalities. In [18–20], first-order stochastic partial differential equations have been studied by the fixed point theory. This paper focuses on second-order systems, which are different from the previously studied systems. In addition, the inequalities established in [14–17] cannot be applied here. Instead, we generalize the first-order systems to second-order stochastic partial differential equations and discuss the stability of the second-order systems by establishing a new integral inequality. Essentially, we generalize these existing results to cover more general high-order systems.

4 Example

In the section, we give an example to verify the effectiveness of the results. Consider the following second-order stochastic neutral partial functional differential equations driven by impulsive noises:

$$\begin{cases} \mathrm{d}[x'(t) - u_1 x_t - v_1] = \left[\dfrac{\partial^2}{\partial z^2} x(t) + u_2 x_t + v_2\right] \mathrm{d}t + (u_3 x_t + v_3)\mathrm{d}w(t),\ 0 \leqslant z \leqslant \pi,\ t \geqslant 0,\ t \neq t_\ell, \\ \Delta x(t_\ell) = \dfrac{v_4}{\ell^2} x(t_\ell^-),\ \ell = 1, 2, \ldots, \\ \Delta x'(t_\ell) = \dfrac{v_5}{\ell^2} x(t_\ell^-),\ \ell = 1, 2, \ldots, \\ x(s) = \varphi(s) \in \mathfrak{C}^b_{\mathcal{F}_0}([-\tau, 0], L^0[0, \pi]),\ x'(0) = \psi,\ x(t, 0) = x(t, \pi) = 0, \end{cases} \tag{20}$$

where $u_k > 0,\ k = 1, 2, 3, 4$, and $v_l \geqslant 0,\ l = 1, 2, \ldots, 5$ are constants. $w(t)$ is the standard one-dimensional Wiener process.

Let $\Upsilon = L^2[0, \pi]$ and $\Gamma = \Upsilon^1_0(0, \pi) \cap \Upsilon^2(0, \pi)$. Now we define the operator $A : \Upsilon \to \Upsilon$ by $Ax = \frac{\partial^2 x}{\partial z^2} \in \Upsilon$ with domain $D(A) = \Upsilon^1_0(0, \pi) \cap \Upsilon^2(0, \pi)$, where $\Upsilon^1_0(0, \pi) = \{\omega \in L^2[0, \pi] : \frac{\partial \omega}{\partial z} \in L^2[0, \pi], \omega(0) = \omega(\pi) = 0\}$ and $\Upsilon^2(0, \pi) = \{\omega \in L^2[0, \pi] : \frac{\partial \omega}{\partial z}, \frac{\partial^2 \omega}{\partial z^2} \in L^2[0, \pi]\}$. Then A is the infinitesimal generator of a strongly continuous cosine family of bounded linear operators $C(t),\ t \geqslant 0$ and the corresponding sine family $S(t),\ t \geqslant 0$ in Υ, that satisfies $\|C(t)\| \leqslant \mathrm{e}^{-\pi^2 t}$ and $\|S(t)\| \leqslant \mathrm{e}^{-\pi^2 t}$ for $t \geqslant 0$. Thus it is easy to see that all the conditions are satisfied with $M = 1,\ \tau = 1,\ \alpha = \beta = \pi,\ K_1 = u_1,\ K_2 = u_2,\ K_3 = u_3$, $\kappa = \max\{v_1, v_2, v_3\} = v,\ a_\ell = \frac{v_4}{\ell^2}$, and $b_\ell = \frac{v_5}{\ell^2}$. Thus by Corollary 1, system (20) is mean square exponentially stable provided

$$4 \times 2^{\mathrm{sgn}(\kappa)} u_1^2 \pi^{-2} + 4 \times 2^{\mathrm{sgn}(\kappa)} u_2^2 \pi^{-2} + 4 \times 2^{\mathrm{sgn}(\kappa)} u_3^2 \pi^{-1} + \frac{4 v_4^2 \pi^4}{9} + \frac{4 v_5^2 \pi^4}{9} < 1.$$

Especially, if $\kappa = v = 0$, then system (20) is mean square exponentially stable provided $4u_1^2\pi^{-2} + 4u_2^2\pi^{-2} + 4u_3^2\pi^{-1} + \frac{4v_4^2\pi^4}{9} + \frac{4v_5^2\pi^4}{9} < 1$. If $\kappa = v = v_4 = v_5 = 0$, then by Corollary 2 system (20) without impulses is mean square exponentially stable provided $4u_1^2\pi^{-2} + 4u_2^2\pi^{-2} + 4u_3^2\pi^{-1} < 1$.

5 Conclusion

In this paper, we have examined second-order stochastic neutral partial functional differential equations driven by impulsive noises. We first establish a new integral inequality, which is effective for studying impulsive systems. Based on the new inequality, we establish the exponential stability criteria. Our results generalize and improve some existing work.

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