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Sinusoidal disturbance induced topology identification of Hindmarsh-Rose neural networks

Junchan ZHAO[1,2*], M. A. AZIZ-ALAOUI[3], Cyrille BERTELLE[4] & Nathalie CORSON[3]

[1] *The School of Mathematics and Statistics, Hunan University of Commerce, Changsha* 410205*, China;*
[2] *Key Laboratory of Hunan Province for Mobile Business Intelligence, Hunan University of Commerce, Changsha* 410205*, China;*
[3] *UNIHAVRE, LMAH, Normandie University, FR-CNRS-3335, ISCN,* 76600 *Le Havre, France;*
[4] *UNIHAVRE, LITIS, Normandie University, FR-CNRS-3638, ISCN,* 76600 *Le Havre, France*

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Abstract Topology identification of complex networks is an important problem. Existing research shows that the synchronization of network nodes is an obstacle in the identification of network topology. Identification of the structure of the network presents an interesting challenge during the synchronization of complex networks. We developed a new method using the sinusoidal disturbance to identify the topology when the complex network achieves synchronization. Compared with the disturbance of all the nodes, the disturbance of the key nodes alone can achieve a very good effect. Finally, numerical simulation data are provided to validate our hypothesis.

Keywords complex network, topology identification, sinusoidal disturbance, persistently exciting condition, Hindmarsh-Rose system

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1 Introduction

Complex networks, such as transportation and phone networks, the Internet, wireless networks, and the World Wide Web, play an important role in our life. Significant progress has been made in understanding complex networks since the discovery of their small-world [1] and scale-free [2] characteristics. Recently, synchronization of complex networks has evoked broad interest. Two basic tools are used to analyze various synchronization problems encountered in complex networks. The first is the master stability function, proposed by Pecora and Carroll [3], and the other is the connection graph stability method, proposed by Belykh et al. [4, 5].

However, the above conclusion is based on the condition that the topology of a complex network is known. When the topology is unknown, it is important to estimate it in the study of complex networks. Topology identification, as an inverse problem, has received widespread attention from the systems science community. Many effective methods such as adaptive synchronization, Granger causality test, compressive sensing sourced from different fields of research are used to study the identification problem [6–13].

* Corresponding author (email: junchanzhao@163.com)

In this study, we focused on topology identification using outer synchronization between networks. Using a complex dynamic network with unknown topology as a drive system, researchers have constructed a response system and adaptive controllers to estimate the unknown topology. Note that one condition is very important during the process of topology identification, which is the linearly independent condition on synchronization manifold. In our previous study, we used the persistently excite (PE) condition to replace the above condition. Topology identification of a complex network is successful when PE condition is satisfied, otherwise it would be failed. We also analyzed the relationship between the two conditions. However, it is difficult for some complex networks to satisfy these conditions when the coupling strength exceeds a certain threshold. For example, the synchronization threshold is very small for fully connected networks or star-shaped networks.

Hindmarsh-Rose (HR) model, is a popular neuron model, which has received wide attention in the past years. The model is a Hodgkin-Huxley-type physiologically realistic model describing the signal transmission across axons in neurons. Recently, the dynamic behavior of the HR system in complex networks has been investigated in detail [14–18]. By formulating a master stability equation for the time-delayed networks of HR neurons, Mukeshwar et al. [14] showed that there is always an extended region of stable synchronous activity corresponding to low coupling strengths. Wang et al. [15] investigated the lag synchronization of multiple identical HR neuron systems coupled in a ring structure. Jia et al. [16] reported several significant results using a bridging network and employing the adaptive law to estimate the topology of the system. However, their results are debatable, because they ignored the PE condition. Using the popular HR neuronal model, Ehrich et al. [17] demonstrated that the dynamics could be complicated even for identical neurons and simple coupling. Li et al. [18] investigated the effects of network structure on the synchronizability of a nonlinearly coupled dynamic network of HR neurons with a sigmoidal coupling function. As complex networks usually contain a large number of nodes, the choice of nodes to be controlled is critical. In recent years, several researchers have proposed the pinning strategy to control complex networks [19–24].

In this paper, considering HR neural as a network node with nonlinear coupling, we investigated the structure identification problem of the complex network. Our results show that when the nodes of the network achieve inner synchronization, it is difficult to identify the structure of the network. This is because when the network nodes reach synchronization, the PE condition cannot be satisfied, and the topology identification fails. Therefore, we focused on how to identification of the network structure when the PE condition is not satisfied. We propose a new method for identifying network structure by adding sinusoidal signals to the network nodes. Considering the heterogeneity of node degrees, we investigated the structural identification of a star-shaped network. Our results indicate that the sinusoidal disturbance introduced into the network nodes can induce topology identification successfully. In particular, as opposed to the disturbance of all nodes, the disturbance of key nodes is a very effective method.

The remainder of this paper is organized as follows. Section 2 introduces HR models and provides the mathematical background for this work. The main results on topology identification are presented in Section 3. In Section 4, topology identification of a complex network, using a sinusoidal disturbance, is discussed. Finally, our concluding remarks are presented in Section 5.

2 Preliminaries

In this section, the HR model and the mathematical background of this study are introduced.

2.1 HR model

In 1952, a mathematical model that describes neuron activity was proposed by two neurophysiologists, Hodgkin and Huxley [25]. Based on their model, several neuron models have been developed over the last few decade [26–29]. We primarily consider one of the well-known HR models, which can be described by

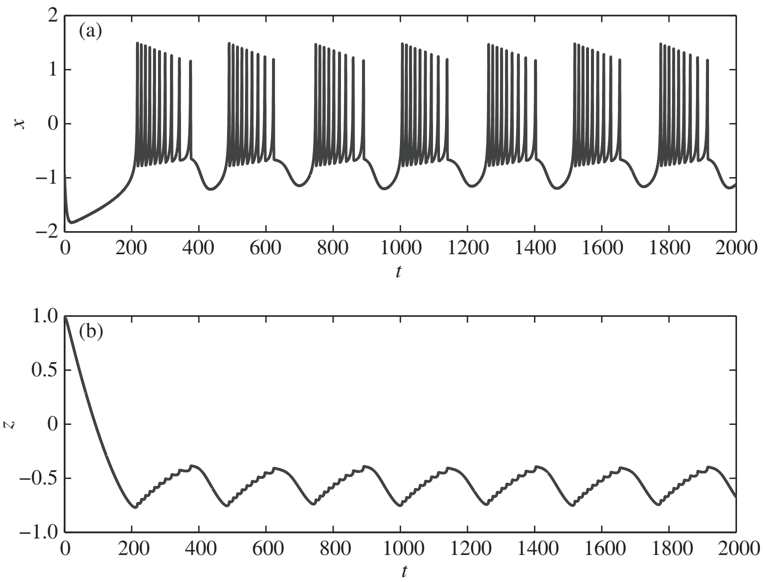

Figure 1 Time evolution of the fast (a) and slow (b) variables in the HR system showing a bursting phase (1) (Parameters used are $a = 2.8, \alpha = 1.6, \rho = 0.001, b = 9$, and $c = 5$).

$$\begin{cases} \dot{x} \, = \, ax^2 - x^3 - y - z, \\ \dot{y} \, = \, (a+\alpha)x^2 - y, \\ \dot{z} \, = \, \rho(bx + c - z), \end{cases} \tag{1}$$

where x represents the membrane potential, and y and z are associated with fast and slow currents, respectively. With appropriate parameter settings, the HR neuron model can exhibit most of biological neuron behavior, such as spiking or bursting, as shown in Figure 1. The HR system can be denoted as $\dot{X} = f(X)$, where $X = (x, y, z)$ and $f(X)$ represents the right functions.

2.2 Mathematical background

In order to derive the main results, the following mathematical expressions are required.

Definition 1 ([30, 31])**.** A function $\varphi : R^+ \to R^{n\times m}$ is PE if there exist $T_0, \delta_1, \delta_2 > 0$ such that

$$\delta_1 I_n \leqslant \int_t^{t+T_0} \varphi(\tau)\varphi^{\mathrm{T}}(\tau)\mathrm{d}\tau \leqslant \delta_2 I_n \tag{2}$$

holds for all $t \geqslant 0$.

Remark 1. The PE condition is a key factor in the topology identification of complex networks. Some researchers also propose the linearly independent condition on synchronization manifold. The first condition can imply the second one (refer to the proof in [8]).

Assumption 1 ([32])**.** There exists a positive definite matrix P and a constant L, such that

$$(\tilde{X} - X)^{\mathrm{T}} P(f(t, \tilde{X}) - f(t, X)) \leqslant L(\tilde{X} - X)^{\mathrm{T}}(\tilde{X} - X) \tag{3}$$

for any $X, \tilde{X} \in R^m$ and $t \geqslant t_0$.

Lemma 1 ([33])**.** Given a system of the following form:

$$\begin{cases} \dot{e}_1 \, = \, g(t)e_2 + f_1(t); \; e_1 \in R^p, e_2 \in R^q, \\ \dot{e}_2 \, = \, f_2(t), \end{cases} \tag{4}$$

such that

(i) $\lim_{t\to\infty}\|e_1(t)\|=0$, $\lim_{t\to\infty}\|f_1(t)\|=0$, $\lim_{t\to\infty}\|f_2(t)\|=0$;
(ii) $g(t),\dot{g}(t)$ are bounded, and $g^{\mathrm{T}}(t)$ is PE;
then $\lim_{t\to\infty}\|e_2(t)\|=0$.

3 Topology identification of a complex network using a nonlinear coupling function

Let us consider a network comprising HR systems with nonlinear coupling, which is described by

$$\begin{cases}\dot{x}_i = ax_i^2 - x_i^3 + y_i - z_i + \sum_{j=1}^N A_{ij}h(x_i,x_j) + r_1(X_i),\\ \dot{y}_i = (a+\alpha)x_i^2 - y_i + r_2(X_i),\\ \dot{z}_i = \rho(bx_i + c - z_i) + r_3(X_i),\end{cases} \tag{5}$$

where $X_i=(x_i,y_i,z_i)$ and the coupling function h is given by

$$h(x_i,x_j)=\epsilon\frac{x_i-V}{1+\exp(-\lambda(x_j-\Theta))}, \tag{6}$$

where ϵ is the coupling strength, $\Theta=-0.25$ is the threshold reached by every action potential for a neuron, $V=2$ is the reversal potential and $\lambda=10$. $R(X_i)=(r_1(X_i),r_2(X_i),r_3(X_i))$ is a disturbance function. Suppose the system is bounded, we can easily derive the following assumption using the mean value theorem for the function h.

Assumption 2. There exist positive constants L_1 and L_2 such that

$$|h(\tilde{x}_i,\tilde{x}_j)-h(x_i,x_j)|\leqslant L_1|\tilde{x}_i-x_i|+L_2|\tilde{x}_j-x_j|, \tag{7}$$

for all x_i and x_j.

We design the response network as

$$\begin{cases}\dot{\tilde{x}}_i = a\tilde{x}_i^2 - x_i^3 + \tilde{y}_i - \tilde{z}_i + \sum_{j=1}^N d_{ij}h(\tilde{x}_i,\tilde{x}_j) - k_i(\tilde{x}_i-x_i) + r_1(\tilde{X}_i),\\ \dot{\tilde{y}}_i = (a+\alpha)\tilde{x}_i^2 - \tilde{y}_i - k_i(\tilde{y}_i-y_i) - k_i(\tilde{y}_i-y_i) + r_2(\tilde{X}_i),\\ \dot{\tilde{z}}_i = \rho(b\tilde{x}_i + c - \tilde{z}_i) - k_i(\tilde{z}_i-z_i) - k_i(\tilde{z}_i-z_i) + r_3(\tilde{X}_i),\end{cases} \tag{8}$$

where $\tilde{X}_i=(\tilde{x}_i,\tilde{y}_i,\tilde{z}_i)$, with the adaptive laws

$$\begin{cases}\dot{d}_{ij} = -\delta_{ij}(\tilde{x}_i-x_i)h(\tilde{x}_i,\tilde{x}_j),\\ \dot{k}_i = s_ie_i^{\mathrm{T}}e_i, \qquad i=1,\dots,N,\end{cases} \tag{9}$$

where $e_i=(\tilde{x}_i-x_i,\tilde{y}_i-y_i,\tilde{z}_i-z_i)^{\mathrm{T}}$.

Theorem 1. Suppose that the HR system functions $f(X)$ and the disturbance functions $R(X)$ satisfy Assumption 1, and the coupling function satisfies Assumption 2. If for any i, $g_i^{\mathrm{T}}(t)=[h(x_i,x_1),\dots, h(x_i,x_N)]^{\mathrm{T}}$ is PE, and $\dot{g}_i(t)$ is bounded, then $\lim_{t\to\infty}(d_{ij}(t)-A_{ij})=0$.

Proof. Choose a Lyapunov candidate as

$$V=\frac{1}{2}\sum_{i=1}^N e_i^{\mathrm{T}}e_i+\frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N\frac{1}{\delta_{ij}}(d_{ij}(t)-A_{ij})^2+\frac{1}{2}\sum_{i=1}^N\frac{1}{s_i}(k_i(t)-k^*)^2,$$

where k^* is a positive constant to be determined. Moreover, the derivative of V along trajectories of the dynamical network (5) and (8) is given by

$$\dot{V}=\sum_{i=1}^N e_i^{\mathrm{T}}(f(\tilde{X}_i)-f(X_i))+\sum_{i=1}^N\sum_{j=1}^N d_{ij}e_{ix}h(\tilde{x}_i,\tilde{x}_j)-\sum_{i=1}^N\sum_{j=1}^N A_{ij}e_{ix}h(x_i,x_j)$$

$$
\begin{aligned}
&-\sum_{i=1}^{N}\sum_{j=1}^{N} e_{ix}((d_{ij}-A_{ij})h(\tilde{x}_i,\tilde{x}_j)+\sum_{i=1}^{N} e_i^{\mathrm{T}}(R(\tilde{X}_i)-R(X_i))-\sum_{i=1}^{N} k^* e_i^{\mathrm{T}} e_i\\
&\leqslant \sum_{i=1}^{N}(L+\tilde{L})e_i^{\mathrm{T}}e_i+\sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij}e_{ix}(h(\tilde{x}_i,\tilde{x}_j)-h(x_i,x_j))-k^*\sum_{i=1}^{l} e_i^{\mathrm{T}}e_i\\
&\leqslant \sum_{i=1}^{N}(L+\tilde{L})e_i^{\mathrm{T}}e_i+\sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij}e_{ix}(L_1|e_{ix}|+L_2|e_{jx}|)-k^*\sum_{i=1}^{N} e_i^{\mathrm{T}}e_i\\
&\leqslant \sum_{i=1}^{N}(L+\tilde{L})e_i^{\mathrm{T}}e_i+\sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij}[(L_1+\frac{L_2}{2})e_{ix}^2+\frac{L_2}{2}e_{jx}^2]-k^*\sum_{i=1}^{N} e_i^{\mathrm{T}}e_i.
\end{aligned}
$$

As elements of the matrix A are constant, we denote $A_M=\max_{i,j=1,\ldots,N}\{|A_{ij}|\}$, and obtain

$$
\dot{V}\leqslant\sum_{i=1}^{N}\left(NA_M(L_1+L_2)+L+\tilde{L}-k^*\right)e_i^{\mathrm{T}}e_i. \tag{10}
$$

Thus, letting $NA_M(L_1+L_2)+L+\tilde{L}+1=k^*$, we obtain

$$
\dot{V}\leqslant-\sum_{i=1}^{N}e_i^{\mathrm{T}}e_i, \tag{11}
$$

which means that $\|E\|$ and $|d_{ij}(t)-A_{ij}|$ are bounded, where $E^{\mathrm{T}}(t)=(e_1,e_2,\ldots,e_N)$. From Eq. (11), if for any t,

$$
\int_0^t E^{\mathrm{T}}(\tau)E(\tau)\mathrm{d}\tau\leqslant-\int_0^t\dot{V}(\tau)\mathrm{d}\tau=(V(0)-V(t))\leqslant V(0),
$$

then $E(t)\in L^2$ and thus $e_i(t)\in L^2$.

From Assumptions 1 and 2, it is seen that $\dot{e}_i$ $(i=1,\ldots,N)$ are bounded, it follows from Barbalat's lemma [30] that $\lim_{t\to\infty}\|e_i(t)\|=0$ for $i=1,\ldots,N$.

Note that

$$
\begin{aligned}
\dot{e}_i&=f(\tilde{X}_i)-f(X_i)+R(\tilde{X}_i)-R(X_i)+\sum_{j=1}^{N}A_{ij}(h(\tilde{x}_i,\tilde{x}_j)-h(x_i,x_j))\\
&\quad-k_ie_i+[h(\tilde{x}_i,\tilde{x}_1),\ldots,h(\tilde{x}_i,\tilde{x}_N)]\zeta_i,\\
\dot{\zeta}_i&=-[\delta_{i1}e_{1x}h(\tilde{x}_i,\tilde{x}_1),\ldots,\delta_{iN}e_{ix}h(\tilde{x}_i,\tilde{x}_N)]^{\mathrm{T}},
\end{aligned}
$$

where $\zeta_i=[d_{i1}(t)-A_{i1},\ldots,d_{iN}(t)-A_{iN}]^{\mathrm{T}}$. Let $\eta_i(t)=f(\tilde{X}_i)-f(X_i)+R(\tilde{X}_i)-R(X_i)+\sum_{j=1}^{N}A_{ij}(h(\tilde{x}_i,\tilde{x}_j)$ $-h(x_i,x_j))-k_ie_i$. It is obvious that $\lim_{t\to\infty}\eta_i(t)=0$ as $\lim_{t\to\infty}[\tilde{X}_i(t)-X_i(t)]=\lim_{t\to\infty}e_i(t)=0$.

Because $g_i^{\mathrm{T}}(t)=[h(\tilde{x}_i,\tilde{x}_1),\ldots,h(\tilde{x}_i,\tilde{x}_N)]^{\mathrm{T}}$ is PE, and $\dot{g}_i(t)$ is bounded. From Lemma 1, we can conclude that $\lim_{t\to\infty}(d_{ij}(t)-A_{ij})=0$ for all $i,j=1,2,\ldots,N$.

Remark 2. From (11), we only obtain that $\dot{V}$ is semi-definition. Therefore, it does not imply that $|d_{ij}(t)-A_{ij}|\to 0$. In order to show that the identification error tends to zero, we used Lemma 1 to complete the proof.

Remark 3. As the coupling strength increases, the drive network (5) can achieve inner synchronization. Therefore, the PE condition is not satisfied and the identification fails.

4 Topology identification of a complex network using disturbance

From Theorem 1, we know that the PE condition plays an important role in topology identification. However, estimating the unknown topology of complex networks when the condition is not satisfied is a problem that requires a solution.

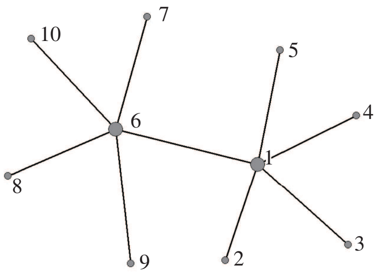

Figure 2 Star-shaped network.

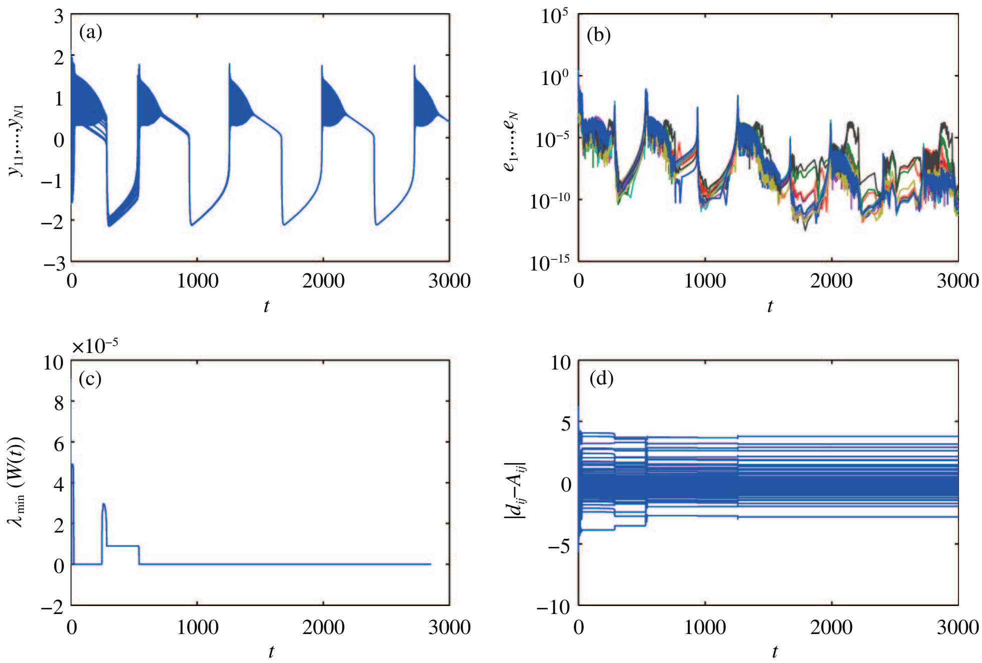

Figure 3 (Color online) Network size is $N = 10$ without disturbance, and we use the parameters of HR models which are given by Eq. (12). (a) The trajectory of the first variable in the response network, where $\epsilon = 0.5$ in (5); (b) the synchronization error between the drive network and the response network; (c) the minimum eigenvalue of the matrix $W_i(t)$; (d) parameters identification errors.

In order to test the application of sinusoidal disturbances in topology identification, we simulated a network of $N = 10$ bidirectionally coupled HR systems. The structure of the network is shown in Figure 2. There are two important nodes in this network and we call it a star-shaped network. To illustrate the necessity of pinning control, we discuss three perturbation styles that induce topology identification: pinning disturbance on the smallest nodes, pinning disturbance on the largest nodes, and pinning disturbance on all the nodes. The parameters of HR models used throughout the numerical simulations are as follows:

$$a = 2.8, \quad \alpha = 1.6, \quad c = 5, \quad b = 9, \tag{12}$$

$$V = 2, \quad \lambda = 10, \quad \Theta = 0.25. \tag{13}$$

First, consider the complex network comprising 10 HR models without disturbance. The initial condition of the drive and response networks are randomly assigned in the interval $[-2, 2]$ and let $k_i(0) = 0, d_{ij}(0) = 0$. By the adaptive control, Eqs. (5) and (8) can achieve inner synchronization with time, as shown in Figure 3(a). Figure 3(b) shows that the HR models reach synchronization at a very small coupling strength ($\epsilon = 0.1$) in the drive complex network (5). Similar to previous studies [7,8], we observed that the inner synchronization of the drive network is an obstacle faced during the topology

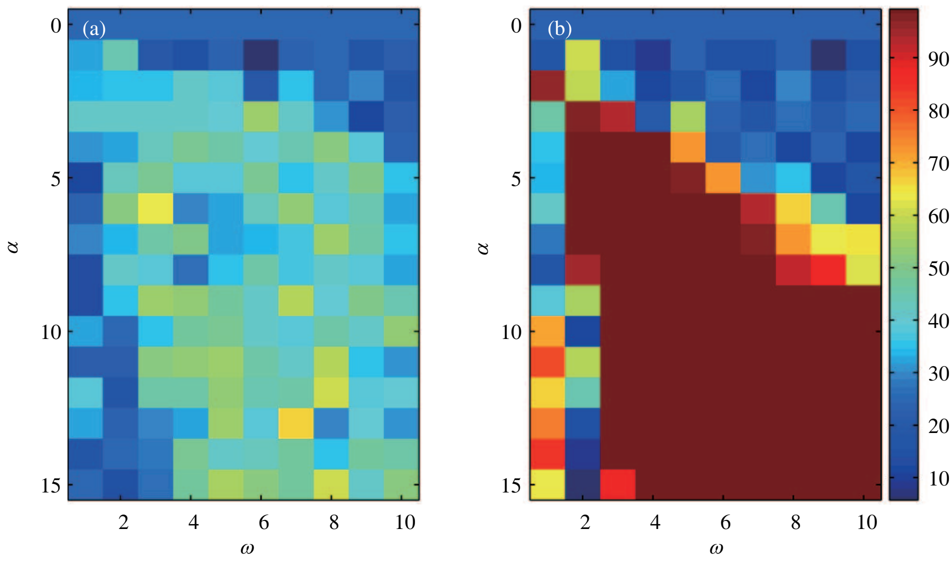

Figure 4 (Color online) Identification rate under the different values of the parameters α, β. (a) Disturb two smallest nodes; (b) disturb two largest nodes.

identification of complex networks. Define $W_i(t) = \int_t^{t+T_0} g_i(\tau)g_i^{\rm T}(\tau){\rm d}\tau$. Figure 3(c) shows that the minimum value of matrix $W_i(t)$ is very close to zero; thus the PE condition is not satisfied. Therfore, the identification procedure is unsuccessful, as shown in Figure 3(d).

Then, using the nonlinear disturbance, $R(X_i) = (\alpha\sin(x_i/\beta), 0, 0)$, only the first variable is disturbed for each node. We now demonstrate how the disturbance leads to a successful identification of the topology. To analyze the influence of the parameters α, β, we calculated the identification success ratio for different values. The different colors represent different the identification rates. Compared with the standard values on the right side of Figure 4, we can easily get the identification success rate. From Figure 4(a), it is seen that identification cannot be achieved by disturbing the two smallest nodes alone. However, when the two largest nodes are disturbed, the topology can be successfully identified, see the result in Figure 4(b). With an increase in the value of α, the amplitude of the sine function increases. The difference among the variables becomes large, which makes it difficult to achieve inner synchronization in the drive network. Therefore, the PE condition is satisfied, and identification is successful. However, if the value of parameter α is very large, the disturbance term will have a major role in the HR system, and will change the nature of the system. Moreover, when the value of the parameter β increases, x/β remains between 0 and $\pi/2$, and the sine function is an increasing function. Further, the difference between the variables increases, and make it easy to satisfy the PE condition, thus making the identification is more successful. However when the value of parameter β becomes very large, the function $\sin(x/\beta)$ approaches zero and the disturbance vanishes. Therefore, the topology identification fails.

In order to demonstrate the effect of pinning disturbance, we disturbed all nodes in the network, and the resultant identification success ratio is shown in Figure 5(a). Compared with complete disturbance, we observed that only pinning two key nodes could result in a 70% identification success ratio. This shows that the target disturbance is an effective method. Furthermore, the adaptive law d_{ij} is robust. If the network is attacked at some point in time, some links are broken. However, the topology is still identified successfully, as shown in Figure 5(b).

5 Conclusion

In this work, we investigated topology identification of complex networks with HR systems. Topology identification of the complex networks with nonlinear coupling is shown. Especially, induced topology identification using sine disturbance when the PE condition is not satisfied is discussed. Note that the

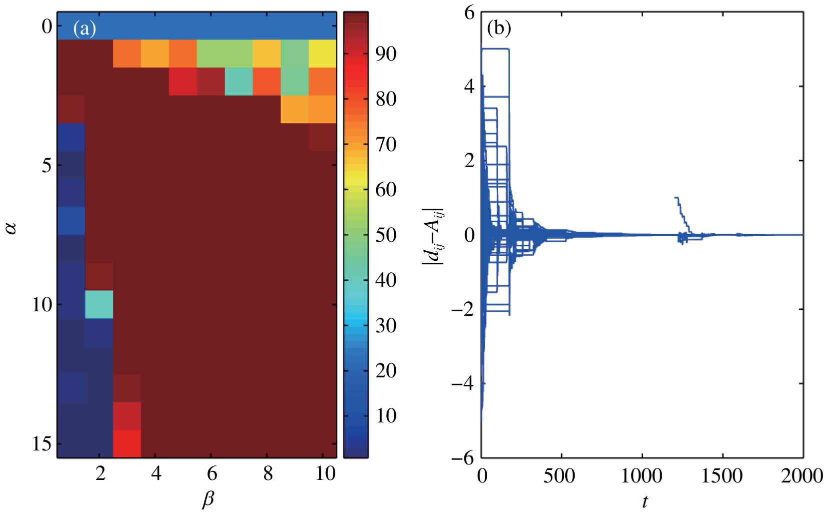

Figure 5 (Color online) (a) Identification rate under the different values of the parameters α, β by disturbing the total nodes; (b) one link of the network is broken at $t = 1200$ s.

disturbance of key nodes is an effective method to achieve identification.

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Conflict of interest The authors declare that they have no conflict of interest.

References

1 Watts D J, Strogatz S H. Collective dynamics of 'small-world' networks. Nature, 1998, 391: 440–442

2 Barabasi A L, Albert R. Emergence of scaling in random networks. Science, 1999, 286: 509–512

3 Pecora L M, Carroll T L. Master stability function for synchronized coupled systems. Phys Rev Lett, 1998, 80: 2109–2112

4 Belykh V N, Belykh I V, Hasler M. Connection graph stability method for synchronized coupled chaotic systems. Phys D, 2004, 195: 159–187

5 Belykh I V, Belykh V N, Hasler M. Generalized connection graph method for synchronization in asymmetrical networks. Phys D, 2006, 224: 42–51

6 Lu J, Cao J D. Synchronization-based approach for parameters identification in delayed chaotic neural networks. Phys A, 2007, 382: 672–682

7 Chen L, Lu J, Tse C K. Synchronization: an obstacle to identification of network topology. IEEE Trans Circ Syst-II, 2009, 56: 310–314

8 Zhao J C, Li Q, Lu J A, et al. Topology identification of complex dynamical networks. Chaos, 2010, 20: 023119

9 Ren J, Wang W, Li B, et al. Noise bridges dynamical correlation and topology in coupled oscillator networks. Phys Rev Lett, 2010, 104: 058701

10 Wu X Q, Wang W H, Zheng W X. Inferring topologies of complex networks with hidden variables. Phys Rev E, 2012, 86: 046106

11 Zhang S N, Wu X Q, Lu J A, et al. Recovering structures of complex dynamical networks based on generalized outer synchronization. IEEE Trans Circ Syst I, 2014, 61: 3216–3224

12 Li G J, Wu X Q, Liu J, et al. Recovering network topologies via Taylor expansion and compressive sensing. Chaos, 2015, 25: 043102

13 Wu X Q, Zhao X Y, Lü J H, et al. Identifying topologies of complex dynamical networks with stochastic perturbations. IEEE Trans Control Netw Syst, 2015, 21: 1–10

14 Mukeshwar D, Jirsa V, Ding M. Enhancement of neural synchrony by time delay. Phys Rev Lett, 2004, 92: 074104

15 Wang Z, Shi X. Lag synchronization of multiple identical Hindmarsh-Rose neuron models coupled in a ring structure. Nonlinear Dyn, 2010, 60: 375–383

16 Jia C, Wang J, Deng B, et al. Estimating and adjusting abnormal networks with unknown parameters and topology. Chaos, 2011, 21: 013109

17 Ehrich S, Pikovsky A, Rosenblum M. From complete to modulated synchrony in networks of identical Hindmarsh-Rose neurons. Eur Phys J Spec Top, 2013, 222: 2407–2416

18 Li C H, Yang S Y. Effects of network structure on the synchronizability of nonlinearly coupled Hindmarsh-Rose neurons. Phys Lett A, 2015, 379: 2541–2548

19 Li X, Wang X F, Chen G R. Pinning a complex dynamical network to its equilibrium. IEEE Trans Circ Syst I, 2004, 51: 2074–2087

20 Chen T, Liu X, Lu W. Pinning complex networks by a single controller. IEEE Trans Circ Syst I, 2007, 54: 1317–1326

21 Yu W W, Chen G R, Lü J H. On pinning synchronization of complex dynamical networks. Automatica, 2009, 45: 429–435

22 Xiang L Y, Liu Z X, Chen Z Q, et al. Pinning weighted complex networks with heterogeneous delays by a small number of feedback controllers. Sci China Ser F-Inf Sci, 2008, 51: 511–523

23 Zhao J C, Lu J A, Wu X Q. Pinning control of general complex dynamical networks with optimization. Sci China Inf Sci, 2010, 53: 813–822

24 Zheng C, Cao J. Robust synchronization of coupled neural networks with mixed delays and uncertain parameters by intermittent pinning control. Neurocomputing, 2014, 141: 153–159

25 Hodgkin A L, Huxley A F. A quantitative description of membrane current and its application to conduction and excitation in nerve. J Phys, 1952, 117: 500–544

26 Hindmarsh J L, Rose R M. A model of the nerve impulse using two first-order differential equations. Nature, 1982, 296: 162–164

27 Hindmarsh J L, Rose R M. A model of neuronal bursting using three coupled first order differential equations. Proc Royal Soc London, 1984, 221: 87–102

28 Corson N, Aziz-Alaoui M A. Asymptotic dynamics for slow-fast Hindmarsh-Rose neuronal system, dynamics of continuous. Discrete Impulsive Syst Ser, 2009, 16: 535–549

29 Corson N, Aziz-Alaoui M A. Ghnemat R, et al. Modelling the dynamics of complex interaction systems: from morphogenesis to control. Int J Bifurcation Chaos, 2012, 51: 112–120

30 Khalil H. Nonlinear Systems. 2nd ed. Englewood Cliffs: Prentice Hall, 1996

31 Sastry S, Bodson M. Adaptive Control-Stability, Convergence, and Robustness. Englewood Cliffs: Prentice Hall, 1989

32 Lu W L, Chen T P, Chen G R. Synchronization analysis of linearly coupled systems described by differential equations with a coupling delay. Phys D, 2006, 221, 118–134

33 Besançon G. Remarks on nonlinear adaptive observer design. Syst Control Lett, 2000, 41: 271–280