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Pricing the spare bandwidth: towards maximizing data center's profit

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Abstract To provide tenants with predictable network performance, tenants are allowed to purchase specified amount of data center network (DCN) resources (e.g., bandwidth). Then, data center will reserve the purchased resources for the corresponding tenants. Meanwhile, some DCNs are work conserving, which means that the spare bandwidth will be fairly shared by all active tenants. Even though the amount of the spare bandwidth is stochastic and uncertain, DCN providers are not likely to give away it for free. Thus, data center with work conserving may impose extra payment on tenants for using the spare bandwidth. In this paper, we propose a suitable tariff to charge for the usages of DCN resources, which includes a bill for the usage of the spare bandwidth. Through theoretical analysis and simulation, we demonstrate that our tariff can incentivize tenants to adjust their purchases of bandwidths, which can lead to the improvement of data center's profit by 27.4% without impairing the social welfare.

Keywords spare bandwidth, data center network (DCN), bandwidth reservation, work conserving, pricing/tariff

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1 Introduction

Traditionally, data center provides its tenants with specified number of central processing unit (CPU), volatile memory capacity (MEM) and non-volatile storage capacity (DSK) (e.g., Amazon EC2). Since data center network (DCN) resources are constituted by multiple individual resources (e.g., bandwidth) and shared by all tenants, it is difficult to reserve specified amount of DCN resources for tenants while remaining high resource utilization. Thus, most data centers do not offer bandwidth guarantee, which will lead to unpredictable network performance [1,2]. However, for some applications (e.g., trading), they need guaranteed network performance to ensure that their quality of service (QoS) requirements can be met.

To attract tenants with strict QoS requirements, there is a booming trend of research about realizing predictable network performance (i.e., min-bandwidth guarantee) [3,4]. Most of them need pre-assumption or pre-testing of tenants' demands. Then, each tenant can make an agreement on a service

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contract with data center, which may contain the tenant's QoS requirements and the tariff (i.e., the pricing policy for data center services). According to the terms described in the contract, data center will pre-allocate/pre-reserve specified amount of DCN resources for the tenant before processing its jobs. Note, however, most tenants usually are not familiar with their demands for bandwidth [5]. Thus, we assume that data center is responsible for pre-testing tenants' demands.

In this paper, bandwidth of data center is divided into two parts: guaranteed/purchased/reserved bandwidth and spare bandwidth. The former is purchased by tenants and is reserved for them. The tenants can utilize their purchased bandwidth irrespective of other tenants' behaviors. The latter is either the leftover/unsold bandwidth or the bandwidth that is purchased but not used. For data center that is work conserving, its links are either fully utilized, or they satisfy the tenants' demands [6]. In this case, the spare bandwidth will be fairly shared by all tenants. Apparently, data center's resource utilization can be improved by employing work conserving. However, if data center gives away the spare bandwidth for free, the greedy tenants may be trained to expect the free resource [7]. Namely, the tenants may strategically purchase less bandwidth but utilize as much spare bandwidth as possible. Therefore, data center is not likely to give away its spare bandwidth for free.

We propose a compound tariff for data center to charge for the usage of bandwidth. It contains two parts: flat rate pricing that is used to charge for the purchase of guaranteed bandwidth, and usage based pricing that is used to charge for the usage of spare bandwidth. With our tariff, tenants are allowed to purchase specified amount of the guaranteed bandwidth. For any rational tenant, it will choose an optimal purchase of the guaranteed bandwidth that can minimize its cost while maximizing its utility (i.e., tenant's valuation for data center services). Namely, rational tenant will try to maximize its surplus (i.e., utility minus cost). Our main contributions in the remaining of this paper are listed as follows:

- Proposing a novel modeling of a tenant's utility, which not only depends on the sending rate of the tenant's data, but relates to the probability that the tenant's demand for bandwidth is fully satisfied.
- Analyzing a tenant's purchase of the guaranteed bandwidth when it can obtain stochastic amount of the spare bandwidth. We demonstrate how a tenant's purchase of the guaranteed bandwidth can be effected by its utility factors, the available spare bandwidth and the prices of guaranteed and spare bandwidths.
- Evaluating our tariff that is used to charge for the usage of bandwidth via theoretical study and simulations. The results show that our proposed tariff can improve a data center's profit while remaining the social welfare unimpaired.

2 Related work

In recent years, many researches about the characteristics of load of data center [8,9] and traffic of Cloud applications [5] have emerged. In [8], Ersoz et al. found that the inter-arrival rates of requests and the message sizes usually follow the log-normal distribution. Benson discovered that the traffic of the edge of data center is periodical ON and OFF and the periods of ON and OFF and the traffic arrival interval could be modeled by log-normal distribution [9].

To help data center pre-allocate DCN resources, Ref. [3] proposed the Proteus system. Before joining data center, tenants' applications will be profiled under different configurations. After profiling, data center provides tenants with different configurations and the corresponding tariffs. Furthermore, Niu et al. announced that a tenant's demand for bandwidth could be forecasted as a Gaussian process [5]. Therefore, we believe that data center has the expertise to understand and model the demands of specified tenants or applications.

With precise prediction of tenants' demands, data center can set up an adaptive mechanism for pre-allocating DCN resources. However, it also needs a suitable pricing to bill tenants for the usage of DCN resources. Generally, the pricing can be simply categorized to static pricing and dynamic pricing [10]. Traditionally, tenants prefer the static pricing such as flat-rate based and usage based, since it is easy to understand and predict the cost. Additionally, complicated tariffs such as dynamic pricing may be tend to be manipulated by data center operators [11]. Thus, the tariff proposed in this paper is static.

3 Utility function

In this paper, we adopt hose modeled data center (e.g., ElasticSwitch [4] and Gatekeeper [12]). Before processing tenants' jobs, data center needs to profile tenants'/applications' demands for bandwidth and report the results to the corresponding tenants (such as [3, 5]). Then, tenants purchase specific amount of the guaranteed bandwidth for each of their purchased VMs according to their "willingness to pay" [13] and the estimations of their demands. Note that, if a tenant's purchase exceeds the amount of unsold bandwidth, the request for the guaranteed bandwidth will be rejected. Meanwhile, data center may set a positive gap between the network capacity and the maximum amount of bandwidth that can be reserved to relieve congestion (e.g., Elasticswitch [4]). In this case, there are always some spare bandwidth.

We analyse a data center with N tenants, $\forall i \in \{1, \dots, N\}$. Let G_i (Mbps) denote tenant i's purchase of guaranteed bandwidth. Meanwhile, the spare bandwidth that can be obtained by tenant i is denoted by X_i (Mbps), which is modeled by a random variable in this paper. Let D_i (Mbps) be tenant i's demands for bandwidth. Like some prior work [14], we assume that the utility function of tenant i is proportional to the bandwidth used by this tenant, and the tenant's utility is denoted by U_i. Apparently, the spare bandwidth and the guaranteed bandwidth can contribute the same to the sending rate. Additionally, G_i+X_i may be higher than D_i. In this case, tenant i will consume only D_i (Mbps) bandwidth. Therefore, we suppose that U_i is proportional to the minimum of the expectation of $G_i + X_i$ and D_i.

Further, we believe that the utility of tenant i monotonously increase with the possibility that its demand is fully satisfied. Let the possibility of $G_i + X_i \geqslant D_i$ be $\Pr(G_i + X_i \geqslant D_i)$ and associate tenant i's utility U_i with it. Then, an assumption is made as follows: $\forall i \in \{1, \dots, N\}$, the utility function $U_i(\Pr(G_i + X_i \geqslant D_i))$: $[0, 1] \to \Re_+$ of tenant i is increasing, bounded, convex and differentiable. For example, a web search application can obtain linear utility gain from expectation of obtained bandwidth (i.e., $\min(E(G_i + X_i), D_i)$) and the possibility (i.e., $\Pr(G_i + X_i \geqslant D_i)$). Thus, we define the utility function by Eq. (1), where $a_i \geqslant 0$ and $A_i \geqslant 0$ are both utility factors and decided by tenant itself,

$$U_i(G_i) = a_i \min(E(G_i + X_i), D_i) + A_i \Pr(G_i + X_i \geqslant D_i). \tag{1}$$

For simplicity, we let G_i and D_i be constant. While admittedly naive, we believe it is a good first-step of analysis of tenants' purchases of the guaranteed bandwidth. In this case,

$$E(G_i + X_i) = G_i + E(X_i),$$

$$\Pr(G_i + X_i \geqslant D_i) = 1 - F_{X_i}(D_i - G_i),$$

where $F_{X_i}(x)$ is the cumulative distribution function (CDF) of X_i. Meanwhile, we use $R_i(G_i)$ to denote the amount of bandwidth that is used by tenant i. Apparently,

$$R_i(G_i) = \min(G_i + E(X_i), D_i).$$

Obviously, $E(X_i) \geqslant 0$, $G_i \geqslant 0$ and $D_i \geqslant 0$. Additionally, $R_i(G_i)$ may change with time.

We charge tenant i for data center services based on Eq. (2), where $P_{\rm g}$ ($ per Mbps per hour) is the price of the guaranteed bandwidth and $P_{\rm s}$ ($ per Mbps per hour) is the price of the spare bandwidth. Meanwhile, both $P_{\rm g}$ and $P_{\rm s}$ are constant and decided by data center.

$$P_i(G_i) = P_{\rm g} G_i + P_{\rm s} \max(R_i(G_i) - G_i, 0). \tag{2}$$

Combining Eqs. (1) and (2), we can get tenant i's surplus $S_i(G_i)$, which is defined by Eq. (3):

$$S_i(G_i) = U_i(G_i) - P_i(G_i). \tag{3}$$

We assume that all tenants are rational. Namely, the goal of each tenant is to maximize its surplus, and the optimal purchase of the guaranteed bandwidth is defined as

$$G_i^* = \arg \max_{G_i \in [0,\infty)} S_i(G_i). \tag{4}$$

Next, we assume that the profit of data center Π can be calculated by Eq. (5), where $C(\sum_i G_i)$ is the cost of bandwidth reservation,

$$\Pi = \sum_i P_i(G_i) - C\Big(\sum_i G_i\Big). \tag{5}$$

Like [5], we relax the linear cost assumption $C(\sum_i G_i) = k\sum_i G_i$. In this case, we define the social welfare W as in Eq. (6):

$$W = \sum_i S_i(G_i) + \Pi = \sum_i U_i(G_i) - k\sum_i G_i. \tag{6}$$

4 Purchase optimization

In this paper, we analyse tenants' optimal purchases with the available spare bandwidth X_i being modeled by log-normal distribution, which is widely used to model the traffic of data center [8,9]. If the available spare bandwidth can be modeled by other random distribution, we can use the same method to analyse tenants' optimal purchases.

When X_i follows the log-normal distribution $\mathrm{In}\mathcal{N}(\mu_i, \sigma_i^2)$, the expectation of X_i is

$$E(X_i) = \mathrm{e}^{\mu_i + \sigma_i^2/2},$$

where μ_i is the expectation of X_i's natural logarithm and σ_i is its natural logarithm's standard deviation. Meanwhile,

$$F_{X_i}(x_i; \mu_i, \sigma_i) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\mathrm{In}(x_i) - \mu_i}{\sigma_i\sqrt{2}}\right)\right],$$

where $\mathrm{erf}(x)$ is the error function and defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-t^2}\mathrm{d}t.$$

Combining these equations with Eq. (1), we can obtain that tenant i's utility $U_i(G_i)$ is as shown in Eq. (7):

$$\begin{cases} a_i(G_i + E(X_i)) + A_i[1 - F_{X_i}(D_i - G_i)], & G_i \in [0, D_i - E(X_i)], \\ a_i D_i + A_i[1 - F_{X_i}(D_i - G_i)], & G_i \in [D_i - E(X_i), D_i), \\ a_i D_i + A_i, & G_i \in [D_i, \infty). \end{cases} \tag{7}$$

Note that, when $E(X_i) > D_i$. the domain $[0, D_i - E(X_i)]$ is empty set $\emptyset$.

Theorem 1. $\forall i \in \{1, \dots, N\}$, all of 0, $D_i - E(X_i)$, G_i' and D_i are possible optimal points of optimization problem:

$$\max\ S_i(G_i) \qquad \text{s.t. } G_i \geqslant 0, \tag{8}$$

where G_i' is defined by

$$\frac{\partial S_i(G_i')}{\partial G_i'} = 0. \tag{9}$$

By solving (9), we get

$$G_i' = D_i - \mathrm{e}^{\mu_i - \sigma_i^2}\mathrm{e}^{-\sigma_i\sqrt{\sigma_i^2 - 2\mu_i - 2\mathrm{In}\frac{\sqrt{2\pi}(P_\mathrm{g} - P_\mathrm{s})\sigma_i}{A_i}}}.$$

Note, however, if $D_i - E(X_i)$ or G_i' is not in the domain of $[0, \infty)$, it is not a potential optimal point of this optimization problem.

Theorem 2. $\forall i \in \{1, \ldots, N\}$,

$$G_i^* = \begin{cases} 0, & \text{if } S_i^* = S_i(0), \\ D_i - E(X_i), & \text{if } S_i^* = S_i(D_i - E(X_i)), \\ G_i', & \text{if } S_i^* = S_i(G_i'), \\ D_i, & \text{if } S_i^* = S_i(D_i), \end{cases} \tag{10}$$

with S_i^* being defined as the maximum surplus of the following cases:

$$\begin{cases} S_i(0) = (a_i - P_{\mathrm{s}}) \min(E(X_i), D_i) + A_i[1 - F(D_i)], \\ S_i(D_i - E(X_i)) = a_i D_i + A_i[1 - F(E(X_i))] - P_{\mathrm{g}}(D_i - E(X_i)) - P_{\mathrm{s}} E(X_i), \\ S_i(G_i') = a_i D_i + A_i[1 - F(D_i - G_i')] - P_{\mathrm{g}}(G_i') - P_{\mathrm{s}}(D_i - G_i'), \\ S_i(D_i) = a_i D_i + A_i - P_{\mathrm{g}} D_i. \end{cases} \tag{11}$$

Note, if $D_i - E(X_i) \notin [0, \infty)$, $S_i(D_i - E(X_i))$ is set to $-\infty$. Identically, if $G_i' \notin [0, \infty)$, $S_i(G_i') = -\infty$. Meanwhile, if $S_i^* < 0$, the tenant will not join the data center.

Proof [Proof of Theorem 1]. We divide the domain of optimization problem (8) into three parts: $\mathcal{D}_1 = [0, D_i - E(X_i)]$, $\mathcal{D}_2 = [D_i - E(X_i), D_i)$ and $\mathcal{D}_3 = [D_i, \infty)$. Further, the optimization problem (8) can be divided into three sub-optimization problems, accordingly. Apparently, all domains $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ are convex set and the corresponding sub-objective function is twice differentiable.

As seen, when $G_i \in \mathcal{D}_1 \bigcup \mathcal{D}_2$,

$$\frac{\partial^2 S_i(G_i)}{\partial G_i^2} = \frac{A_i \mathrm{e}^{-\left(\frac{\mathrm{In}(D_i - G_i) - \mu_i}{\sqrt{2}\sigma_i}\right)^2} (\mathrm{In}(D_i - G_i) - \mu_i + \sigma^2)}{\sqrt{2\pi}\sigma_i^3 (D_i - G_i)^2}. \tag{12}$$

Thus, we can get

$$\frac{\partial^2 S_i(G_i)}{\partial G_i^2} \begin{cases} > 0, & G_i < D_i - \mathrm{e}^{\mu_i - \sigma_i^2}, \\ = 0, & G_i = D_i - \mathrm{e}^{\mu_i - \sigma_i^2}, \\ < 0, & G_i > D_i - \mathrm{e}^{\mu_i - \sigma_i^2}. \end{cases} \tag{13}$$

Based on this knowledge, we investigate the possible optimal points of the sub-optimization problem when $G_i \in \mathcal{D}_1$ and $D_i \geqslant E(X_i)$ (otherwise, $\mathcal{D}_1 = \emptyset$). Apparently, in this case, $\partial^2 S_i(G_i)/\partial G_i^2 > 0$. Thus, all possible relationships between $S_i(G_i)$ and G_i are illustrated by Figure 1.

Specifically, when $G_i = 0$, if $\partial S_i(G_i)/\partial G_i \geqslant 0$, i.e., $a_i + H(D_i) - P_{\mathrm{g}} \geqslant 0$, where

$$H(x) = \frac{A_i}{\sqrt{2\pi}\sigma_i x} \mathrm{e}^{-\left(\frac{Inx - \mu_i}{\sqrt{2}\sigma_i}\right)^2}.$$

$S_i(G_i)$ is monotone increasing, which can be represented by Figure 1(a). In this case, the optimal point of this sub-optimization problem is $D_i - E(X_i)$. On the contrary, if $\partial S_i(G_i)/\partial G_i \leqslant 0$ when $G_i = D_i - E(X_i)$, i.e., $a_i + H(E(X_i)) - P_{\mathrm{g}} \leqslant 0$, the best choice for the tenant is to buy no guaranteed bandwidth (refer to Figure 1(b)). Further, if the function $S_i(G_i)$ is not monotonous (e.g., as in Figure 1(c)), both 0 and $D_i - E(X_i)$ can be the optimal point of sub-optimization problem 1. In this case, we just need to compare $S_i(0)$ with $S_i(D_i - E(X_i))$ to get the optimal point. For example, if $S_i(0) > S_i(D_i - E(X_i))$, tenant will not buy any guaranteed bandwidth.

For the sub-optimization problem when $G_i \in \mathcal{D}_2$, $\partial S_i(G_i)/\partial G_i$ can be either greater or less than 0. However, since $G_i' < D_i$, we can ensure that $\partial S_i(G_i)/\partial G_i < 0$ when $G_i = D_i$. To better understand the feasible relationships between $S_i(G_i)$ and G_i when $G_i \in \mathcal{D}_2$, we use Figure 2 to illustrate them.

To be specific, if $\partial S_i(G_i)/\partial G_i \leqslant 0$ when $G_i = D_i - \exp(\mu_i - \sigma_i^2)$ (i.e., $H(\mathrm{e}^{\mu_i - \sigma_i^2}) - P_{\mathrm{g}} + P_{\mathrm{s}} \leqslant 0$), $S_i(G_i)$ is monotone decreasing function of G_i (refer to Figure 2(a)). Obviously, the optimal point is $D_i - E(X_i)$. Identically, if $\partial S_i(G_i)/\partial G_i \geqslant 0$ when $G_i = D_i - E(X_i)$ (i.e., $H(E(X_i)) - P_{\mathrm{g}} + P_{\mathrm{s}} \geqslant 0$), the optimal point

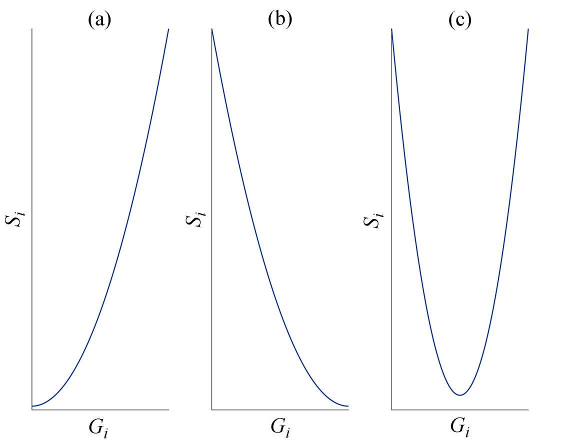

Figure 1 (Color online) Relationship between $S_i(G_i)$ and G_i when $G_i \in \mathcal{D}_1$ and (a) $\partial S_i(G_i)/\partial G_i \geqslant 0$ when $G_i = 0$; (b) $\partial S_i(G_i)/\partial G_i \leqslant 0$ when $G_i = D_i - E(X_i)$; (c) otherwise.

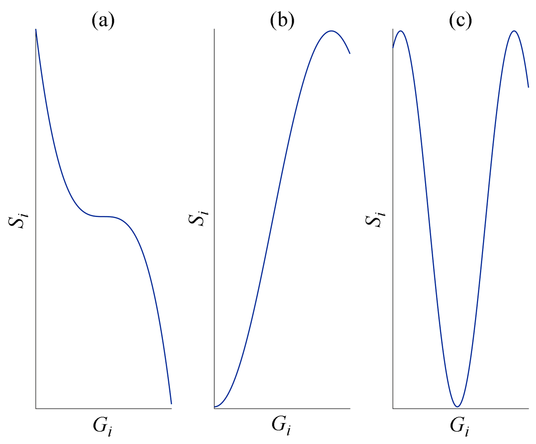

Figure 2 (Color online) Relationship between $S_i(G_i)$ and G_i when $G_i \in \mathcal{D}_2$ and (a) $\partial S_i(G_i)/\partial G_i \leqslant 0$ when $G_i = D_i - \exp(\mu_i - \sigma_i^2)$; (b) $\partial S_i(G_i)/\partial G_i \geqslant 0$ when $G_i = D_i - E(X_i)$; (c) otherwise.

will be G_i' as shown in Figure 2(b). Otherwise, if $H(E(X_i)) - P_{\rm g} + P_{\rm s} < 0$, there are two possible optimal points: $D_i - E(X_i)$ and G_i' as in Figure 2(c). In this case, when $S_i(D_i - E(X_i)) \geqslant S_i(G_i')$, tenant should purchase $D_i - E(X_i)$ guaranteed bandwidth and vice versa. Note, however, G_i' may not be valid (i.e., $G_i' \notin \mathcal{D}_2$). In this case, the optimal point is $D_i - E(X_i)$.

Finally, we analyse the optimal point of sub-optimization problem when $G_i \in \mathcal{D}_3$, where the objective function can be written as

$$S_i(G_i) = a_i D_i - P_{\rm g} G_i.$$

Since its feasible region $\mathcal{D}_3$ is convex set, it is a linear programming. Thus, when $P_{\rm g} > 0$, we can easily get its optimal point, which is D_i.

Proof [Proof of Theorem 2]. We assume that $\exists \tilde{G}_i \in [0, \infty)$, $\tilde{G}_i \notin \{0, D_i - E(X_i), G_i', D_i\}$ and $\tilde{G}_i = \arg\max_{G_i \in [0,\infty)} S_i(G_i)$. Firstly, we assume that $\tilde{G}_i \in \mathcal{D}_1$. According to Proof 1, $\tilde{G}_i = 0$ or $\tilde{G}_i = D_i - E(X_i)$, which violates the conditions of $\tilde{G}_i \neq 0$ and $\tilde{G}_i \neq D_i - E(X_i)$. Identically, we can prove that $\tilde{G}_i \notin \mathcal{D}_2$ and $\tilde{G}_i \notin \mathcal{D}_3$. Thus, there are no other possible optimal point of this optimization problem except for 0, $D_i - E(X_i)$, G_i' and D_i. In this case, we can calculate the optimal point based on Theorem 2.

5 Analysis of the optimal purchase

To better understand tenants' optimal purchases, we first evaluate the relationship between G_i^* and the factors such as the available spare bandwidth and the corresponding tenant's utility (i.e., μ_i, σ_i, a_i and A_i), the results are shown in Figure 3. By analyzing Figure 3, we can find the following information:

- Since tenants' demands for bandwidth are limited, when the amount of the spare bandwidth is high enough, tenants' purchases will monotonously decrease with the increasing of the available spare bandwidth.
- If a_i is relatively small, tenant may buy no guaranteed bandwidth but rely on the unreliable spare bandwidth. However, if the tenant does want to ensure that its demand for bandwidth can be fully satisfied (i.e., A_i is extremely high), it will try not to use the spare bandwidth but buy enough guaranteed bandwidth.

Then, we investigate the relationship between G_i^* and the prices (i.e., $P_{\rm g}$ and $P_{\rm s}$). Figure 4 shows the results. Apparently, we can obtain that:

- With the increase of $P_{\rm g}$, G_i^* will gradually decrease from D_i to 0. We can see that, when $P_{\rm g}$ is low enough, $S_i^* = S_i(D_i)$. With the increase of $P_{\rm g}$, both $S_i(D_i)$ and $S_i(D_i - E(X_i))$ are reduced. However,

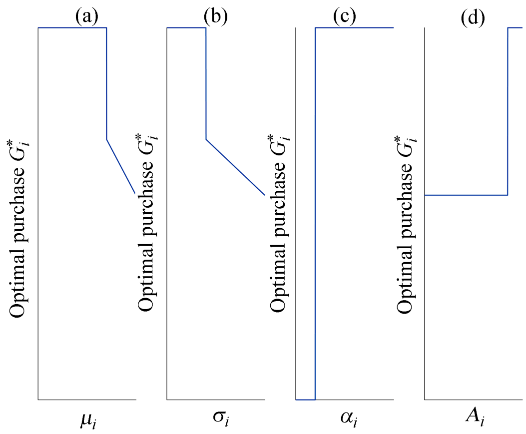

Figure 3 (Color online) Relationship between G_i^* and factors that can effect G_i^*: (a) μ_i; (b) σ_i; (c) a_i; (d) A_i.

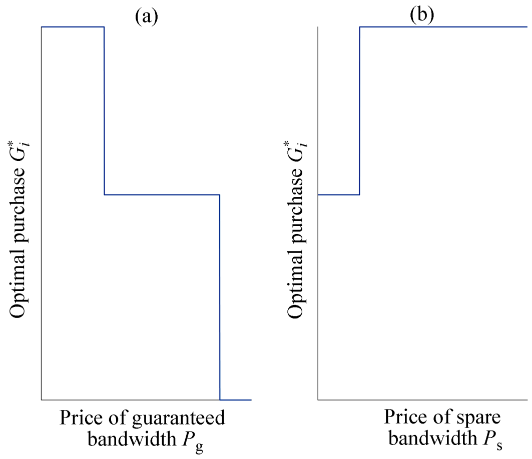

Figure 4 (Color online) Relationship between G_i^* and the pricing policy: (a) price of the guaranteed bandwidth $P_{\rm g}$; (b) price of the spare bandwidth $P_{\rm s}$.

the decrease rate of $S_i(D_i - E(X_i))$ is lower. Thus, when $P_{\rm g}$ is relatively large, $S_i^* = S_i(D_i - E(X_i))$. Since only $S_i(0)$ will not be impacted by the change of $P_{\rm g}$, when $P_{\rm g}$ is large enough, $S_i^* = S_i(0)$.

- With the increase of $P_{\rm s}$, G_i^* can be gradually increased. We can see that, when $P_{\rm s}$ is low enough, $S_i^* = S_i(D_i - E(X_i))$. Since only $S_i(D_i)$ is not impacted by the change of $P_{\rm s}$, when $P_{\rm s}$ is large enough, $S_i^* = S_i(D_i)$.

6 Simulation

We evaluate our tariff via simulations. The results show that our tariff can improve data center's profit without impairing the social welfare.

6.1 Simulation setup

We build a simulator with 10000 machines. In our simulator, each machine can hold 40 VMs and it is connected to the top of rack (ToR) switch via a 1 Gbps link. Our simulator involves 20000 potential tenants, each with different utility factors and demands (i.e., $\forall i, j \in \{1, \ldots, N\}$, if $i \neq j$, $a_i \neq a_j$ or/and $A_i \neq A_j$ and $D_i \neq D_j$). Note, however, tenants are always in and out of the data center (i.e., when a tenant's optimal surplus less than 0, the tenant will temporarily stop employing the data center's services). Meanwhile, when a tenant is out of data center, its cost and utility are both 0.

Before processing tenants' jobs, data center will not only profile the tenants' demands for bandwidth and report the results to the corresponding tenants, but also provide its tenants with the characteristics of the available spare bandwidth (e.g., the expectation and deviation of the amount of the available spare bandwidth). Then, tenants will decide their purchases of the guaranteed bandwidth. Especially, if the available spare bandwidth can be modeled by log-normal distribution, tenants can calculate their optimal purchases based on Eq. (10).

Note, the characteristics of the available spare bandwidth may be variable (e.g., its expectation and deviation may change with time) and tenants' demands for bandwidth can be variable, too. In this case, by adjusting tenants' purchases, the tenants' surpluses may can be improved. However, frequent adjustments of tenants' purchases are costly and impractical due to the complexity of management of service contract and bandwidth reservation. Thus, tenants will not allow to adjust their purchases when they see a upsurge or downturn of the available spare bandwidth or their real anticipated loads. Meanwhile, the modeling of the available spare bandwidth provided by data center should be obtained based on the analysis of the historical data instead of the real time network state.

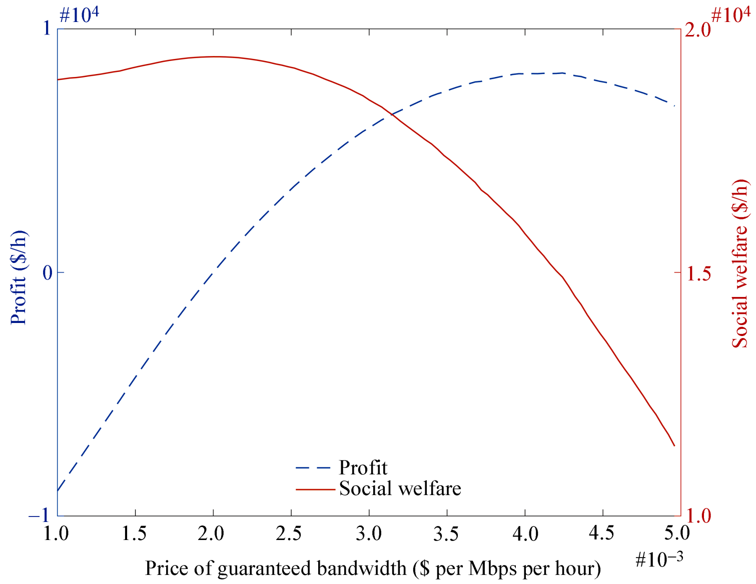

Figure 5 (Color online) The profit of data center with different valuations of $P_{\rm g}$.

We compare our mechanism against Baseline. Here, Baseline allows tenants to purchase guaranteed bandwidth at the price of $P_{\rm g}$ ($ per Mbps per hour) and the spare bandwidth is for free (i.e., $P_{\rm s}$ = $0 per Mbps per Hour). We assume that the provider of Baseline is a monopolist and its goal is to optimize its profit Π. According to [15], the profit is optimized when

$$\sum_i \left(G_i + P_{\rm g} \frac{\partial G_i}{\partial P_{\rm g}} - k \frac{\partial G_i}{\partial P_{\rm g}} \right) = 0. \tag{14}$$

However, data center always is not familiar with its tenants' utility functions. Thus, it cannot get the optimal price according to Eq. (14). In this paper, we assume that the marginal cost of bandwidth reservation is k = $0.002 per Mbps per hour (the marginal cost of bandwidth reservation can be set to other numbers, which will lead to the same conclusion).

With our simulator, we try hundreds of thousands valuations of $P_{\rm g}$ to find the optimal price of the guaranteed bandwidth that can maximize data center's profit. The result is shown in Figure 5. Therefore, we can find that the optimal profit of Baseline can be obtained when $P_{\rm g}$ = $0.00424 per Mbps per hour. Obviously, Baseline has to sacrifice the social welfare for maximizing its profit.

6.2 Results analysis

We first evaluate data center's profit and the social welfare when the available spare bandwidth's characteristics remain unchanged (i.e., $\forall i \in \{1, \ldots, N\}$, μ_i and σ_i are constant). Figure 6 shows the results. Here, Baseline's sets $P_{\rm g}$ = $0.00424 per Mbps per hour, which can maximize its profit. Moreover, Tariff 1 and Tariff 2 represent the tariff proposed in this paper, i.e., both Tariff 1 and Tariff 2 have imposed extra payment on tenants for utilizing the spare bandwidth, each with different prices of guaranteed and spare bandwidths.

Through analyzing Figure 6, we can get the following information:

- With our tariff, one can improve the social welfare without decreasing data center's profit (comparing Tariff 1 with Baseline).
- Identically, with our tariff, data center can improve its profit while remaining the social welfare unimpaired (comparing Tariff 2 with Baseline).

Then, we let the characteristics of the available spare bandwidth be variable (i.e., $\forall i \in \{1, \ldots, N\}$, μ_i and σ_i will change with time). Note, tenants are not allowed to change their purchases of the guaranteed bandwidth during running our simulator. Figure 7 shows the results, which shows that our tariff can

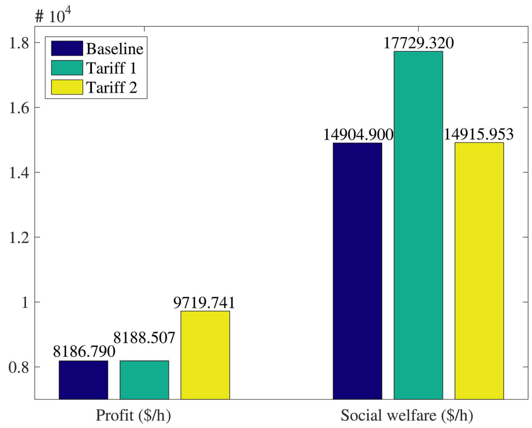

Figure 6 (Color online) Average profit and social welfare when the characteristics of the available spare bandwidth remains unchanged.

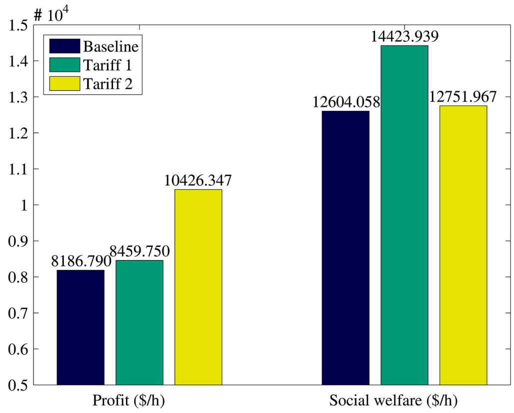

Figure 7 (Color online) Average profit and social welfare when the characteristics of the available spare bandwidth is variable.

also outperforms Baseline when the characteristics of the available spare bandwidth are variable. For example, data center with Tariff 2 can obtain 27.4% more profit and 1.2% more social welfare than the one with Baseline.

7 Conclusion

In this paper, a novel utility function is defined to measure tenant's valuation of data center services. Meanwhile, we propose a suitable tariff to charge for the usage of bandwidth, which is divided into two parts: a fee for bandwidth reservation and a fee for the usage of the spare bandwidth. Then, we investigate the tenant's optimal purchase of the guaranteed bandwidth when the available spare bandwidth is uncertain. Through theoretical analysis and simulation, we demonstrate that our tariff can improve data center's profit without impairing the social welfare.

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Conflict of interest The authors declare that they have no conflict of interest.

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