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Achievable degrees of freedom of MIMO two-way X relay channel with delayed CSIT

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Abstract The previous work on interference alignment for multiple-input-multiple-output (MIMO) two-way X relay channel assumes perfect channel state information at the transmitter (CSIT), which is reasonable in slow fading channel. However, in fast fading scenario, this assumption is impractical. In this paper, assuming that each node has delayed CSIT, we study the achievable degrees of freedom (DOF) for MIMO two-way X relay channel in frequency division duplex (FDD) systems. Specifically, in the broadcast (BC) phase, we propose a new multiple-stage transmission (MST) scheme, which utilizes retrospective interference alignment for physical layer network coding (PLNC). We show that MST can achieve significant DOF gain and tremendous power gain over other schemes. When the number of antennas for each user, N, is smaller than the number of the relays, M, the time division multiple access (TDMA) scheme can only achieve an ergodic sum-rate increase by N bps/Hz for every increasing of 3 dB of signal-to-noise power ratio (SNR), while the proposed MST scheme can achieve an ergodic sum-rate increase by M bps/Hz.

Keywords degrees of freedom, physical layer network coding, interference alignment, delayed CSIT, multiple-stage transmission

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1 Introduction

Wireless relay is an enabling technology for coverage extension, power reduction, throughput improvement, etc. It has been attracting intensive attention from both research community and industry. Nowadays, relaying technique is becoming one of the key ingredients of modern mobile communication systems (e.g., LTE-A, 5G) [1]. Nevertheless, the fundamental limits of relay networks (such as capacity characterization) have been long-standing open problems since its early studies [2–4], even in the simplest one-way relay setup. The high signal-to-noise ratio (SNR) approximation of channel capacity in the form of degrees-of-freedom (DOF) or multiplexing gain enables us to gain some insights with more tractable analysis.

In multi-user multi-hop wireless relay networks, how to deal with interference is the key to understand the fundamental limits of relaying. Xing et al. investigated a unified linear minimum mean-square-error

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(LMMSE) transceiver design framework, which can be applied to a variety of wireless networks, including multi-hop relay networks [5]. Later, Xing et al. considered the robust linear transceiver design for multi-hop amplify-and-forward (AF) multiple-input-multiple-output (MIMO) relay channels with channel estimation errors [6]. Recently, two interference mitigation techniques, interference alignment (IA) and physical layer network coding (PNC), were proposed to boost network performance of X channels and relay channels respectively at high SNR. Specifically, IA was first proposed to achieve the maximum DOF for the MIMO X channel [7, 8]. On the other hand, compared with conventional storage-and-forward scheme, PNC can double the spectrum utilization efficiency by performing simple bit 'xor' operation [9] at the relay. Recently, an interesting network information flow called MIMO two-way X relay channel was proposed by Xiang et al. [10], considering the combination of X and relay settings. In this MIMO two-way X relay channel model, there are two user groups and each group contains two users; each user in one group can exchange messages with all users in another group via a relay node, but cannot exchange messages with the other user in the same group. The DOF analysis of MIMO two-way X relay channel was given in [10] under the assumption that instantaneous global channel state information at the transmitter (CSIT) is available at all nodes.

However, in practical wireless systems, obtaining global and instantaneous CSIT is often challenging, especially when the transmitters are distributed and the channels are time-varying. As such, it is difficult to obtain channel state information via feedback simultaneously. Feedback delay is one of the major concerns. Given the fact that the current channel is probably changed when the channel feedback arrives at transmitters, it seems such delayed feedback is useless. Surprisingly, Maddah-Ali and Tse showed that such delayed feedback is still very useful even if it is totally obsolete and independent of the current one, by proposing a novel concept of retrospective interference alignment [11, 12]. Along the same line, DOF characterizations were done recently for a variety of channel settings with delayed CSIT, such as single-input-single-output (SISO) X/interference channels [13], MIMO broadcast/interference channels [14–16], layered two-unicast networks [17], MIMO X channels [18], and MIMO Y channels [19, 20].

In this paper, we will study the DOF of MIMO two-way X relay networks with delayed CSIT, where achievable DOFs will be given for all possible antenna configurations. Compared with the schemes in literature, our proposed multiple-stage transmission (MST) scheme achieves both significant DOF gain and tremendous power gain. The organization of the paper is as follows. Section 2 first describes the system model. Then, we analyze the achievable DOF under different transmission schemes in Section 3, followed by an example on retrospective interference alignment in Section 4. In Section 5, we provide an MST scheme to achieve the DOF. DOF and sum rate of different transmission schemes are compared in Section 6. Section 7 concludes this paper.

2 System model

We consider a wireless network, as shown in Figure 1, where a group of two users (cf. users 1 and 2) want to exchange messages with another group of two users (cf. users 3 and 4) via a relay. Being equipped with M antennas at the relay and N antennas at each user, this network setup can be categorized into the MIMO two-way X relay channel. Similar to the state-of-the-art in MIMO two-way X relay networks [10], we assume that the source nodes and relay operate in full-duplex mode so that the transmission is implemented in simultaneous multiple access (MAC) phase and broadcast (BC) phase, and decode-and-forward policy is employed at the relay. We follow the setting of delayed CSIT as for example in [11], where all nodes have global CSIT with one unit time (i.e. channel coherence time) delay in both MAC and BC phases. As such, the delayed CSIT is independent of the current one. In addition, perfect channel state information at the receiver (CSIR) is assumed in both phases.

In the MAC phase, unlike [10], signal space alignment (i.e. align signal streams of different user pairs at the relay) seems impossible in the absence of current CSIT. Thus, all users simply send data symbols without precoding. As shown in Figure 1, we denote the message from user i to user j as $\boldsymbol{s}_{j,i}$. At the receiver side, by using zero-forcing (ZF) decoding algorithm, the relay node recovers $\boldsymbol{s}_{3,1}$, $\boldsymbol{s}_{4,1}$, $\boldsymbol{s}_{3,2}$, $\boldsymbol{s}_{4,2}$,

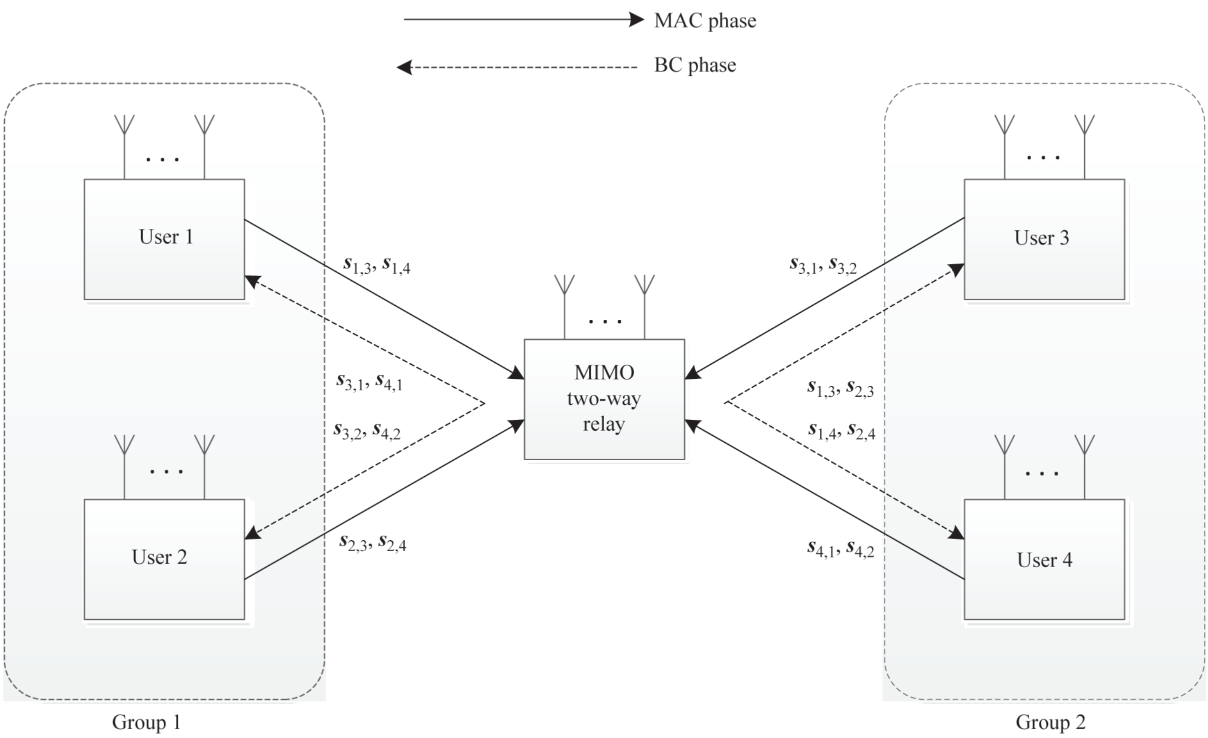

Figure 1 System model of MIMO two-way X relay channel.

$\boldsymbol{s}_{1,3}$, $\boldsymbol{s}_{2,3}$, $\boldsymbol{s}_{1,4}$, $\boldsymbol{s}_{2,4}$ directly. In the BC phase, The MST scheme built upon physical layer network coding and retrospective interference alignment will be used at the relay, which will be clear in Section 5.

Note that: (1) multiple time slots are necessary in both MAC and BC phases; (2) all symbols decoded by the relay during the current MAC phase can only be sent out in the form of PNC symbols during the next BC phase.

3 DOF results

Given transmitting power P and the achievable rate R_P, the DOF (a.k.a. multiplexing gain) is defined as

$$\eta \triangleq \lim_{P\to\infty} \frac{R_P}{\log_2 P}. \tag{1}$$

Throughout this paper, we will omit all noise terms because they have no impact on DOF analysis in high SNR regime.

If ZF decoding is used in the MAC phase, the sum DOF of MIMO two-way X relay channel is defined as the minimum sum DOF in MAC and BC phase,

$$\eta = \min\{\eta^{\mathrm{MAC}}, \eta^{\mathrm{BC}}\}. \tag{2}$$

This is because (1) the rate of each message is calculated as the minimum rate in MAC and BC phase (see [10, Eq. (42)]), thus the DOF of each message is defined as the minimum DOF in MAC and BC phase; (2) due to symmetry, each message will achieve the same DOF.

In the MAC phase, all nodes form an $M \times 4N$ super MIMO system. We assume all channel uses are generic, i.e., any channel matrix will have rank equal to the minimum number of its rows and columns. By using ZF decoding, η^{MAC} is given by

$$\eta^{\mathrm{MAC}} = \min\{M, 4N\}. \tag{3}$$

In the BC phase, we consider three schemes: Vaze's bound (w/o. PNC), TDMA scheme (w/o. PNC), and MST scheme (w. PNC). In particular, Vaze et al. in [14] obtained an upper bound for the DOF region of a K-user MIMO broadcast channel with delayed CSIT. Based on this result, we have the following proposition for $K = 4$.

Proposition 1. The DOF of four-user MIMO BC channel with delayed CSIT is upper bounded by

$$\eta^{\mathrm{BC}} \leqslant \eta^{\mathrm{BC}}_{\mathrm{Vaze}} = \frac{4}{\frac{1}{\min(M,N)} + \frac{1}{\min(M,2N)} + \frac{1}{\min(M,3N)} + \frac{1}{\min(M,4N)}}. \tag{4}$$

The right-hand side of (4) can be explicitly expressed as

$$\eta^{\mathrm{BC}}_{\mathrm{Vaze}} = \begin{cases} M, & M \leqslant N, \\ \dfrac{4MN}{M+3N}, & N < M \leqslant 2N, \\ \dfrac{8MN}{3M+4N}, & 2N < M \leqslant 3N, \\ \dfrac{24MN}{11M+6N}, & 3N < M \leqslant 4N, \\ \dfrac{48}{25}N, & M > 4N. \end{cases} \tag{5}$$

When TDMA is used, η_{BC} can be summarized as

$$\eta^{\mathrm{BC}}_{\mathrm{TDMA}} = \min(M, N). \tag{6}$$

By using MST in Section 5, η^{BC} is given by

$$\eta^{\mathrm{BC}}_{\mathrm{MST}} = \begin{cases} 2M, & M \leqslant N, \\ \dfrac{8MN}{3M+N}, & N < M \leqslant 3N, \\ \dfrac{12}{5}N, & M > 3N. \end{cases} \tag{7}$$

Based on (2), (3), and (5)–(7), note that $\eta^{\mathrm{BC}}_{\mathrm{Vaze}} \leqslant \eta^{\mathrm{MAC}}$ and $\eta^{\mathrm{BC}}_{\mathrm{TDMA}} \leqslant \eta^{\mathrm{MAC}}$, our main results are given as follows.

Theorem 1. For a four-user MIMO two-way X relay channel described in Section 2, by using Vaze's bound in the BC phase[1)], $\eta \leqslant \eta^{\mathrm{BC}}_{\mathrm{Vaze}}$; by using TDMA in the BC phase, $\eta^{\mathrm{BC}}_{\mathrm{TDMA}} = \min(M, N)$; by using MST, η is given as

$$\eta = \begin{cases} M, & M \leqslant \dfrac{7}{3}N, \\ \dfrac{8MN}{3M+N}, & \dfrac{7}{3}N < M \leqslant 3N, \\ \dfrac{12}{5}N, & M > 3N. \end{cases} \tag{8}$$

4 Retrospective interference alignment

Retrospective interference alignment is the basis of MST scheme. The key idea is to use delayed CSIT to reconstruct and align the interference in multiple time slots. Here, we take a simple example to show how retrospective interference alignment is achieved by using delayed CSIT.

We consider a two-user MISO (multiple-input-single-output) broadcast channel, where the base station (BS) is equipped with two antennas while each user is equipped with one antenna. We assume all nodes have global CSIT with one unit time (i.e. channel coherence time) feedback delay in BC phase. As such,

1) In many cases of DOF analysis, the DOF upper bound is difficult to be achieved. In this paper, we do not care whether Vaze's bound can be achieved or not. We use Vaze's bound to demonstrate the DOF benefits of PNC in the BC phase.

the delayed CSIT is independent of the current one. In addition, perfect CSIR is assumed at all nodes. Under this CSI setting, the DOF is given by $\eta = \frac{4}{3}$ and can be achieved as follows.

In time slot 1, the BS sends a 2×1 vector $\boldsymbol{q}_1 = [\boldsymbol{s}_1^1, \boldsymbol{s}_1^2]^{\mathrm{T}}$, which contains two streams for user 1. Thus, the received signals of users 1 and 2 can be denoted as

$$\begin{bmatrix} \boldsymbol{y}_1^1 \\ \boldsymbol{y}_2^1 \end{bmatrix} = \begin{bmatrix} h_{11}^1 & h_{12}^1 \\ h_{21}^1 & h_{22}^1 \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^1 \\ \boldsymbol{s}_1^2 \end{bmatrix}, \tag{9}$$

where h_{kj}^t is the channel coefficient from the jth antenna of the BS to user k in time slot t, $\boldsymbol{y}_i^t$ is the received signal of user i in time slot t and $\boldsymbol{s}_i^j$ denotes the jth stream of user i.

Since user 1 has only one antenna, it can not decode two streams simultaneously. Note that each node has delayed global CSIT, user 1 will get h_{21}^1 and h_{22}^1 with one unit time delay. A natural idea is that if the BS can deliver $\boldsymbol{y}_2^1$ to user 1, user 1 can decode $\boldsymbol{q}_1$ as

$$\begin{bmatrix} \boldsymbol{s}_1^1 \\ \boldsymbol{s}_1^2 \end{bmatrix} = \begin{bmatrix} h_{11}^1 & h_{12}^1 \\ h_{21}^1 & h_{22}^1 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{y}_1^1 \\ \boldsymbol{y}_2^1 \end{bmatrix}. \tag{10}$$

In time slot 2, the BS sends a 2×1 vector $\boldsymbol{q}_2 = [\boldsymbol{s}_2^1, \boldsymbol{s}_2^2]^{\mathrm{T}}$, which contains two streams for user 2. The received signals of user 1 and user 2 can be denoted as

$$\begin{bmatrix} \boldsymbol{y}_1^2 \\ \boldsymbol{y}_2^2 \end{bmatrix} = \begin{bmatrix} h_{11}^2 & h_{12}^2 \\ h_{21}^2 & h_{22}^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_2^1 \\ \boldsymbol{s}_2^2 \end{bmatrix}. \tag{11}$$

Similar to (10), if the BS can deliver $\boldsymbol{y}_1^2$ to user 2, user 2 can decode $\boldsymbol{q}_2$ as

$$\begin{bmatrix} \boldsymbol{s}_2^1 \\ \boldsymbol{s}_2^2 \end{bmatrix} = \begin{bmatrix} h_{11}^2 & h_{12}^2 \\ h_{21}^2 & h_{22}^2 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{y}_1^2 \\ \boldsymbol{y}_2^2 \end{bmatrix}. \tag{12}$$

Note that the BS has delayed global CSIT, so it will get $\boldsymbol{y}_2^1$ and $\boldsymbol{y}_1^2$. In time slot 3, the BS sends $\boldsymbol{q}_3 = \boldsymbol{y}_2^1 + \boldsymbol{y}_1^2$ with just one antenna (i.e. the other antenna keeps silent). The received signals of user 1 and user 2 can be denoted as

$$\begin{bmatrix} \boldsymbol{y}_1^3 \\ \boldsymbol{y}_2^3 \end{bmatrix} = \begin{bmatrix} h_{11}^3 \\ h_{21}^3 \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_2^1 + \boldsymbol{y}_1^2 \end{bmatrix} \tag{13}$$

$$= \begin{bmatrix} h_{11}^3 h_{21}^1 & h_{11}^3 h_{22}^1 & h_{11}^3 h_{11}^2 & h_{11}^3 h_{12}^2 \\ h_{21}^3 h_{21}^1 & h_{21}^3 h_{22}^1 & h_{21}^3 h_{11}^2 & h_{21}^3 h_{12}^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^1 \\ \boldsymbol{s}_2^1 \\ \boldsymbol{s}_1^2 \\ \boldsymbol{s}_2^2 \end{bmatrix}. \tag{14}$$

Take user 1 for instance, we show how interference alignment is achieved by using delayed CSIT as follows.

$$\begin{bmatrix} \boldsymbol{y}_1^1 \\ \boldsymbol{y}_1^2 \\ \boldsymbol{y}_1^3 \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11}^2 & h_{12}^2 \\ 0 & 0 \\ h_{11}^3 h_{21}^1 & h_{11}^3 h_{22}^1 \end{bmatrix}}_{\text{rank=2}} \begin{bmatrix} \boldsymbol{s}_1^1 \\ \boldsymbol{s}_2^1 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ h_{11}^2 & h_{12}^2 \\ h_{11}^3 h_{11}^2 & h_{11}^3 h_{12}^2 \end{bmatrix}}_{\text{rank=1}} \begin{bmatrix} \boldsymbol{s}_1^2 \\ \boldsymbol{s}_2^2 \end{bmatrix}. \tag{15}$$

We can see from (15) that, for user 1, interference signals are aligned within a subspace with a size (rank) of 1, while desired signals occupied a subspace with a size of 2. Hence, user 1 can recover its desired signal as

$$\begin{bmatrix} \boldsymbol{s}_1^1 \\ \boldsymbol{s}_2^1 \end{bmatrix} = \begin{bmatrix} h_{11}^2 & h_{12}^2 \\ h_{11}^3 h_{21}^1 & h_{11}^3 h_{22}^1 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{y}_1^1 \\ \boldsymbol{y}_1^3 - h_{11}^3 \boldsymbol{y}_1^2 \end{bmatrix}. \tag{16}$$

The same analysis can be generalized to user 2. Note that two users can recover four desired streams within three consecutive time slots, so the DOF is given by $\frac{4}{3}$.

5 MST scheme

The idea of MST scheme is to combine PNC and retrospective interference alignment. Specifically, we use delayed CSIT to reconstruct and align PNC symbols in multiple time slots. To facilitate our discussion, we use the following notations: (1) $\boldsymbol{s}_{j,k}^{i}$ denotes the ith element of $\boldsymbol{s}_{j,k}$, which is decoded by the relay during the previous MAC phase; (2) $\{\boldsymbol{s}_{j,k}^{i}\}_{i=k_1}^{k_2}$ denotes a k_2-k_1+1 vector $(\boldsymbol{s}_{j,k}^{k_1}, \boldsymbol{s}_{j,k}^{k_1+1}, \ldots, \boldsymbol{s}_{j,k}^{k_2})^{\mathrm{T}}$, where T denotes transpose operation; (3) $\{\boldsymbol{s}_{j,k}^{i}\}_{i=k_1}^{k_2} \oplus \{\boldsymbol{s}_{k,j}^{i}\}_{i=k_1}^{k_2}$ denotes a k_2-k_1+1 vector $(\boldsymbol{s}_{j,k}^{k_1} \oplus \boldsymbol{s}_{k,j}^{k_1}, \boldsymbol{s}_{j,k}^{k_1+1} \oplus \boldsymbol{s}_{k,j}^{k_1+1}, \ldots, \boldsymbol{s}_{j,k}^{k_2} \oplus \boldsymbol{s}_{k,j}^{k_2})^{\mathrm{T}}$, where '$\oplus$' denotes bit 'xor' operation; (4) the subscript of $\boldsymbol{q}$ denotes the index of time slot; (5) order-k symbol/vector denotes a symbol/vector containing k users' common information; (6) $\boldsymbol{H}_{k,r}^{(t)}$ denotes the channel matrix from the relay to user k in time slot t; (7) $\boldsymbol{y}_{k}^{t}$ is the received signal of user k in time slot t.

5.1 $M \leqslant N$

In BC phase, let the relay transmit a PNC vector per time slot. For example, in time slot 1, the relay sends

$$\boldsymbol{q}_1 = \{\boldsymbol{s}_{1,3}^{i}\}_{i=1}^{M} \oplus \{\boldsymbol{s}_{3,1}^{i}\}_{i=1}^{M}. \tag{17}$$

The received signals of user 1 and user 3 in time slot 1 can be denoted as

$$\boldsymbol{y}_1^1 = \boldsymbol{H}_{1,r}^{(1)}\boldsymbol{q}_1, \quad \boldsymbol{y}_3^1 = \boldsymbol{H}_{3,r}^{(1)}\boldsymbol{q}_1. \tag{18}$$

Let we choose the first M rows of $\boldsymbol{y}_1^1$, $\boldsymbol{H}_{1,r}^{(1)}$, $\boldsymbol{y}_3^1$, and $\boldsymbol{H}_{3,r}^{(1)}$, and denote it as $\overline{\boldsymbol{y}}_1^1$, $\overline{\boldsymbol{H}}_{1,r}^{(1)}$, $\overline{\boldsymbol{y}}_3^1$, and $\overline{\boldsymbol{H}}_{3,r}^{(1)}$, respectively. By multiplying the inverse of $\overline{\boldsymbol{H}}_{1,r}^{(1)}$ (resp. $\overline{\boldsymbol{H}}_{3,r}^{(1)}$) with $\overline{\boldsymbol{y}}_1^1$ (resp. $\overline{\boldsymbol{y}}_3^1$), user 1 (resp. user 3) can perfectly decode $\boldsymbol{q}_1$. After decoding $\boldsymbol{q}_1$, note that $\{\boldsymbol{s}_{3,1}^{i}\}_{i=1}^{M}$ (resp. $\{\boldsymbol{s}_{1,3}^{i}\}_{i=1}^{M}$) is the side information of user 1 (resp. user 3), users 1 and 3 can recover their desired information as

$$\{\boldsymbol{s}_{1,3}^{i}\}_{i=1}^{M} = \boldsymbol{q}_1 \oplus \{\boldsymbol{s}_{3,1}^{i}\}_{i=1}^{M}, \quad \{\boldsymbol{s}_{3,1}^{i}\}_{i=1}^{M} = \boldsymbol{q}_1 \oplus \{\boldsymbol{s}_{1,3}^{i}\}_{i=1}^{M}. \tag{19}$$

This process can be generalized to user pair $\{1,4\}$, $\{2,3\}$, and $\{2,4\}$. In each time slot, two users can recover their desired symbols, thus

$$\eta_{\mathrm{MST}}^{\mathrm{BC}} = 2M. \tag{20}$$

5.2 $N < M \leqslant 3N$

To send $8MN$ order-1 symbols over $3M+N$ time slots, we divide the whole BC phase into two stages ($4N$ and $3(M-N)$ time slots in stages 1 and 2, respectively). In each stage, we consider two cases: $N < M \leqslant 2N$ and $2N < M \leqslant 3N$.

- First stage

The transmitted vectors over $4N$ time slots are given as

$$\begin{aligned}
\boldsymbol{q}_{4k-3} &= \{\boldsymbol{s}_{1,3}^{i}\}_{i=(k-1)M+1}^{kM} \oplus \{\boldsymbol{s}_{3,1}^{i}\}_{i=(k-1)M+1}^{kM},\\
\boldsymbol{q}_{4k-2} &= \{\boldsymbol{s}_{1,4}^{i}\}_{i=(k-1)M+1}^{kM} \oplus \{\boldsymbol{s}_{4,1}^{i}\}_{i=(k-1)M+1}^{kM},\\
\boldsymbol{q}_{4k-1} &= \{\boldsymbol{s}_{2,3}^{i}\}_{i=(k-1)M+1}^{kM} \oplus \{\boldsymbol{s}_{3,2}^{i}\}_{i=(k-1)M+1}^{kM},\\
\boldsymbol{q}_{4k} &= \{\boldsymbol{s}_{2,4}^{i}\}_{i=(k-1)M+1}^{kM} \oplus \{\boldsymbol{s}_{4,2}^{i}\}_{i=(k-1)M+1}^{kM},
\end{aligned} \tag{21}$$

where $k = 1, 2, \ldots, N$.

Case I. $N < M \leqslant 2N$

Let we show how order-2 symbols are constructed. In time slot $4k-3$, the received signals of users 1, 2, and 3 can be denoted as

$$\boldsymbol{y}_1^{4k-3} = \boldsymbol{H}_{1,r}^{(4k-3)}\boldsymbol{q}_{4k-3}, \quad \boldsymbol{y}_2^{4k-3} = \boldsymbol{H}_{2,r}^{(4k-3)}\boldsymbol{q}_{4k-3}, \quad \boldsymbol{y}_3^{4k-3} = \boldsymbol{H}_{3,r}^{(4k-3)}\boldsymbol{q}_{4k-3}. \tag{22}$$

Let we choose the first $M-N$ rows of $\boldsymbol{y}_2^{4k-3}$ and $\boldsymbol{H}_{2,r}^{(4k-3)}$, and denote it as $\overline{\boldsymbol{y}}_2^{4k-3}$ and $\overline{\boldsymbol{H}}_{2,r}^{(4k-3)}$, respectively. Note that each node will have delayed global CSIT, user 1 (resp. user 3) will know $\overline{\boldsymbol{H}}_{2,r}^{(4k-3)}$ with one unit time delay. Similar to (10), if the relay can deliver $\overline{\boldsymbol{y}}_2^{4k-3}$ to users 1 and 3, user 1 can decode $\boldsymbol{q}_{4k-3}$ as

$$\boldsymbol{q}_{4k-3} = \begin{bmatrix} \boldsymbol{H}_{1,r}^{(4k-3)} \\ \overline{\boldsymbol{H}}_{2,r}^{(4k-3)} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{y}_1^{4k-3} \\ \overline{\boldsymbol{y}}_{2,r}^{4k-3} \end{bmatrix}, \tag{23}$$

and user 3 can decode $\boldsymbol{q}_{4k-3}$ as

$$\boldsymbol{q}_{4k-3} = \begin{bmatrix} \boldsymbol{H}_{3,r}^{(4k-3)} \\ \overline{\boldsymbol{H}}_{2,r}^{(4k-3)} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{y}_3^{4k-3} \\ \overline{\boldsymbol{y}}_{2,r}^{4k-3} \end{bmatrix}. \tag{24}$$

After decoding $\boldsymbol{q}_{4k-3}$, users 1 and 3 can recover their desired signals as shown in (19). We can construct order-2 vectors for user pair $\{1,3\}$ as

$$\boldsymbol{u}_{1,3}^{4k-3} = \overline{\boldsymbol{y}}_2^{4k-3}. \tag{25}$$

Similarly, we choose the first $M-N$ rows of $\boldsymbol{y}_3^{4k-2}$, $\boldsymbol{y}_4^{4k-1}$ and $\boldsymbol{y}_1^{4k}$, and denote it as $\overline{\boldsymbol{y}}_3^{4k-2}$, $\overline{\boldsymbol{y}}_4^{4k-1}$ and $\overline{\boldsymbol{y}}_1^{4k}$, respectively. we can construct order-2 vectors for user pairs $\{1,4\}$, $\{2,3\}$ and $\{2,4\}$ as

$$\boldsymbol{u}_{1,4}^{4k-2} = \overline{\boldsymbol{y}}_3^{4k-2}, \quad \boldsymbol{u}_{2,3}^{4k-1} = \overline{\boldsymbol{y}}_4^{4k-1}, \quad \boldsymbol{u}_{2,4}^{4k} = \overline{\boldsymbol{y}}_1^{4k}. \tag{26}$$

Let we stack all $\boldsymbol{u}_{1,3}^{4k-3}$ (resp. $\boldsymbol{u}_{1,4}^{4k-2}$, $\boldsymbol{u}_{2,3}^{4k-1}$, and $\boldsymbol{u}_{2,4}^{4k}$) into a super vector $\boldsymbol{U}_{1,3}$ (resp. $\boldsymbol{U}_{1,4}$, $\boldsymbol{U}_{2,3}$, $\boldsymbol{U}_{2,4}$) with a size of $N(M-N)$. Hence, in the end of the first stage, the relay need to transmit $4N(M-N)$ order-2 symbols. We then split these super vectors into small vectors with a size of N:

$$\begin{aligned} &\boldsymbol{u}_2(l) = \{\boldsymbol{U}_{1,3}^i\}_{i=(l-1)N+1}^{lN}, \quad \boldsymbol{u}_3(l) = \{\boldsymbol{U}_{1,4}^i\}_{i=(l-1)N+1}^{lN}, \\ &\boldsymbol{u}_4(l) = \{\boldsymbol{U}_{2,3}^i\}_{i=(l-1)N+1}^{lN}, \quad \boldsymbol{u}_1(l) = \{\boldsymbol{U}_{2,4}^i\}_{i=(l-1)N+1}^{lN}, \end{aligned} \tag{27}$$

where $l = (1,\ldots,M-N)$.

Case II. $2N < M \leqslant 3N$

Now we show how order-2 symbols are constructed. In time slot $4k-3$, the received signals of users 1, 3, 2, and 4 can be denoted as

$$\begin{aligned} &\boldsymbol{y}_1^{4k-3} = \boldsymbol{H}_{1,r}^{(4k-3)}\boldsymbol{q}_{4k-3}, \quad \boldsymbol{y}_3^{4k-3} = \boldsymbol{H}_{3,r}^{(4k-3)}\boldsymbol{q}_{4k-3}, \\ &\boldsymbol{y}_2^{4k-3} = \boldsymbol{H}_{2,r}^{(4k-3)}\boldsymbol{q}_{4k-3}, \quad \boldsymbol{y}_4^{4k-3} = \boldsymbol{H}_{4,r}^{(4k-3)}\boldsymbol{q}_{4k-3}. \end{aligned} \tag{28}$$

Let we choose the first $M-2N$ rows of $\boldsymbol{y}_4^{4k-3}$ and $\boldsymbol{H}_{4,r}^{(4k-3)}$, and denote it as $\overline{\boldsymbol{y}}_4^{4k-3}$ and $\overline{\boldsymbol{H}}_{4,r}^{(4k-3)}$, respectively. If the relay can deliver $\boldsymbol{y}_2^{4k-3}$ and $\overline{\boldsymbol{y}}_4^{4k-3}$ to user pair $\{1, 3\}$, user 1 can decode $\boldsymbol{q}_{4k-3}$ as

$$\boldsymbol{q}_{4k-3} = \begin{bmatrix} \boldsymbol{H}_{1,r}^{(4k-3)} \\ \boldsymbol{H}_{2,r}^{(4k-3)} \\ \overline{\boldsymbol{H}}_{4,r}^{(4k-3)} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{y}_1^{4k-3} \\ \boldsymbol{y}_2^{4k-3} \\ \overline{\boldsymbol{y}}_{4,r}^{4k-3} \end{bmatrix}, \tag{29}$$

and user 3 can decode $\boldsymbol{q}_{4k-3}$ as

$$\boldsymbol{q}_{4k-3} = \begin{bmatrix} \boldsymbol{H}_{3,r}^{(4k-3)} \\ \boldsymbol{H}_{2,r}^{(4k-3)} \\ \overline{\boldsymbol{H}}_{4,r}^{(4k-3)} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{y}_3^{4k-3} \\ \boldsymbol{y}_2^{4k-3} \\ \overline{\boldsymbol{y}}_{4,r}^{4k-3} \end{bmatrix}. \tag{30}$$

We can construct order-2 vectors for user pair $\{1,3\}$ as

$$\boldsymbol{u}_{1,3}^{4k-3} = \begin{bmatrix} \boldsymbol{y}_2^{4k-3} \\ \overline{\boldsymbol{y}}_4^{4k-3} \end{bmatrix}. \tag{31}$$

Similarly, we choose the first $M-2N$ rows of $\boldsymbol{y}_2^{4k-2}$, $\boldsymbol{y}_1^{4k-1}$ and $\boldsymbol{y}_3^{4k}$, and denote it as $\overline{\boldsymbol{y}}_2^{4k-2}$, $\overline{\boldsymbol{y}}_1^{4k-1}$ and $\overline{\boldsymbol{y}}_3^{4k}$, respectively. Then we can construct order-2 vectors for user pairs $\{1,4\}$, $\{2,3\}$ and $\{2,4\}$ as

$$\boldsymbol{u}_{1,4}^{4k-2}=\begin{bmatrix}\boldsymbol{y}_3^{4k-2}\\ \overline{\boldsymbol{y}}_2^{4k-2}\end{bmatrix},\quad \boldsymbol{u}_{2,3}^{4k-1}=\begin{bmatrix}\boldsymbol{y}_4^{4k-1}\\ \overline{\boldsymbol{y}}_1^{4k-1}\end{bmatrix},\quad \boldsymbol{u}_{2,4}^{4k}=\begin{bmatrix}\boldsymbol{y}_1^{4k}\\ \overline{\boldsymbol{y}}_3^{4k}\end{bmatrix}. \tag{32}$$

We then stack all $\boldsymbol{y}_2^{4k-3}$ (resp. $\boldsymbol{y}_3^{4k-2}$, $\boldsymbol{y}_4^{4k-1}$, and $\boldsymbol{y}_1^{4k}$) into a super vector $\boldsymbol{U}_2$ (resp. $\boldsymbol{U}_3$, $\boldsymbol{U}_4$, and $\boldsymbol{U}_1$) with a size of N^2, and stack all $\overline{\boldsymbol{y}}_4^{4k-3}$ (resp. $\overline{\boldsymbol{y}}_2^{4k-2}$, $\overline{\boldsymbol{y}}_1^{4k-1}$ and $\overline{\boldsymbol{y}}_3^{4k}$) into a super vector $\overline{\boldsymbol{U}}_4$ (resp. $\overline{\boldsymbol{U}}_2$, $\overline{\boldsymbol{U}}_1$, and $\overline{\boldsymbol{U}}_3$) with a size of $N(M-2N)$. Hence, in the end of the first stage, the relay need to transmit $4N(M-N)$ order-2 symbols. We then split these super vectors into smaller vectors with a size of N:

$$\begin{aligned}
&\boldsymbol{u}_1(l)=\{\boldsymbol{U}_1^i\}_{i=(l-1)N+1}^{lN},\quad \boldsymbol{u}_2(l)=\{\boldsymbol{U}_2^i\}_{i=(l-1)N+1}^{lN},\\
&\boldsymbol{u}_3(l)=\{\boldsymbol{U}_3^i\}_{i=(l-1)N+1}^{lN},\quad \boldsymbol{u}_4(l)=\{\boldsymbol{U}_4^i\}_{i=(l-1)N+1}^{lN},\\
&\overline{\boldsymbol{u}}_1(l')=\{\overline{\boldsymbol{U}}_1^i\}_{i=(l'-1)N+1}^{l'N},\quad \overline{\boldsymbol{u}}_2(l')=\{\overline{\boldsymbol{U}}_2^i\}_{i=(l'-1)N+1}^{l'N},\\
&\overline{\boldsymbol{u}}_3(l')=\{\overline{\boldsymbol{U}}_3^i\}_{i=(l'-1)N+1}^{l'N},\quad \overline{\boldsymbol{u}}_4(l')=\{\overline{\boldsymbol{U}}_4^i\}_{i=(l'-1)N+1}^{l'N},
\end{aligned} \tag{33}$$

where $l=(1,\ldots,N)$ and $l'=(1,\ldots,M-2N)$.

- Second stage

In this stage, N antennas will be used at the relay. Over three consecutive time slots, the relay will send three random combinations of four order-2 vectors. Thus, it will take $\frac{4N(M-N)\times 3}{4N}=3(M-N)$ time slots to transmit all order-2 vectors.

Case I. $N<M\leqslant 2N$

The transmitted vectors over $3(M-N)$ time slots are given as

$$\begin{aligned}
\boldsymbol{q}_{3l+4N-2}&=\boldsymbol{P}_1\boldsymbol{u}_1(l)+\boldsymbol{P}_2\boldsymbol{u}_2(l)+\boldsymbol{P}_3\boldsymbol{u}_3(l)+\boldsymbol{P}_4\boldsymbol{u}_4(l),\\
\boldsymbol{q}_{3l+4N-1}&=\boldsymbol{P}_5\boldsymbol{u}_1(l)+\boldsymbol{P}_6\boldsymbol{u}_2(l)+\boldsymbol{P}_7\boldsymbol{u}_3(l)+\boldsymbol{P}_8\boldsymbol{u}_4(l),\\
\boldsymbol{q}_{3l+4N}&=\boldsymbol{P}_9\boldsymbol{u}_1(l)+\boldsymbol{P}_{10}\boldsymbol{u}_2(l)+\boldsymbol{P}_{11}\boldsymbol{u}_3(l)+\boldsymbol{P}_{12}\boldsymbol{u}_4(l),
\end{aligned} \tag{34}$$

where $l=\{1,2,\ldots,M-N\}$. $\boldsymbol{P}_1,\boldsymbol{P}_2,\ldots$, and $\boldsymbol{P}_{12}$ are different full rank random matrices and they are shared with all nodes.

Now we take user 1 for instance to show how it can decode $\boldsymbol{u}_2(l)$, $\boldsymbol{u}_3(l)$, and $\boldsymbol{u}_4(l)$. Over three consecutive time slots, the received signals of user 1 can be denoted as

$$\boldsymbol{y}_1^{3l+4N-2}=\boldsymbol{H}_{1,r}^{(3l+4N-2)}\boldsymbol{q}_{3l+4N-2},\ \boldsymbol{y}_1^{3l+4N-1}=\boldsymbol{H}_{1,r}^{(3l+4N-1)}\boldsymbol{q}_{3l+4N-1},\ \boldsymbol{y}_1^{3l+4N}=\boldsymbol{H}_{1,r}^{(3l+4N)}\boldsymbol{q}_{3l+4N}. \tag{35}$$

First, by using ZF decoding, user 1 can recover $\boldsymbol{q}_{3l+4N-2}$, $\boldsymbol{q}_{3l+4N-1}$, and $\boldsymbol{q}_{3l+4N}$ as

$$\begin{aligned}
\boldsymbol{q}_{3l+4N-2}&=(\boldsymbol{H}_{1,r}^{(3l+4N-2)})^{-1}\boldsymbol{y}_1^{3l+4N-2},\\
\boldsymbol{q}_{3l+4N-1}&=(\boldsymbol{H}_{1,r}^{(3l+4N-1)})^{-1}\boldsymbol{y}_1^{3l+4N-1},\\
\boldsymbol{q}_{3l+4N}&=(\boldsymbol{H}_{1,r}^{(3l+4N)})^{-1}\boldsymbol{y}_1^{3l+4N}.
\end{aligned} \tag{36}$$

Note that $\boldsymbol{P}_1,\boldsymbol{P}_2,\ldots$, and $\boldsymbol{P}_{12}$ are shared with all nodes and $\boldsymbol{u}_1(l)$ is the side information of user 1, user 1 can decode $\boldsymbol{u}_2(l)$, $\boldsymbol{u}_3(l)$, and $\boldsymbol{u}_4(l)$ as

$$\begin{bmatrix}\boldsymbol{u}_2(l)\\ \boldsymbol{u}_3(l)\\ \boldsymbol{u}_4(l)\end{bmatrix}=\begin{bmatrix}\boldsymbol{P}_2&\boldsymbol{P}_3&\boldsymbol{P}_4\\ \boldsymbol{P}_6&\boldsymbol{P}_7&\boldsymbol{P}_8\\ \boldsymbol{P}_{10}&\boldsymbol{P}_{11}&\boldsymbol{P}_{12}\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{q}_{3l+4N-2}-\boldsymbol{P}_1\boldsymbol{u}_1(l)\\ \boldsymbol{q}_{3l+4N-1}-\boldsymbol{P}_5\boldsymbol{u}_1(l)\\ \boldsymbol{q}_{3l+4N}-\boldsymbol{P}_9\boldsymbol{u}_1(l)\end{bmatrix}. \tag{37}$$

Similarly, user 2 can decode $\boldsymbol{u}_1(l)$, $\boldsymbol{u}_3(l)$, and $\boldsymbol{u}_4(l)$; user 3 can decode $\boldsymbol{u}_1(l)$, $\boldsymbol{u}_2(l)$, and $\boldsymbol{u}_4(l)$; user 4 can decode $\boldsymbol{u}_1(l)$, $\boldsymbol{u}_2(l)$, and $\boldsymbol{u}_3(l)$.

That means, user pairs {1,3}, {1,4}, {2,3}, and {2,4} can get $\boldsymbol{u}_2(l)$, $\boldsymbol{u}_3(l)$, $\boldsymbol{u}_4(l)$, and $\boldsymbol{u}_1(l)$, respectively. Therefore, we have

$$\eta_{\rm MST}^{\rm BC} = \frac{8MN}{3M+N}. \tag{38}$$

Case II. $2N < M \leqslant 3N$

The transmitted vectors over the first $3N$ time slots can be given as

$$\begin{aligned} \boldsymbol{q}_{3l+4N-2} &= \boldsymbol{P}_1\boldsymbol{u}_1(l) + \boldsymbol{P}_2\boldsymbol{u}_2(l) + \boldsymbol{P}_3\boldsymbol{u}_3(l) + \boldsymbol{P}_4\boldsymbol{u}_4(l), \\ \boldsymbol{q}_{3l+4N-1} &= \boldsymbol{P}_5\boldsymbol{u}_1(l) + \boldsymbol{P}_6\boldsymbol{u}_2(l) + \boldsymbol{P}_7\boldsymbol{u}_3(l) + \boldsymbol{P}_8\boldsymbol{u}_4(l), \\ \boldsymbol{q}_{3l+4N} &= \boldsymbol{P}_9\boldsymbol{u}_1(l) + \boldsymbol{P}_{10}\boldsymbol{u}_2(l) + \boldsymbol{P}_{11}\boldsymbol{u}_3(l) + \boldsymbol{P}_{12}\boldsymbol{u}_4(l), \end{aligned} \tag{39}$$

where $l = \{1, 2, \ldots, N\}$, $\boldsymbol{P}_1, \boldsymbol{P}_2, \ldots$, and $\boldsymbol{P}_{12}$ are different full rank random matrices and they are shared with all nodes.

The transmitted vectors over the last $3(M-2N)$ time slots can be denoted as

$$\begin{aligned} \boldsymbol{q}_{3l'+5N-2} &= \boldsymbol{P}_1\overline{\boldsymbol{u}}_1(l') + \boldsymbol{P}_2\overline{\boldsymbol{u}}(l') + \boldsymbol{P}_3\overline{\boldsymbol{u}}_3(l') + \boldsymbol{P}_4\overline{\boldsymbol{u}}_4(l'), \\ \boldsymbol{q}_{3l'+5N-1} &= \boldsymbol{P}_5\overline{\boldsymbol{u}}_1(l') + \boldsymbol{P}_6\overline{\boldsymbol{u}}_2(l') + \boldsymbol{P}_7\overline{\boldsymbol{u}}_3(l') + \boldsymbol{P}_8\overline{\boldsymbol{u}}_4(l'), \\ \boldsymbol{q}_{3l'+5N} &= \boldsymbol{P}_9\overline{\boldsymbol{u}}_1(l') + \boldsymbol{P}_{10}\overline{\boldsymbol{u}}_2(l') + \boldsymbol{P}_{11}\overline{\boldsymbol{u}}_3(l') + \boldsymbol{P}_{12}\overline{\boldsymbol{u}}_4(l'), \end{aligned} \tag{40}$$

where $l' = \{1, 2, \ldots, M-2N\}$.

The same process of Case I can be generalized to this case. Thus, user 1 can recover all $\boldsymbol{u}_2(l)$, $\boldsymbol{u}_3(l)$, $\boldsymbol{u}_4(l)$, $\overline{\boldsymbol{u}}_2(l')$, $\overline{\boldsymbol{u}}_3(l')$ and $\overline{\boldsymbol{u}}_4(l')$; user 2 can recover all $\boldsymbol{u}_1(l)$, $\boldsymbol{u}_3(l)$, $\boldsymbol{u}_4(l)$, $\overline{\boldsymbol{u}}_1(l')$, $\overline{\boldsymbol{u}}_3(l')$, and $\overline{\boldsymbol{u}}_4(l')$; user 3 can recover all $\boldsymbol{u}_1(l)$, $\boldsymbol{u}_2(l)$, $\boldsymbol{u}_4(l)$, $\overline{\boldsymbol{u}}_1(l')$, $\overline{\boldsymbol{u}}_2(l')$, and $\overline{\boldsymbol{u}}_4(l')$; user 4 can recover all $\boldsymbol{u}_1(l)$, $\boldsymbol{u}_2(l)$, $\boldsymbol{u}_3(l)$, $\overline{\boldsymbol{u}}_1(l')$, $\overline{\boldsymbol{u}}_2(l')$, and $\overline{\boldsymbol{u}}_3(l')$. That means, user pair {1,3} can get $\boldsymbol{u}_2(l)$ and $\overline{\boldsymbol{u}}_4(l')$; {1,4} can get $\boldsymbol{u}_3(l)$ and $\overline{\boldsymbol{u}}_2(l')$; {2,3} can get $\boldsymbol{u}_4(l)$ and $\overline{\boldsymbol{u}}_1(l')$; {2,4} can get $\boldsymbol{u}_1(l)$ and $\overline{\boldsymbol{u}}_3(l')$. Therefore, similar to the previous case, we have

$$\eta_{\rm MST}^{\rm BC} = \frac{8MN}{3M+N}. \tag{41}$$

5.3 $M > 3N$

When $M > 3N$, its DOF is trivially equal to the case of $M = 3N$. To see this, let us re-visit equation [28] for user pair {1,3}. For user pair {1,3}, note that users 2 and 4 can provide up to N equations, respectively. If M antennas are used at the relay, the number of overhead equations (i.e. the equations provided by users 2 and 4) for user pair {1,3} is not sufficient to decode the message. The same analysis can be generalized to user pairs {1,4}, {2,3} and {2,4}. In other word, in the case of $M > 3N$, we only activate $M' = 3N$ antennas at the relay. Therefore, we have

$$\eta_{\rm MST}^{\rm BC} = \frac{8M'N}{3M'+N} = \frac{12}{5}N. \tag{42}$$

6 Simulation result and analysis

To gain more intuitive insight into the DOF behavior of different transmission schemes, similar to [16], we define the normalized sum DOF as $\eta_{\rm sum}^{\rm norm} = \frac{\eta}{N}$. Also, we define $m = \frac{M}{N}$. By doing so, the normalized DOF can be seen as a function of m, thus can be plotted. The normalized DOF of MST can be denoted as

$$\eta_{\text{Norm-MST}} = \begin{cases} m, & m \leqslant \dfrac{7}{3}, \\ \dfrac{8m}{3m+1}, & \dfrac{7}{3} < m \leqslant 3, \\ \dfrac{12}{5}, & m > 3. \end{cases} \tag{43}$$

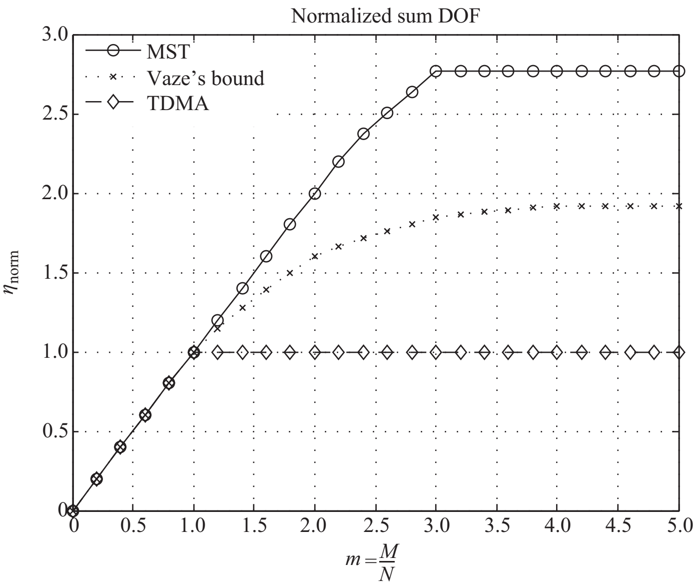

Figure 2 Normalized sum DOF.

The normalized DOF of Vaze's bound can be denoted as

$$\eta_{\text{Norm-Vaze}} = \begin{cases} m, & m \leqslant 1, \\ \dfrac{4m}{m+3}, & 1 < m \leqslant 2, \\ \dfrac{8m}{3m+4}, & 2 < m \leqslant 3, \\ \dfrac{24m}{11m+6}, & 3 < m \leqslant 4, \\ \dfrac{48}{25}, & m > 4. \end{cases} \tag{44}$$

The normalized DOF of TDMA can be denoted as

$$\eta_{\text{Norm-TDMA}} = \min(m, 1). \tag{45}$$

From the results in Figure 2, we have the following conclusion: when $M = \frac{7}{3}N$, MST scheme offers a significant 37.5% sum DOF gain over Vaze's bound. This is because each bit of a PNC symbol contains two users' information, such that the number of time slots needed in BC phase can be reduced when transmitting the same number of order-1 symbol.

In addition to DOF, ergodic sum rate is another important metric for wireless systems. In this section, we provide the sum rate performance of MST and TDMA through simulations. Figures 3 and 4 illustrate the performance of ergodic sum rate of $N = 2$ and $N = 4$, respectively. We can see that, MST scheme can provide tremendous power gain over TDMA scheme. Specifically, we can always observe that, when $N \leqslant M$, an ergodic sum-rate increase of N bps/Hz for every 3 dB can be achieved by using TDMA scheme, while M bps/Hz increase for every 3 dB can be achieved by using MST scheme. These simulation results validate our DOF analysis.

Note that, the base of logarithmic function in DOF definition is 2, while the base of logarithmic function in 'dB' definition is 10. Thus, the sum rate increase for every 3 dB is the DOF.

7 Conclusion

In this paper, under the assumption that each node has delayed CSIT, we investigate the achievable DOF for MIMO two-way X relay channel. We discuss three schemes: our proposed MST, Vaze's bound, and

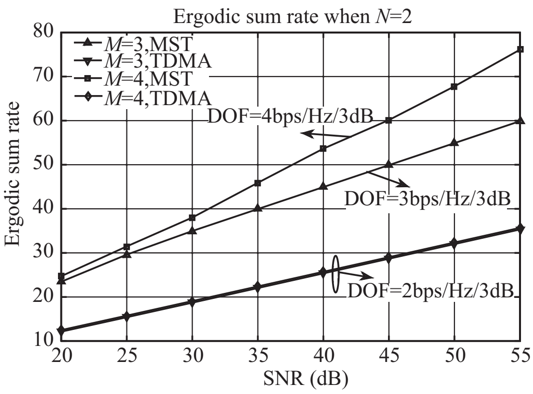

Figure 3 Ergodic sum rate when N=2.

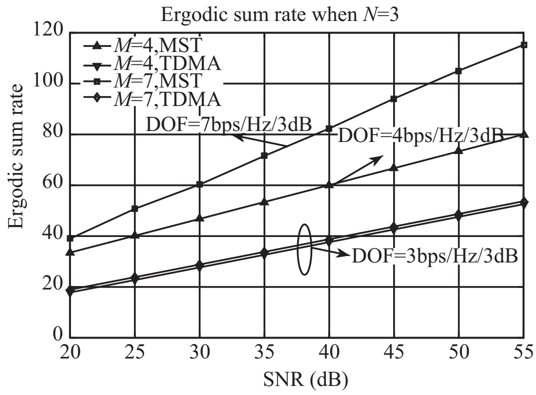

Figure 4 Ergodic sum rate when N=3.

TDMA. DOF analysis shows that MST can achieve significant DOF gain over other two schemes. The sum rate simulation illustrates MST can provide tremendous power gain over TDMA. However, with delayed CSIT, the DOF characterization of more complex multi-way relay channels are still unsolved. We will extend our MST to more specific multi-way relay channels in future study.

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Conflict of interest The authors declare that they have no conflict of interest.

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