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Further results on quantized stabilization of nonlinear cascaded systems with dynamic uncertainties

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Abstract This article studies the quantized partial-state feedback stabilization of a class of nonlinear cascaded systems with dynamic uncertainties. Under the assumption that the dynamic uncertainties are input-to-state practically stable, a novel recursive design method is developed for quantized stabilization by taking into account the influence of quantization and using the small-gain theorem. When the dynamic uncertainty is input-to-state stable, asymptotic stabilization can be achieved with the proposed quantized control law.

Keywords quantized control, nonlinear systems, dynamic uncertainties, asymptotic stabilization

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1 Introduction

The convergence of controls and communications motivates the study of quantized control, which is highly relevant to engineering applications such as smart grids, intelligent transportation systems, robotic networks, and other cyber-physical systems. In a quantized control system, the control signals and/or the measured outputs are processed by quantizers, which map the signals from continuous spaces to discrete sets. The study of the effect of quantization to control systems can be tracked back to the work [1] by Kalman. Recent results on quantized control of linear systems include Ref. [2] based on a discrete-event model, Refs. [3–5] studying dynamic quantization (dynamically scaling the quantization levels), and Refs. [6, 7] considering the coarsest quantizer and numerous references therein.

This article focuses on the quantized control of nonlinear systems. Significant development has also been achieved in this direction. Ref. [8] extended the result in [6] using a control Lyapunov function method. In [4], dynamic quantization was realized for input-to-state stabilizable nonlinear systems. In [9], an n-bit quantized state feedback control law was developed for n-dimensional feed-forward systems. The conditions under which a logarithmic quantizer does not cancel the stabilizing effect of a continuous feedback control law were studied in [10] for dissipative systems. Recently, we have contributed some

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novel solutions to the quantized state feedback stabilization problem of strict-feedback systems [11–13] and the quantized output-feedback control of output-feedback systems [14, 15]. See also [16] for a recent survey of the literature. These systems have been widely studied in the literature of nonlinear controls; see [17–19] and the references therein. However, our previous results do not address dynamic uncertainties and do not guarantee asymptotic convergence. The research presented here is motivated by relaxing the limitations of our previous work.

In this article, we consider the quantized stabilization problem of a general class of nonlinear cascaded systems with dynamic uncertainties. To the best of our knowledge, this problem has not been fully studied in the quantized control literature. Based on the concepts of input-to-state stability (ISS) [20] and input-to-state practical stability (ISpS) [21], this article contributes a new small-gain design, which can be readily used to handle the quantized control problem in the presence of dynamic uncertainties. Also, when the ISpS property of the dynamic uncertainty becomes ISS, asymptotic stabilization can be achieved for the closed-loop system with the proposed quantized control law. The interested reader should consult [20] for a tutorial of ISS and [21] for the ISS small-gain theorem and applications. This article can be considered as a quantized version of the asymptotic stabilization result in [22], though we only consider ISS (instead of IOS) dynamic uncertainties.

The rest of the paper is organized as follows. Section 2 gives the problem formulation. In Section 3, we present a design ingredient based on which the quantized control problem can be solved recursively. The main result of the paper is given in Section 4. In Section 5, we employ a design example to show the effectiveness of the proposed method. Section 6 contains some concluding remarks.

Some notations and definitions that are commonly used in the article are given here. $\mathbb{R}^n$, $\mathbb{R}_+$ and $\mathbb{Z}_+$ represent the n-dimensional Euclidean space, the set of nonnegative real numbers and the set of nonnegative integers, respectively. $|x|$ represents the Euclidean norm of a vector $x \in \mathbb{R}^n$. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be positive definite if $\alpha(0) = 0$ and $\alpha(s) > 0$ for $s > 0$. A continuous function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be a class $\mathcal{K}$ function, denoted by $\alpha \in \mathcal{K}$, if it is strictly increasing and $\alpha(0) = 0$; it is said to be a class $\mathcal{K}_\infty$ function, denoted by $\alpha \in \mathcal{K}_\infty$, if it is a class $\mathcal{K}$ function and satisfies $\alpha(s) \to \infty$ as $s \to \infty$.

2 Problem formulation

The class of nonlinear cascaded systems studied in this article takes the form:

$$\dot{z}_i = p_i(X_i, Z_i), \tag{1}$$

$$\dot{x}_i = f_i(X_i, Z_i) + g_i(X_i, Z_i)x_{i+1}, \tag{2}$$

for $i = 1, \ldots, n$, where $[z_1^{\mathrm{T}}, x_1, \ldots, z_n^{\mathrm{T}}, x_n]^{\mathrm{T}}$ with $z_i \in \mathbb{R}^{m_i}$ and $x_i \in \mathbb{R}$ represent the state, $x_{n+1} := u \in \mathbb{R}$ is the control input, and $x_1 := y$ is the output, $X_i = [x_1, \ldots, x_i]^{\mathrm{T}}$ and $Z_i = [z_1^{\mathrm{T}}, \ldots, z_i^{\mathrm{T}}]^{\mathrm{T}}$. The maps p_i, f_i, g_i are locally Lipschitz. It is assumed that $z_1, \ldots, z_n$, the states of dynamic uncertainty, are not available for feedback control design. Our objective is to develop a quantized control law to stabilize systems (1) and (2) using the quantized state $Q(X) := [q_1(x_1), \ldots, q_n(x_n)]^{\mathrm{T}}$, where $X := [x_1, \ldots, x_n]^{\mathrm{T}}$ and $q_i(x_i)$ is the quantized signal of x_i.

Assumptions 1–4 are made on systems (1) and (2) throughout the article.

Assumption 1. For each $i = 1, \ldots, n$, there exist an $\alpha_{p_i} \in \mathcal{K}_\infty$ being Lipschitz on compact sets and a constant $\delta_{p_i} \geqslant 0$, such that for all X_i, Z_i,

$$|p_i(X_i, Z_i)| \leqslant \alpha_{p_i}(|[X_i^{\mathrm{T}}, Z_i^{\mathrm{T}}]^{\mathrm{T}}|) + \delta_{p_i}. \tag{3}$$

Assumption 2. For each $i = 1, \ldots, n$, there exist an $\alpha_{f_i} \in \mathcal{K}_\infty$ being Lipschitz on compact sets and a constant $\delta_{f_i} \geqslant 0$, such that for all X_i, Z_i,

$$|f_i(X_i, Z_i)| \leqslant \alpha_{f_i}(|[X_i^{\mathrm{T}}, Z_i^{\mathrm{T}}]^{\mathrm{T}}|) + \delta_{f_i}. \tag{4}$$

Remark 1. It should be noted that for any locally Lipschitz function $\varphi : \mathbb{R}^m \to \mathbb{R}$, there exists $\alpha_\varphi \in \mathcal{K}_\infty$ being Lipschitz on compact sets and $\delta_\varphi \geqslant 0$, such that $|\varphi(x)| \leqslant \alpha_\varphi(|x|) + c_\varphi$, for all $x \in \mathbb{R}^m$. This can be proved by defining $c_\varphi = |\varphi(0)|$ and finding an $\alpha_\varphi \in \mathcal{K}_\infty$, such that $\alpha_\varphi \geqslant \alpha_\varphi^0$ with $\alpha_\varphi^0(s) := \max_{|x|\leqslant s}\{|\varphi(x) - \varphi(0)|\}$, for $s \geqslant 0$. Also, if $\varphi(0) = 0$, then we can set $c_\varphi = 0$. Thus, Assumptions 1 and 2 are not restrictive.

Assumption 3. For each $i = 1, \ldots, n$, there exists a known constant $\underline{g}_i > 0$, such that for all X_i, Z_i,

$$|g_i(X_i, Z_i)| \geqslant \underline{g}_i. \tag{5}$$

We recall the definition of ISpS from [21] to make this article self-contained.

Definition 1. The nonlinear system $\dot{x} = f(x, u)$ with state $x \in \mathbb{R}^n$ and input $u \in \mathbb{R}^m$ is ISpS if there exist a $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}$ and a constant $\delta \geqslant 0$, such that for each initial time t_0, each initial condition $x(t_0)$ and each piecewise continuous bounded control $u : [t_0, \infty) \to \mathbb{R}^m$, the solution of the system exists on $[t_0, \infty)$ and is such that

$$|x(t)| \leqslant \max\{\beta(|x(t_0)|, t - t_0), \gamma(\|u\|_{[t_0,t]}), \delta\}, \quad \text{for all } t \geqslant t_0. \tag{6}$$

If Eq. (6) holds with $\delta = 0$, ISpS reduces to Sontag's ISS [20]. γ is called ISS gain of system $\dot{x} = f(x, u)$.

Remark 2. If system $\dot{x} = f(x, u)$ is ISpS, then system $\dot{x} = f(x, 0)$ is practically stable. See, e.g., [23, Definition 1] for the definition of practical stability.

Assumption 4. For $i = 1, \ldots, n$, each z_i-system (1) with $[X_i^{\mathrm{T}}, Z_{i-1}^{\mathrm{T}}]^{\mathrm{T}}$ as the input is ISpS with an ISS gain being Lipschitz on compact sets.

Remark 3. The assumption on the Lipschitz property of the ISS gains is not restrictive. If the ISS gain γ defined in Eq. (6) is of class $\mathcal{K}$ but not Lipschitz on compact sets, then we can replace γ with a $\hat{\gamma} \in \mathcal{K}_\infty$ being Lipschitz on compact sets, such that $\hat{\gamma}(s) \geqslant (1 + \rho)(\gamma(s) - \epsilon)$ and replace δ with $\hat{\delta} := \max\{(1 + \rho)\epsilon/\rho, \delta\}$, where constants ρ, ϵ can be arbitrarily small. It can be verified that $\max\{\gamma(s), \delta\} \leqslant \max\{\hat{\gamma}(s)/(1 + \rho) + \epsilon, \delta\} \leqslant \max\{\hat{\gamma}(s), (1 + \rho)\epsilon/\rho, \delta\} = \max\{\hat{\gamma}, \hat{\delta}\}$ for all $s \geqslant 0$.

In this article, we consider state quantizers q_i that satisfy the sector bound property.

Assumption 5. For $i = 1, \ldots, n$, each quantizer q_i is a piecewise constant function, and there exist known constants $0 \leqslant b_i < 1$ and $a_i \geqslant 0$, such that for all $x_i \in \mathbb{R}$,

$$|q_i(x_i) - x_i| \leqslant b_i|x_i| + (1 - b_i)a_i. \tag{7}$$

Remark 4. The quantizers with the sector bound property like in Eq. (7) have been considered in several existing results on quantized control of linear and nonlinear systems; see, e.g., Refs. [6, 7, 10, 12]. In particular, our recent article [12] studied the quantized control for nonlinear systems without dynamic uncertainties.

3 An ingredient of recursive control design

The basic idea of this article is to develop a design ingredient that can be readily used to solve the quantized control problem. We consider the following system:

$$\dot{\zeta}_0 = h(\zeta_0, \zeta_1), \tag{8}$$

$$\dot{\zeta}_1 = f(\zeta_0, \zeta_1) + g(\zeta_0, \zeta_1)\zeta_2, \tag{9}$$

where $[\zeta_0^{\mathrm{T}}, \zeta_1]^{\mathrm{T}}$, with $\zeta_0 \in \mathbb{R}^{n_0}$ and $\zeta_1 \in \mathbb{R}$, is the state, $\zeta_2 \in \mathbb{R}$ is the input, $h : \mathbb{R}^{n_0} \times \mathbb{R} \to \mathbb{R}^{n_0}$ and $f, g : \mathbb{R}^{n_0} \times \mathbb{R} \to \mathbb{R}$ are locally Lipschitz functions.

The following Assumptions 6–9 are made on systems (8) and (9). In Section 4, we will transform the quantized control problem for systems (1) and (2) into a recursive control design problem for systems in the form of Eqs. (8) and (9) satisfying Assumptions 6–9.

Assumption 6. There exist an $\alpha_h \in \mathcal{K}_\infty$ being Lipschitz on compact sets and a constant $\delta_h \geqslant 0$, such that for all ζ_0, ζ_1,

$$|h(\zeta_0, \zeta_1)| \leqslant \alpha_h(|[\zeta_0^{\mathrm{T}}, \zeta_1]^{\mathrm{T}}|) + \delta_h. \tag{10}$$

Assumption 7. There exist an $\alpha_f \in \mathcal{K}_\infty$ being Lipschitz on compact sets and a constant $\delta_f \geqslant 0$, such that for all ζ_0, ζ_1,

$$|f(\zeta_0, \zeta_1)| \leqslant \alpha_f(|[\zeta_0^{\mathrm{T}}, \zeta_1]^{\mathrm{T}}|) + \delta_f. \tag{11}$$

Assumption 8. There exists a known constant $\underline{g} > 0$, such that

$$|g(\zeta_0, \zeta_1)| \geqslant \underline{g}, \tag{12}$$

for all ζ_0, ζ_1.

Assumption 9. There exists a set-valued map $S_0 : \mathbb{R}^{n_0} \rightsquigarrow \mathbb{R}$ with $\max S_0$ and $\min S_0$ continuously differentiable almost everywhere, such that the system described by differential inclusion:

$$\dot{\zeta}_0 \in H(\zeta_0, \tilde{\zeta}_1) := \left\{ h(\zeta_0, \hat{\zeta}_1 + \tilde{\zeta}_1) : \hat{\zeta}_1 \in S_0(\zeta_0) \right\}, \tag{13}$$

with $\tilde{\zeta}_1$ as the input is ISpS with an ISS gain being Lipschitz on compact sets. Also, there exist an $\alpha_{S_0} \in \mathcal{K}_\infty$ being Lipschitz on compact sets and a constant $\delta_{S_0} \geqslant 0$, such that for all $\zeta_0 \in \mathbb{R}^{n_0}$,

$$\max\{|\max S_0(\zeta_0)|, |\min S_0(\zeta_0)|\} \leqslant \alpha_{S_0}(|\zeta_0|) + \delta_{S_0}. \tag{14}$$

Remark 5. Assumption 9 means that the ζ_0-system can be input-to-state practically stabilized by considering ζ_1 as the "virtual" control input and using the feedback control law $\zeta_1 \in Q_0(\zeta_0, \tilde{\zeta}_1) := \{\hat{\zeta}_1 + \tilde{\zeta}_1 : \hat{\zeta}_1 \in S_0(\zeta_0)\}$. A special case is that the ζ_0-system is ISpS by itself. In this case, by defining $S_0(\zeta_0) \equiv \{0\}$, the ζ_0-system still satisfies Assumption 9 with $\hat{\zeta}_1 = 0$ and $\tilde{\zeta}_1 = \zeta_1$.

Lemma 1. Under Assumptions 6–9, there exists a set-valued map $S_1 : \mathbb{R}^{n_0+1} \rightsquigarrow \mathbb{R}$ in the form of

$$\begin{aligned} S_1(\zeta_0, \zeta_1) = \Bigg\{ & d_{12}\kappa_1(\zeta_1 - \zeta_1^* + d_{11}) : \zeta_1^* \in \hat{S}_0(\zeta_0),\ |d_{11}| \leqslant b_1|\zeta_1| + (1 - b_1)a_1, \\ & \frac{1}{1 + b_2} \leqslant d_{12} \leqslant \frac{1}{1 - b_2} \Bigg\}, \end{aligned} \tag{15}$$

where $\kappa_1 : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and satisfies $\kappa_1(0) = 0$,

$$\hat{S}_0(\zeta_0) = \left\{ \zeta_1^* : d_{02}\zeta_1^* \in S_0(\zeta_0) \text{ for } \frac{1}{1 + b_1} \leqslant d_{02} \leqslant \frac{1}{1 - b_1} \right\}, \tag{16}$$

and constants a_1, b_1, b_2 satisfy $a_1 \geqslant 0$, $0 \leqslant b_1 < 1$, $0 \leqslant b_2 < 1$, such that the system

$$\begin{bmatrix} \dot{\zeta}_0 \\ \dot{\zeta}_1 \end{bmatrix} \in \left\{ \begin{bmatrix} h(\zeta_0, \zeta_1) \\ f(\zeta_0, \zeta_1) + g(\zeta_0, \zeta_1)(\hat{\zeta}_2 + \tilde{\zeta}_2) \end{bmatrix} : \hat{\zeta}_2 \in S_1(\zeta_0, \zeta_1) \right\}, \tag{17}$$

with $\tilde{\zeta}_2$ as the input is ISpS with an ISS gain being Lipschitz on compact sets. Also, if Assumptions 6 and 7 are satisfied with $\delta_h = \delta_f = 0$ and Assumption 9 is satisfied with the ζ_0-system being ISS and $\delta_{S_0} = 0$, then the (ζ_0, ζ_1)-system can be designed to be ISS by choosing an S_1 in the form of Eq. (15) with $a_1 = 0$.

Proof. For $r \in \mathbb{R}$ and closed $S \subset \mathbb{R}$, denote $\vec{d}(r, S) = r - \operatorname{argmin}_{r' \in S}\{|r - r'|\}$. Define $\tilde{\zeta}_1 = \vec{d}(\zeta_1, S_0(\zeta_0))$. Clearly, $\zeta_1 - \tilde{\zeta}_1 \in S_0(\zeta_0)$. With such definition, the ζ_0-subsystem with $\tilde{\zeta}_1$ as the input is ISpS with an ISS gain being Lipschitz on compact sets.

When $\tilde{\zeta}_1 > 0$, $\tilde{\zeta}_1 = \zeta_1 - \max S_0(\zeta_0)$. By taking the derivative of $\tilde{\zeta}_1$, we have

$$\dot{\tilde{\zeta}}_1 = \dot{\zeta}_1 - \nabla \max S_0(\zeta_0)\dot{\zeta}_0$$

$$= f(\zeta_0,\zeta_1)+g(\zeta_0,\zeta_1)\zeta_2-\nabla\max S_0(\zeta_0)h(\zeta_0,\zeta_1)$$
$$= f(\zeta_0,\zeta_1)+g(\zeta_0,\zeta_1)(\zeta_2-\tilde{\zeta}_2)+g(\zeta_0,\zeta_1)\tilde{\zeta}_2-\nabla\max S_0(\zeta_0)h(\zeta_0,\zeta_1), \tag{18}$$

wherever $\nabla\max S_0$ exists. Note that $\max S_0$ is continuously differentiable almost everywhere. Thus, $\nabla\max S_0$ may be discontinuous and the $\tilde{\zeta}_1$-system may be a discontinuous system. We represent the solutions of the $\tilde{\zeta}_1$-system with a differential inclusion by defining

$$\partial\max S_0(\zeta_0)=\bigcap_{\epsilon>0}\bigcap_{\mu(\tilde{\mathcal{M}})=0}\overline{\mathrm{co}}\left\{\nabla\max S_0\left(\mathcal{B}_\epsilon(\zeta_0)\backslash\tilde{\mathcal{M}}\right)\right\}, \tag{19}$$

where $\mathcal{B}_\epsilon(\eta)$ represents the ball of radius ϵ around ζ_0, and $\tilde{\mathcal{M}}$ represents all the sets belonging to $\mathbb{R}^{n_0}$ of zero measure (i.e., $\mu(\tilde{\mathcal{M}})=0$). Thus, $\partial\max S_0$ is convex, compact and upper semi-continuous.

Then, in the case of $\tilde{\zeta}_1>0$, the $\tilde{\zeta}_1$-system can be represented by

$$\dot{\tilde{\zeta}}_1\in\left\{g(\zeta_0,\zeta_1)(\zeta_2-\tilde{\zeta}_2)+\phi:\phi\in\Phi^*(\zeta_0,\zeta_1,\tilde{\zeta}_2)\right\}, \tag{20}$$

where

$$\Phi^*(\zeta_0,\zeta_1,\tilde{\zeta}_2)=\left\{f(\zeta_0,\zeta_1)+g(\zeta_0,\zeta_1)\tilde{\zeta}_2+\phi_0h(\zeta_0,\zeta_1):\phi_0\in\partial\max S_0(\zeta_0)\right\}. \tag{21}$$

Under Assumptions 6 and 7, using the definition of $\tilde{\zeta}_1$ and property (14), there exist an $\alpha_{\Phi^*}\in\mathcal{K}_\infty$ being Lipschitz on compact sets and a constant $\delta_{\Phi^*}\geqslant 0$, such that for all $\zeta_0,\tilde{\zeta}_1,\tilde{\zeta}_2$,

$$\max\left\{|\max\Phi^*(\zeta_0,\zeta_1,\tilde{\zeta}_2)|,|\min\Phi^*(\zeta_0,\zeta_1,\tilde{\zeta}_2)|\right\}\leqslant\alpha_{\Phi^*}(|[\zeta_0^{\mathrm{T}},\tilde{\zeta}_1,\tilde{\zeta}_2]^{\mathrm{T}}|)+\delta_{\Phi^*}. \tag{22}$$

Note that the set-valued map $S_1(\zeta_0,\zeta_1)$ defined in Eq. (15) is determined by κ_1. We construct the set-valued map by finding an appropriate κ_1. For any $\iota>0$, any $0<c<1$ and any $\gamma_{\tilde{\zeta}_1}^{\zeta_0},\gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2},\chi\in\mathcal{K}_\infty$ with $\left(\gamma_{\tilde{\zeta}_1}^{\zeta_0}\right)^{-1},\left(\gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2}\right)^{-1},\chi^{-1}$ being Lipschitz on compact sets, one can find a $\kappa_1^0:\mathbb{R}_+\to\mathbb{R}_+$ which is positive, nondecreasing, and continuously differentiable on $(0,\infty)$ and satisfies

$$\frac{\underline{g}(1-b_1)(1-c)}{1+b_2}\kappa_1^0\left((1-b_1)(1-c)s\right)s$$
$$\geqslant\iota s+\alpha_{\Phi^*}\left(\left(\gamma_{\tilde{\zeta}_1}^{\zeta_0}\right)^{-1}(s)+\left(\gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2}\right)^{-1}(s)+s\right)+\chi^{-1}(s), \tag{23}$$

for all $s>0$. The existence of such κ_1^0 is guaranteed by Ref. [22, Lemma 1]. Define

$$\kappa_1(r)=-\kappa_1^0(|r|)r, \tag{24}$$

for $r\in\mathbb{R}$. Then, κ_1 is continuously differentiable on $(-\infty,\infty)$ and $\kappa_1(0)=0$.

Define $V_{\tilde{\zeta}_1}(\tilde{\zeta}_1)=|\tilde{\zeta}_1|$ as an ISS-Lyapunov function candidate for the $\tilde{\zeta}_1$-subsystem. We study the case of

$$V_{\tilde{\zeta}_1}(\tilde{\zeta}_1)\geqslant\max\left\{\gamma_{\tilde{\zeta}_1}^{\zeta_0}(|\zeta_0|),\gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2}(|\tilde{\zeta}_2|),\chi(|\delta_{\Phi^*}|),\frac{a_1}{c}\right\}. \tag{25}$$

From the definition of $\hat{S}_0$ in Eq. (16), we have

$$\max S_0(\zeta_0)\geqslant\max\left\{\frac{1}{1+b_1}\max\hat{S}_0(\zeta_0),\frac{1}{1-b_1}\max\hat{S}_0(\zeta_0)\right\}, \tag{26}$$

$$\min S_0(\zeta_0)\leqslant\min\left\{\frac{1}{1+b_1}\min\hat{S}_0(\zeta_0),\frac{1}{1-b_1}\min\hat{S}_0(\zeta_0)\right\}. \tag{27}$$

Consider the following cases for $\zeta_1-\zeta_1^*+d_{11}$ with $\zeta_1^*\in\hat{S}_0(\zeta_0)$, $|d_{11}|\leqslant b_1|\zeta_1|+(1-b_1)a_1$ and $\zeta_1\geqslant 0$:

(i) $\zeta_1 > \max S_0(\zeta_0)$ (i.e., $\tilde{\zeta}_1 > 0$):

$$\begin{aligned}\zeta_1 - \zeta_1^* + d_{11} &\geqslant (1-b_1)\zeta_1 - \max \hat{S}_0(\zeta_0) - (1-b_1)a_1 \\ &= (1-b_1)\left(\zeta_1 - \max\left(\frac{1}{1-b_1}\hat{S}_0(\zeta_0)\right) - a_1\right) \\ &\geqslant (1-b_1)(\zeta_1 - \max S_0(\zeta_0) - a_1) \\ &= (1-b_1)(\tilde{\zeta}_1 - a_1); \end{aligned} \tag{28}$$

(ii) $\zeta_1 < \min S_0(\zeta_0)$ (i.e., $\tilde{\zeta}_1 < 0$):

$$\begin{aligned}\zeta_1 - \zeta_1^* + d_{11} &\leqslant (1+b_1)\zeta_1 - \min \hat{S}_0(\zeta_0) + (1-b_1)a_1 \\ &= (1+b_1)\left(\zeta_1 - \min\left(\frac{1}{1+b_1}\hat{S}_0(\zeta_0)\right)\right) + (1-b_1)a_1 \\ &\leqslant (1-b_1)(\tilde{\zeta}_1 + a_1). \end{aligned} \tag{29}$$

Thus, with $\zeta_1^* \in \hat{S}_0(\zeta_0)$ and $|d_{11}| \leqslant b_1|\zeta_1| + (1-b_1)a_1$, if $\zeta_1 \geqslant 0$,

$$(1-b_1)(\tilde{\zeta}_1 - a_1) \leqslant \zeta_1 - \zeta_1^* + d_{11} \leqslant (1-b_1)(\tilde{\zeta}_1 + a_1). \tag{30}$$

Similarly, property (30) can also be proved for the case of $\zeta_1 < 0$. Note that, in the case of Eq. (25), $|\zeta_0| \leqslant \left(\gamma_{\tilde{\zeta}_1}^{\zeta_0}\right)^{-1}(|\tilde{\zeta}_1|)$, $|\tilde{\zeta}_2| \leqslant \left(\gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2}\right)^{-1}(|\tilde{\zeta}_1|)$, $\delta_{\Phi^*} \leqslant \chi^{-1}(|\tilde{\zeta}_1|)$, and $a_1 \leqslant c|\tilde{\zeta}_1|$. Then, we have

$$|\zeta_1 - \zeta_1^* + d_{11}| \geqslant (1-b_1)(1-c)|\tilde{\zeta}_1|, \quad \operatorname{sgn}(\zeta_1 - \zeta_1^* + d_{11}) = \operatorname{sgn}(\tilde{\zeta}_1). \tag{31}$$

Thus, in the case of Eq. (25), for any $|d_{11}| \leqslant b_1|\zeta_1| + (1-b_1)a_1$, $\frac{1}{1+b_2} \leqslant d_{12} \leqslant \frac{1}{1-b_2}$, $\zeta_1^* \in \hat{S}_0(\zeta_0)$ and $\phi \in \Phi^*(\zeta_0, \zeta_1, \tilde{\zeta}_2)$, it holds that

$$\begin{aligned}&\nabla V_{\tilde{\zeta}_1}(\tilde{\zeta}_1)\left(g(\zeta_0,\zeta_1)d_{12}\kappa_1(\zeta_1 - \zeta_1^* + d_{11}) + \phi\right) \\ &\quad = \operatorname{sgn}(\tilde{\zeta}_1)\left(g(\zeta_0,\zeta_1)d_{12}\kappa_1(\zeta_1 - \zeta_1^* + d_{11}) + \phi\right) \\ &\quad \leqslant -\frac{g(\zeta_0,\zeta_1)}{1+b_2}\kappa_1^0(|\zeta_1 - \zeta_1^* + d_{11}|)|\zeta_1 - \zeta_1^* + d_{11}| + |\phi| \\ &\quad \leqslant -\frac{g}{1+b_2}\kappa_1^0(|\zeta_1 - \zeta_1^* + d_{11}|)|\zeta_1 - \zeta_1^* + d_{11}| + |\phi| \\ &\quad \leqslant -\frac{g}{1+b_2}\kappa_1^0((1-b_1)(1-c)|\tilde{\zeta}_1|)(1-b_1)(1-c)|\tilde{\zeta}_1| + \alpha_{\Phi^*}(|[\zeta_0^{\mathrm{T}}, \tilde{\zeta}_1, \tilde{\zeta}_2]^{\mathrm{T}}|) + \delta_{\Phi^*} \\ &\quad \leqslant -\iota|\tilde{\zeta}_1|, \quad \text{a.e.}\end{aligned} \tag{32}$$

As a result, for any $\gamma_{\tilde{\zeta}_1}^{\zeta_0}, \gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2}, \chi \in \mathcal{K}_\infty$ with $\left(\gamma_{\tilde{\zeta}_1}^{\zeta_0}\right)^{-1}, \left(\gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2}\right)^{-1}, \chi^{-1}$ being Lipschitz on compact sets, we can design κ_1, such that if Eq. (25) holds, then

$$\max_{f_{\tilde{\zeta}_1} \in F_{\tilde{\zeta}_1}(\zeta_0, \zeta_1, \tilde{\zeta}_2)} \nabla V_{\tilde{\zeta}_1}(\tilde{\zeta}_1) f_{\tilde{\zeta}_1} \leqslant -\iota V_{\tilde{\zeta}_1}(\tilde{\zeta}_1), \quad \text{a.e.} \tag{33}$$

where

$$F_{\tilde{\zeta}_1}(\zeta_0, \zeta_1, \tilde{\zeta}_2) := \left\{ g(\zeta_0,\zeta_1)(\zeta_2 - \tilde{\zeta}_2) + \phi : \zeta_2 - \tilde{\zeta}_2 \in S_1(\zeta_0, \zeta_1), \phi \in \Phi^*(\zeta_0, \zeta_1, \tilde{\zeta}_2) \right\}. \tag{34}$$

Thus, $V_{\tilde{\zeta}_1}$ is an ISS-Lyapunov function of the $\tilde{\zeta}_1$-system, and the $\tilde{\zeta}_1$-system has been designed to be ISpS. In particular, there exist $\beta_{\tilde{\zeta}_1} \in \mathcal{KL}$, $\gamma_{\tilde{\zeta}_1}^{\eta_0}, \gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2} \in \mathcal{K}_\infty$ with $\left(\gamma_{\tilde{\zeta}_1}^{\eta_0}\right)^{-1}, \left(\gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2}\right)^{-1}$ being Lipschitz on compact sets and constant $\delta_{\tilde{\zeta}_1} \geqslant 0$, such that

$$|\tilde{\zeta}_1(t)| \leqslant \max\left\{ \beta_{\tilde{\zeta}_1}(|\tilde{\zeta}_1(t_0)|, t - t_0), \gamma_{\tilde{\zeta}_1}^{\zeta_0}(\|\zeta_0\|_{[t_0,t]}), \gamma_{\tilde{\zeta}_1}^{\tilde{\zeta}_2}(\|\tilde{\zeta}_2\|_{[t_0,t]}), \delta_{\tilde{\zeta}_1} \right\}, \tag{35}$$

for all $t \geqslant t_0 \geqslant 0$, where $\delta_{\tilde{\zeta}_1} = \max\{\chi(|\delta_{\Phi^*}|), a_1/c\}$.

Consider the interconnection of the ζ_0-system and the $\tilde{\zeta}_1$-system. Since $\gamma_{\zeta_0}^{\tilde{\zeta}_1}$ is Lipschitz on compact sets, we can design the $\tilde{\zeta}_1$-system, such that both $\gamma_{\tilde{\zeta}_1}^{\zeta_0}$ and $\left(\gamma_{\tilde{\zeta}_1}^{\zeta_0}\right)^{-1}$ are Lipschitz on compact sets and the small-gain condition $\gamma_{\zeta_0}^{\tilde{\zeta}_1} \circ \gamma_{\tilde{\zeta}_1}^{\zeta_0} < \mathrm{Id}$ is satisfied. Using the small-gain theorem for interconnected ISpS systems, we can guarantee the ISpS of the $(\zeta_0, \tilde{\zeta}_1)$-system and thus the (ζ_0, ζ_1)-system. Also, the ISS gain can be proved to be Lipschitz on compact sets. This ends the proof of Lemma 1.

Remark 6. Lemma 1 means that the (ζ_0, ζ)-system can be input-to-state practically stabilized by considering ζ_2 as the "virtual" control input and using the feedback control law $\zeta_2 \in Q_1(\zeta_0, \zeta_1, \tilde{\zeta}_2) := \{\hat{\zeta}_2 + \tilde{\zeta}_2 : \hat{\zeta}_2 \in S_1(\zeta_0, \zeta_1)\}$.

Remark 7. In the discussion of the Lipschitz property of the ISS gains, we used the result that the ISS gain of an interconnected system composed of ISS subsystems is Lipschitz on compact sets if the ISS gains of the subsystems are Lipschitz on compact sets. This result can be verified by directly using the ISS gain estimate (19) in Eq. [21] and the fact that the composition of functions being Lipschitz on compact sets is Lipschitz on compact sets.

Remark 8. Lemma 1 is motivated by [22, Section IV]. Ref. [22, Section IV] studies nonlinear systems composed of two subsystems, one of which is input-to-output practically stable (IOpS). In this article, we focus on ISS small-gain based designs and do not assume IOpS properties. Instead, for direct usability of the ingredient to cope with quantization, we consider nonlinear systems composed of two subsystems, one of which is input-to-state practically stabilizable, and develop a set-valued map design. Such treatment may reduce the complexity of the design procedure.

4 Quantized stabilization

With the help of the design tool presented in Section 3, we can develop a class of quantized control laws for the nonlinear cascaded systems in the form of Eqs. (1) and (2). The main result of this article is given in the following theorem.

Theorem 1. Consider systems (1) and (2). Under Assumptions 1–5, there exists a quantized partial-state feedback control law in the form of $u = u(q_1(x_1), \ldots, q_n(x_n))$, such that all the signals in the closed-loop quantized system are bounded. Also, if the offsets δ_{p_i}'s in Assumption 1, δ_{f_i}'s in Assumption 2, and a_i's in Assumption 5 are equal to zero and each z_i-subsystem is ISS, then the closed-loop quantized system is asymptotically stable.

Proof. We prove Theorem 1 by designing a novel quantized partial-state feedback controller through a step-by-step recursive design procedure.

Initial step. The objective of this step is to find a set-valued map $S_1 : \mathbb{R} \rightsquigarrow \mathbb{R}$, such that if $x_2 \in \{\hat{x}_2 + \tilde{x}_2 : \hat{x}_2 \in S_1(X_1)\}$, then the (Z_2, X_1)-subsystem with $\tilde{x}_2$ as the input is ISpS with an ISS gain being Lipschitz on compact sets.

Consider the (z_1, x_1)-subsystem of systems (1) and (2):

$$\dot{z}_1 = p_1(X_1, Z_1), \tag{36}$$

$$\dot{x}_1 = f_1(X_1, Z_1) + g_1(X_1, Z_1)x_2. \tag{37}$$

Under Assumption 4, the z_1-system is ISpS with x_1 as the input. Using Lemma 1, we define the set-valued map S_1 as

$$S_1(X_1) = \left\{ d_{12}\kappa_1(x_1 + d_{11}) : |d_{11}| \leqslant b_1|x_1| + (1 - b_1)a_1, \frac{1}{1 + b_2} \leqslant d_{12} \leqslant \frac{1}{1 - b_2} \right\}, \tag{38}$$

where $\kappa_1 : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and satisfies $\kappa_1(0) = 0$, such that nonlinear system

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{X}_1 \end{bmatrix} \in \left\{ \begin{bmatrix} p_1(X_1, Z_1) \\ f_1(X_1, Z_1) + g_1(X_1, Z_1)(\hat{x}_2 + \tilde{x}_2) \end{bmatrix} : \hat{x}_2 \in S_1(x_1) \right\} \tag{39}$$

is ISpS with the ISS gain being Lipschitz on compact sets.

For convenience of notations, denote $p_2(x_1, x_2, Z_2) = p_2(X_2, Z_2)$. Define

$$P_2(x_1, \tilde{x}_2, Z_2) = \{p_2(x_1, \hat{x}_2 + \tilde{x}_2, Z_2) : \hat{x}_2 \in S_1(x_1)\}, \tag{40}$$

and consider

$$\dot{z}_2 \in P_2(x_1, \tilde{x}_2, Z_2). \tag{41}$$

Because κ_1 is continuously differentiable and $\kappa_1(0) = 0$, there exist an $\alpha_1 \in \mathcal{K}_\infty$ being Lipschitz on compact sets and a constant $\delta_1 \geqslant 0$, such that for all Z_1, X_2,

$$|[Z_1, X_2^{\mathrm{T}}]^{\mathrm{T}}| \leqslant \alpha_1(|[Z_1, X_1, \tilde{x}_2]^{\mathrm{T}}|) + \delta_1. \tag{42}$$

Under the Assumption 4, the z_2-subsystem with (Z_1, X_2) as the input is ISpS with an ISS gain being Lipschitz on compact sets. Because of property (42) and the definition of P_2 in Eq. (40), it can be verified that system (41) with input $(Z_1, X_1, \tilde{x}_2)$ is also ISpS with an ISS gain being Lipschitz on compact sets.

Consider the (Z_2, X_1)-subsystem of systems (1) and (2), i.e., the cascade connection of the (Z_1, X_1)-system and the z_2-subsystem. The small-gain condition is satisfied automatically, and the (Z_2, X_1)-subsystem with input $\tilde{x}_2$ is ISpS with the ISS gain being Lipschitz on compact sets. If the offsets δ_{p_i}'s in Assumption 1, δ_{f_i}'s in Assumption 2, δ_{z_i}'s in Assumption 4, and a_i's in Assumption 5 are equal to zero, then the closed-loop quantized system is asymptotically stable.

Recursive step. Suppose that there exists a set-valued map $S_{m-1} : \mathbb{R}^{m-1} \rightsquigarrow \mathbb{R}$, such that if $x_m \in \{\hat{x}_m + \tilde{x}_m : \hat{x}_m \in S_{m-1}(X_{m-1})\}$, then the (Z_m, X_{m-1})-subsystem with $\tilde{x}_m$ as the input is ISpS with an ISS gain being Lipschitz on compact sets.

Consider the (Z_m, X_m)-subsystem of systems (1) and (2):

$$\dot{z}_i = p_i(X_i, Z_i), \quad i = 1, \ldots, m-1, \tag{43}$$

$$\dot{x}_i = f_i(X_i, Z_i) + g_i(X_i, Z_i)x_{i+1}, \quad i = 1, \ldots, m-1, \tag{44}$$

$$\dot{z}_m = p_m(X_m, Z_m), \tag{45}$$

$$\dot{x}_m = f_m(X_m, Z_m) + g_m(X_m, Z_m)x_{m+1}. \tag{46}$$

Using Lemma 1, we define a set-valued map $S_m : \mathbb{R}^m \rightsquigarrow \mathbb{R}$ as

$$S_m(X_m) = \Bigg\{d_{m2}\kappa_m(x_m - x_m^* + d_{m1}) : x_m^* \in \hat{S}_{m-1}(X_{m-1}), \quad |d_{m1}| \leqslant b_m|x_m| + (1 - b_m)a_m, \quad \frac{1}{1 + b_{m+1}} \leqslant d_{m2} \leqslant \frac{1}{1 - b_{m+1}}\Bigg\}, \tag{47}$$

where $\kappa_m : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and satisfies $\kappa_1(0) = 0$, and

$$\hat{S}_{m-1}(X_{m-1}) = \left\{x_m^* : d_{(m-1)2}x_m^* \in S_{m-1}(X_{m-1}) \text{ for } \frac{1}{1 + b_m} \leqslant d_{(m-1)2} \leqslant \frac{1}{1 - b_m}\right\}, \tag{48}$$

such that the (Z_m, X_m)-subsystem with $x_{m+1} \in \{\hat{x}_{m+1} + \tilde{x}_{m+1} : \hat{x}_{m+1} \in S_m(X_m)\}$ is ISpS with $\tilde{x}_{m+1}$ as the input, and moreover, the ISS gain is Lipschitz on compact sets.

Consider the (Z_{m+1}, X_m)-subsystem of systems (1) and (2), i.e., the cascade connection of the (Z_m, X_m)-system and the z_{m+1}-subsystem. The small-gain condition is satisfied automatically, and the (Z_{m+1}, X_m)-subsystem with input $\tilde{x}_{m+1}$ is ISpS with the ISS gain being Lipschitz on compact sets.

Final step. By repeating the procedure in the recursive step until $m = n$, we can design a set-valued map $S_n(X_n)$, such that systems (1) and (2) with $x_{n+1} \in \{\hat{x}_{n+1} + \tilde{x}_{n+1} : \hat{x}_{n+1} \in S_n(X_n)\}$ with $b_{n+1} = 0$ are ISpS with $\tilde{x}_{n+1}$ as the input. Note that $x_{n+1} = u$ is the control input of the system. If we can design a quantized control law, such that $u \in S_n(X_n)$, then $\tilde{x}_{n+1} = 0$ and the closed-loop system is practically stable. Because of the appropriately chosen S_i for $i = 1, \dots, n$ in the recursive design procedure, we propose a quantized control law in the form of

$$\hat{x}_2 = \kappa_1(q_1(x_1)), \tag{49}$$

$$\hat{x}_{i+1} = \kappa_i(q_i(x_i) - \hat{x}_{i-1}), \qquad i = 2, \dots, n-1, \tag{50}$$

$$u = \kappa_n(q_n(x_n) - \hat{x}_{n-1}), \tag{51}$$

such that

$$\hat{x}_2 \in \hat{S}_1(X_1) \Rightarrow \cdots \Rightarrow \hat{x}_{i+1} \in \hat{S}_i(X_i) \Rightarrow \cdots \Rightarrow u \in S_n(X_n). \tag{52}$$

If the offsets δ_{p_i}'s in Assumption 1, δ_{f_i}'s in Assumption 2, δ_{z_i}'s in Assumption 4, and a_i's in Assumption 5 are equal to zero, then the closed-loop quantized system is asymptotically stable.

If the offsets δ_{f_i}'s in Assumption 2 and a_i's in Assumption 5 are equal to zero and each z_i-subsystem is ISS, then all the ISpS subsystems discussed above are ISS and the closed-loop quantized system is asymptotically stable. This ends the proof of Theorem 1.

5 A simulation example

To illustrate the basic idea of the set-valued map design, we consider a system in the form of systems (1) and (2) with $n = 1$:

$$\dot{z} = -z^3 + x^2, \tag{53}$$

$$\dot{x} = z^3 + u, \tag{54}$$

where $[z, x]^{\mathrm{T}} \in \mathbb{R}^2$ is the state of the system with $z \in \mathbb{R}$ being the state of the dynamic uncertainty, $u \in \mathbb{R}$ is the control input, and only the quantized signal $q(x)$ of x is available for feedback. We consider the case where the quantizer q satisfies $|q(x) - x| \leqslant b|x| + (1-b)a$ with $b = 0.1$ and $a = 0.01$. For convenience of notations, define $p(x, z) = -z^3 + x^2$ and $f(z) = z^3$. Define $\alpha_f(s) = s^3$. Clearly, $|f(z)| = \alpha_f(|z|)$, for all $z \in \mathbb{R}$.

With $V_z(z) = |z|$ considered as the ISS-Lyapunov function of the z-subsystem, it is directly checked that

$$V_z(z) \geqslant (1.1|x|)^{\frac{2}{3}} \Rightarrow \nabla V_z(z) p(x, z) \leqslant -\frac{1}{11}|z|^3 \quad \text{a.e.}, \tag{55}$$

which implies

$$|z(t)| \leqslant \max\left\{\beta(|z(0)|, t), \gamma_z^x(\|x\|_\infty)\right\}, \tag{56}$$

where $\beta \in \mathcal{KL}$ and $\gamma_z^x(s) = (1.1s)^{2/3}$.

Now, we find a set-valued map $S(x)$ in the form of $S(x) = \{\kappa(x + d) : |d| \leqslant b|x| + (1-b)a\}$, such that a realizable control law $u \in S(x)$ renders the x-subsystem ISS with z and the quantization error as the inputs. Also, to satisfy the small-gain condition, the ISS gain from z to x, denoted by γ_x^z, is assigned, such that $\gamma_x^z \circ \gamma_z^x < \mathrm{Id}$. For this purpose, we choose $\gamma_x^z(s) = s^{3/2}/1.2$ and choose $\kappa(r) = -\kappa^0(|r|)r$ with $\kappa^0 \in \mathcal{K}$ satisfying

$$(1-b)(1-c)\kappa^0((1-b)(1-c)s)s \geqslant \iota s + \alpha_f\left((\gamma_x^z)^{-1}(s)\right), \tag{57}$$

where constant $\iota > 0$ and constant c satisfies $0 < c < 1$. We choose $\iota = 0.1$ and $c = 0.1$. Then, we can choose $\kappa^0(s) = 2.5s + 0.2$ and thus

$$S(x) = \{-2.5|x + d|(x + d) - 0.2(x + d) : |d| \leqslant 0.1|x| + 0.009\}. \tag{58}$$

Then, the quantized control law is designed as

$$u(q(x)) = -2.5|q(x)|q(x) - 0.2q(x). \tag{59}$$

It can be directly checked that $u(q(x)) \in S(x)$.

Figures 1 and 2 show the simulation result with initial state $[z(0), x(0)] = [10, -1]$.

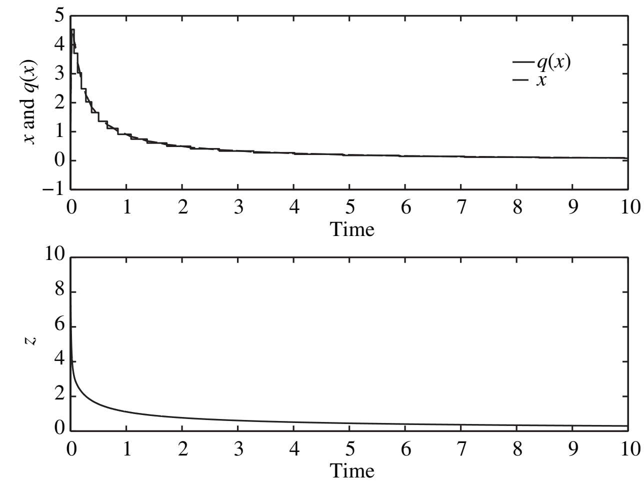

Figure 1 The trajectories of x, $q(x)$ and z.

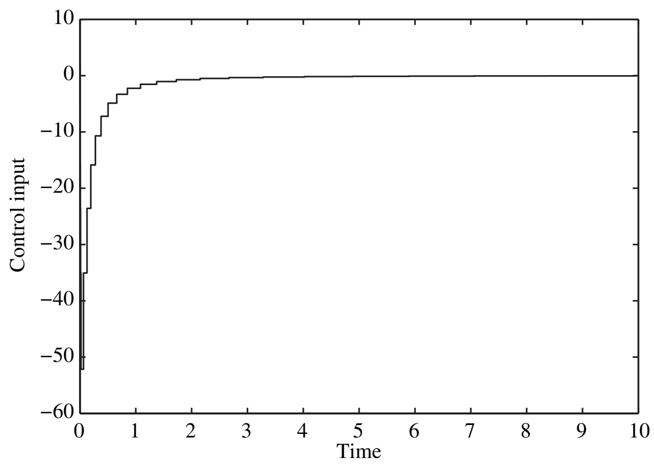

Figure 2 The control input u with quantized feedback.

6 Conclusion

In this article, we have studied the quantized feedback stabilization problem of a general class of nonlinear cascaded systems with dynamic uncertainties. A recursive control design ingredient has been developed based on ISS small-gain theorem and a set-valued map design, which can be readily used for quantized partial-state feedback stabilization. In particular, asymptotic stabilization can be achieved for the closed-loop quantized system if the dynamic uncertainties are ISS with gains being Lipschitz on compact sets.

Along this direction, future research will be directed at the quantized control of nonlinear systems with IOS dynamic uncertainties or even with unstable dynamic uncertainties. While this article and the work of others focus on quantized stabilization, the problem of quantized feedback tracking is of more practical interest and recovers stabilization as a special case. This problem has received practically no attention in the present literature. Closely related to this problem, the output regulation theory [24–26] consists of searching for (unquantized) feedback control laws to achieve asymptotic tracking with disturbance rejection, when the disturbance and reference signals are generated by an exo-system. The quantizers considered in this article have infinite numbers of quantization levels, while practical quantizers have finite numbers of quatization levels. In this case, the design in this article can only guarantee local stabilization. By appropriately adjusting the quantization levels during the control procedure, dynamic quantization is usually used to achieve semi-global stabilization with finite-level quantizers. See [27] for the concept of semi-global stabilization and [28] for a recent result. Another research direction is to apply the quantized control designs to distributed control of nonlinear systems. A recent result for linear systems can be found in [29]. Other related topics including quantized identification [30], the combination of quantized control and sampled-data control [31], and compensator design [32] for quantized systems are also of interest for nonlinear uncertain systems. Other topics of future research include the applications of quantized nonlinear control to smart grids, intelligent transportation systems, robotic networks, and other cyber-physical systems.

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Conflict of interest The authors declare that they have no conflict of interest.

References

1 Kalman R E. Nonlinear aspects of sampled-data control systems. In: Proceedings of the Symposium on Nonlinear Circuit Theory, Brooklyn, 1956, 6: 273–313

2 Lunze J. Qualitative modelling of linear dynamical systems with quantized state measurements. Automatica, 1994, 30: 417–431

3 Brockett R W, Liberzon D. Quantized feedback stabilization of linear systems. IEEE Trans Automat Contr, 2000, 45: 1279–1289

4 Liberzon D. Hybrid feedback stabilization of systems with quantized signals. Automatica, 2003, 39: 1543–1554

5 Tatikonda S, Mitter S. Control under communication constraints. IEEE Trans Automat Contr, 2004, 49: 1056–1068

6 Elia N, Mitter S K. Stabilization of linear systems with limited information. IEEE Trans Automat Contr, 2001, 46: 1384–1400

7 Fu M, Xie L. The sector bound approach to quantized feedback control. IEEE Trans Automat Contr, 2005, 50: 1698–1711

8 Liu J, Elia N. Quantized feedback stabilization of non-linear affine systems. Int J Contr, 2004, 77: 239–249

9 de Persis C. n-bit stabilization of n-dimensional nonlinear systems in feedforward form. IEEE Trans Automat Contr, 2005, 50: 299–311

10 Ceragioli F, de Persis C. Discontinuous stabilization of nonlinear systems: quantized and switching controls. Syst Contr Lett, 2007, 56: 461–473

11 Liu T, Jiang Z P, Hill D J. Quantized stabilization of strict-feedback nonlinear systems based on ISS cyclic-small-gain theorem. Math Contr Signal Syst, 2012, 24: 75–110

12 Liu T, Jiang Z P, Hill D J. A sector bound approach to feedback control of nonlinear systems with state quantization. Automatica, 2012, 48: 145–152

13 Liu T, Jiang Z P, Hill D J. Nonlinear Control of Dynamic Networks. Boca Raton: CRC Press, 2014

14 Liu T, Jiang Z P, Hill D J. Quantized output-feedback control of nonlinear systems: a cyclic-small-gain approach. In: Proceedings of the 30th Chinese Control Conference, Yantai, 2011. 487–492

15 Liu T, Jiang Z P, Hill D J. Small-gain based output-feedback controller design for a class of nonlinear systems with actuator dynamic quantization. IEEE Trans Automat Contr, 2012, 57: 1326–1332

16 Jiang Z P, Liu T. Quantized nonlinear control—a survey. Acta Automat Sin, 2013, 39: 1820–1830

17 Karafyllis I, Jiang Z P. Stability and Stabilization of Nonlinear Systems. London: Springer, 2011

18 Kokotović P V, Arcak M. Constructive nonlinear control: a historical perspective. Automatica, 2001, 37: 637–662

19 Krstić M, Kanellakopoulos I, Kokotović P V. Nonlinear and Adaptive Control Design. Hoboken: John Wiley & Sons, 1995

20 Sontag E D. Input to state stability: basic concepts and results. In: Nonlinear and Optimal Control Theory. Berlin: Springer-Verlag, 2007. 163–220

21 Jiang Z P, Teel A R, Praly L. Small-gain theorem for ISS systems and applications. Math Contr Signal Syst, 1994, 7: 95–120

22 Jiang Z P, Mareels I M Y. A small-gain control method for nonlinear cascade systems with dynamic uncertainties. IEEE Trans Automat Contr, 1997, 42: 292–308

23 Barmish B R, Corless M, Leitmann G. A new class of stabilizing controllers for uncertain dynamical systems. SIAM J Contr Optim, 1983, 21: 246–255

24 Byrnes C I, Priscoli F D, Isidori A. Output Regulation of Uncertain Nonlinear Systems. 1st ed. Boston: Birkhäuser, 1997

25 Chen Z, Huang J. Stabilization and Regulation of Nonlinear Systems: a Robust and Adaptive Approach. Berlin: Springer, 2015

26 Sun W J, Huang J. Output regulation for a class of uncertain nonlinear systems with nonlinear exosystems and its application. Sci China Ser F-Inf Sci, 2009, 52: 2172–2179

27 Lin Z. Semi-global stabilization of linear systems with position and rate limited actuators. Syst Contr Lett, 1997, 30: 1–11
28 Liu X M, Lin Z L. On semi-global stabilization of minimum phase nonlinear systems without vector relative degrees. Sci China Ser F-Inf Sci, 2009, 52: 2153–2162
29 Li T, Xie L. Distributed coordination of multi-agent systems with quantized-observer based encoding-decoding. IEEE Trans Automat Contr, 2012, 57: 3023–3037
30 Guo J, Zhao Y L. Identification of the gain system with quantized observations and bounded persistent excitations. Sci China Inf Sci, 2014, 57: 012205
31 Li T, Zhang J F. Sampled-data based average consensus with measurement noises: convergence analysis and uncertainty principle. Sci China Ser F-Inf Sci, 2009, 52: 2089–2103
32 Guo Y Q, Gui W H, Yang C H. On the design of compensator for quantization-caused input-output deviation. Sci China Inf Sci, 2011, 54: 824–835