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A beamforming design for weighted sidelobe power leakage minimization

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Abstract In this paper, a beamforming scheme for minimizing the weighted sidelobe power leakage while maintaining the norm of the weight vector at unity is proposed. The proposed criterion is very flexible because weighting factors are added to the sidelobes in the object function, and the weighting factors can be adjusted according to any design purpose, e.g., to minimize the interference within a direction of arrival (DoA) range. To acquire the minimum sidelobe power leakage, we first express the sidelobe power through the sidelobe coefficient matrix. Afterwards, the minimization problem can be treated as the 2-norm minimization of the sidelobe coefficient matrix. The optimal weighting vector design is then derived by singular value decomposition (SVD). Simulation results show that the proposed beamformer can decrease the sidelobe power leakage and efficiently suppress interference with barely any increase in the sidelobes; moreover, this beamforming scheme provides good robustness in consideration of the DOA mismatch.

Keywords beamforming, weighted sidelobe power leakage, 2-norm, singular value decomposition (SVD), direction of arrival (DoA)

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1 Introduction

Beamforming is a spatial filtering technique that uses an antenna array to focus a spatial beam in the target direction. In a beamforming system, antennas are weighted by a vector, which determines the gain and phase shift of each antenna. By verifying the weight for each antenna, a beamformer can adaptively adjust the transmitting or receiving pattern, thereby enhancing the system performance. The beamforming technique has been extensively used in a variety of areas in wireless communication, such as physical-layer secure transmission [1], relaying [2], spatial division multiple access [3], and smart antennas [4].

A key issue in beamformer design is the calculation of the weights for each antenna, which is usually solved by first formulating it as a constrained optimization problem. So far, many optimization criteria have been proposed, and various optimal or suboptimal solutions have been obtained [5]. A well-known criterion is to maximize the signal-to-interference-plus-noise ratio (SINR) with a limited array output power [6]. In [7], Capon proposed a data-dependent beamformer to minimize the output variance while

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maintaining the response of the signal of interest (SOI) at a constant value. This approach is usually referred to as minimum variance distortionless response (MVDR) beamforming in the literature. The MVDR approach requires perfect knowledge of the direction of arrival (DOA) or the channel state information (CSI). However, in practice, the MVDR beamformer may suffer from a performance loss due to the DOA mismatch caused by a DOA estimation error, multipath propagation, etc. [8–11].

To solve the DOA mismatch problem, many beamforming methods have been developed on the basis of a variety of criteria. One method is to broaden the main beam pattern, which is referred to as a linearly constrained minimum variance (LCMV) beamformer. On the basis of this method, an adaptive Bayesian beamformer using a weighted sum of MVDR beamformers has been proposed [12]. Another method is based on a matrix analysis, such as the diagonal loading method [13, 14] and eigenspace-based beamformer [15, 16]. In [17], on the basis of an uncertainty model for the SOI and interference, a beamformer was proposed to optimize the worst-case performance, and a closed-form solution as well as low complexity algorithms can be found in [18].

On the other hand, the sidelobe power leakage would create co-channel interference at the end users, which has also attracted attention, as described in [19–23]. In [24], the authors proposed a leakage-based beamformer to minimize the intercell interference in a linear minimum-mean-square-error beamforming scheme. In [25], rather than maximizing the SINR of the users in a multiuser MIMO system, the authors proposed a scheme to achieve maximization of the signal-to-leakage-and-noise ratio (SLNR) for the users. This criterion is used to suppress the interference caused by the sidelobe of the engaged users' signal. In [26], the authors aimed to maximize the mainlobe-to-sidelobe power ratio (MSPR) in Capon beamforming. However, this optimization problem is not convex. The authors introduced a factor to balance the minimum variance constraint and MSPR. An iterative method with Lagrange multipliers was adopted to solve this relaxed MSPR (RMSPR) minimization problem.

In this paper, we propose a novel beamforming design criterion. In contrast to most existing criteria that maximize the output SINR, the proposed criterion minimizes the weighted sidelobe power leakage while maintaining the norm of the weight vector at unity. Under this criterion, more energy is accumulated in the desired directions. In some cases, the proposed criterion is equivalent to traditional criteria, e.g., the maximum SINR and minimum sidelobe power leakage criteria can be seen as two special cases of our design. On the other hand, the proposed criterion provides a flexible means to shape the beamforming, as the weighting factor can be accordingly adjusted for any purpose. For example, in order to reduce the beam gain in a given direction, one can increase the weight in the direction, and similarly, to increase the gain in any direction, one can reduce the weight in the direction. Thus, by adjusting the weights in different directions, the proposed criterion can shape the beam to any pattern as required. According to the proposed criterion, the optimal solution is derived by using singular value decomposition (SVD).

The rest of the paper is organized as follows. The signal model and some previous work on robust beamformers are presented in Section 2. In Section 3, the mathematical model for sidelobe power leakage minimization is established, and the optimal weight design is presented. Some application examples are presented in Section 4. The performance of the proposed scheme is evaluated by numerical results in Section 5. Finally, Section 6 concludes the paper.

Notations: Boldface lowercase and uppercase letters are reserved for vectors and matrices, respectively. The superscript H represents transposition and conjugation, and the superscript T simply denotes transposition. The expectation is represented by $\mathrm{E}[\cdot]$. The rank of the matrix $\boldsymbol{A}$ is expressed as $\mathrm{rank}[\boldsymbol{A}]$. The element in the mth row and nth column of the matrix $\boldsymbol{A}$ is denoted by $[\boldsymbol{A}]_{m,n}$. The 2-norm of an $M\times N$ matrix $\boldsymbol{A}$ is denoted by $\|\boldsymbol{A}\|$, and $\|\boldsymbol{A}\| = \sqrt{\Sigma_{m=1}^{M}\Sigma_{n=1}^{N}|[\boldsymbol{A}]_{m,n}|^2}$. The singular values of $\boldsymbol{A}$ are in descending order, and the ith singular value is $\sigma_i(\boldsymbol{A})$. $[x, y]$ represents the closed interval from x to y.

2 System model

Considering an antenna array system consisting of M elements, the received signal of the array can be given by

$$\boldsymbol{y} = \boldsymbol{s}(\theta)x + \boldsymbol{n}, \tag{1}$$

where x is the SOI, and $\boldsymbol{n}$ denotes the summation of the interference and additive white Gaussian noise (AWGN). $\boldsymbol{s}(\theta) = \left[\mathrm{e}^{\mathrm{j}\theta}, \mathrm{e}^{\mathrm{j}(\theta+\phi_1)}, \ldots, \mathrm{e}^{\mathrm{j}(\theta+\phi_{M-1})}\right]^{\mathrm{T}}$ represents the baseband array response of the SOI, where ϕ_i is the phase shift between the first and ith antennas. After multiplying the complex weight vector of the antenna array, denoted by $\boldsymbol{w} = [w_1, w_2, \ldots, w_M]^{\mathrm{T}}$, we finally obtain the output of the beamformer as $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{y}$.

3 Proposed beamforming design

In this paper, we propose a beamformer that aims to minimize the sidelobe power leakage. We define an observation angle set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_K\}$, which is uniformly sampled within the sidelobe range. The sample interval should be very small to ensure a fine resolution within the sidelobe range. Thus, the sample number, K, would be a large number. The sidelobe output of the beamformer can be rewritten as

$$\boldsymbol{y} = \boldsymbol{\Gamma}\boldsymbol{w}x + \boldsymbol{n}, \tag{2}$$

where $\boldsymbol{\Gamma}$ is a $K \times M$ matrix, and we assume that $\mathrm{rank}[\boldsymbol{\Gamma}] = M$. The element $[\boldsymbol{\Gamma}]_{k,m}$ denotes the sidelobe leakage coefficient in the direction θ_k of the mth elements. $\boldsymbol{w} = \{w_1, w_2, \ldots, w_M\}^{\mathrm{T}}$. Owing to the characteristic that a beamformer transmits the same data over each antenna, the vector, $\boldsymbol{w}$, is equivalent to a linear mapping scheme.

In order to make the beamforming design more flexible, we define a power leakage weighting vector $\boldsymbol{g} = [g(\theta_1), g(\theta_2), \ldots, g(\theta_K)]$, where $g(\theta_k)$ is a nonnegative real number representing the weight in the direction θ_k. The weighted sidelobe power leakage is $P_S = \mathrm{E}\left[\boldsymbol{y}^{\mathrm{H}}\boldsymbol{G}^{\mathrm{H}}\boldsymbol{G}\boldsymbol{y}\right]$, where $\boldsymbol{G} = \mathrm{diag}(\boldsymbol{g})$ is a $K \times K$ diagonal matrix with $\boldsymbol{g}$ on the diagonal. The proposed beamforming design criterion can be formulated as

$$\boldsymbol{w}_{\mathrm{opt}} = \underset{\boldsymbol{w}}{\mathrm{argmin}}\ P_S, \quad \mathrm{s.t.}\ \|\boldsymbol{w}\| = 1. \tag{3}$$

Let $\boldsymbol{A} = \boldsymbol{G}\boldsymbol{\Gamma}$; thus, we have $\mathrm{rank}[\boldsymbol{A}] = M$. The weighted sidelobe can be expressed as $\boldsymbol{r} = \boldsymbol{A}\boldsymbol{w}x$. To solve the optimization problem in (3), we represent the power leakage P_S in a new form. We first decompose the matrix $\boldsymbol{A}\boldsymbol{w}$ by using SVD as

$$\boldsymbol{A}\boldsymbol{w} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^{\mathrm{H}}, \tag{4}$$

where $\boldsymbol{U}$ is a $K \times K$ unitary matrix, and $\boldsymbol{V}\boldsymbol{V}^{\mathrm{H}} = 1$. Actually, $\boldsymbol{A}\boldsymbol{w}$ is a vector; thus, $\boldsymbol{S}$ is a $K \times 1$ vector containing the singular value of $\boldsymbol{A}\boldsymbol{w}$, i.e., $\boldsymbol{S}_{1,1} = \sigma_1\big(\boldsymbol{A}\boldsymbol{w}\big)$. The sample interval in the observation angle set Θ should be very small to provide a fine resolution in the sidelobe region. Therefore, the number of sample points, K, is far larger than the number of the antennas. The power leakage P_S can be expressed as

$$P_S = \mathrm{E}\Big[\boldsymbol{x}^{\mathrm{H}}\boldsymbol{V}\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{x}\Big] = \mathrm{E}\Big[\boldsymbol{x}^{\mathrm{H}}\boldsymbol{V}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{x}\cdot\sigma_1^2(\boldsymbol{A}\boldsymbol{w})\Big] = \mathrm{E}\Big[|x|^2\Big]\cdot\sigma_1^2(\boldsymbol{A}\boldsymbol{w}), \tag{5}$$

$\mathrm{E}[|x|^2] = P$ is the transmit power. Therefore, we have

$$P_S = P\cdot\sigma_i^2(\boldsymbol{A}\boldsymbol{w}) = P\|\boldsymbol{A}\boldsymbol{w}\|. \tag{6}$$

The second equation in (6) holds because the square of the 2-norm is equal to the sum of the squares of the singular values [27]. Moreover, we observe that for the discrete angle set Θ, the average sidelobe leakage power is determined by the 2-norm of $\boldsymbol{A}$. Therefore, the optimization problem in (3) is equivalent to finding an optimal solution that minimizes the 2-norm in (6), i.e.,

$$\boldsymbol{w} = \underset{\boldsymbol{w}}{\mathrm{argmin}}\ \|\boldsymbol{A}\boldsymbol{w}\|, \quad \mathrm{s.t.}\ \|\boldsymbol{w}\| = 1. \tag{7}$$

As $\boldsymbol{w}$ is a normalized orthogonal vector, i.e., $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{w} = 1$, this can be seen as an orthogonal base in an M-dimensional space. Thus, it is possible to find another $R = M - 1$ bases in the M-dimensional space. Considering the $M \times R$ matrix formed by the other R bases, denoted by $\boldsymbol{w}'$, we can construct two matrices:

$\boldsymbol{W}_1 = (\boldsymbol{w}, \boldsymbol{0}_{M\times R})$ and $\boldsymbol{W}_2 = (\boldsymbol{0}_{M\times 1}, \boldsymbol{w}')$, and they form an $M \times M$ unitary matrix $\boldsymbol{W} = (\boldsymbol{W}_1, \boldsymbol{W}_2)$. Apparently, $\boldsymbol{W}_0 = \boldsymbol{W}_1 + \boldsymbol{W}_2$, and $\|\boldsymbol{A}\boldsymbol{w}\| = \|\boldsymbol{A}\boldsymbol{W}_1\|$. Thus, Eq. (7) is equal to

$$\boldsymbol{w}_{\text{opt}} = \underset{\boldsymbol{w}}{\operatorname{argmin}} \ \|\boldsymbol{A}\boldsymbol{W}_0 - \boldsymbol{A}\boldsymbol{W}_2\|, \tag{8}$$

where $\operatorname{rank}[\boldsymbol{A}\boldsymbol{W}_0] = M$ and $\operatorname{rank}[\boldsymbol{A}\boldsymbol{W}_2] = R$. Thus, the optimization problem is converted into approximate $\boldsymbol{A}\boldsymbol{W}_0$ with $\boldsymbol{A}\boldsymbol{W}_2$, which can be solved by using the Eckart–Young low-rank approximation theorem [27, 28]. According to this theorem, if $\operatorname{rank}[\boldsymbol{U}] = u$ and $\operatorname{rank}[\boldsymbol{V}] = v$, where $\boldsymbol{V} = \boldsymbol{U} + \boldsymbol{B}$, the condition that the norm of $\boldsymbol{B}$ is no less than the summation of last $v - u$ singular values, i.e. $\sum_{i=u+1}^{v} \sigma_i^2(\boldsymbol{V}) \leqslant \|\boldsymbol{B}\|^2$ [29], is satisfied. As $\boldsymbol{W}_0$ is an $M \times M$ orthogonal matrix, $\boldsymbol{A}\boldsymbol{W}_0$ and $\boldsymbol{A}$ have the same singular values. Thus, it can be deduced that

$$\min \|\boldsymbol{A}\boldsymbol{w}\| \geqslant \sqrt{\sum_{i=M-1}^{M} \sigma_i^2(\boldsymbol{A})} = |\sigma_M(\boldsymbol{A})|. \tag{9}$$

By exploiting SVD, we can decompose $\boldsymbol{A}$ into $\boldsymbol{A} = \boldsymbol{U}_{\boldsymbol{A}}\boldsymbol{S}_{\boldsymbol{A}}\boldsymbol{V}_{\boldsymbol{A}}^{\text{H}}$. Accordingly, the optimal weighting vector that achieves the minimum norm is

$$\boldsymbol{w}_{\text{opt}} = \boldsymbol{V}_1, \tag{10}$$

where $\boldsymbol{V}_1$ is the M-by-1 submatrix corresponding to the smallest singular value of $\boldsymbol{V}_{\boldsymbol{A}}$. It is very easy to prove that $\boldsymbol{w} = \boldsymbol{V}_1$ is the optimal solution to the norm minimization problem in (9). Let $\boldsymbol{V}_{\boldsymbol{A}} = (\boldsymbol{V}_0, \boldsymbol{V}_1)$, where $\boldsymbol{V}_0$ consists of the first R columns of $\boldsymbol{V}_{\boldsymbol{A}}$, and $\boldsymbol{V}_1$ denotes the last column of $\boldsymbol{V}_{\boldsymbol{A}}$. We can deduce that

$$\boldsymbol{A}\boldsymbol{w}_{\text{opt}} = \boldsymbol{U}_{\boldsymbol{A}}\boldsymbol{S}_{\boldsymbol{A}} \begin{pmatrix} \boldsymbol{V}_0^{\text{H}} \\ \boldsymbol{V}_1^{\text{H}} \end{pmatrix} \boldsymbol{V}_1 = \boldsymbol{U}_{\boldsymbol{A}}\boldsymbol{S}_{\boldsymbol{A}} \begin{pmatrix} \boldsymbol{0}_{R\times 1} \\ \boldsymbol{1} \end{pmatrix} = \boldsymbol{U}_{\boldsymbol{A}}\boldsymbol{S}', \tag{11}$$

where $\boldsymbol{S}'$ is an $M \times 1$ vector with the smallest singular value $\sigma_M(\boldsymbol{A})$. Thus, $\|\boldsymbol{A}\boldsymbol{w}_{\text{opt}}\|^2 = \sigma_M^2(\boldsymbol{A})$, and the sidelobe power leakage after the optimal weighting can be given by

$$P_S = P \cdot \sigma_M^2(\boldsymbol{A}). \tag{12}$$

For a uniformly weighted beamformer, the power leakage $P_S' = P\|\boldsymbol{A}\|^2 = P \cdot \sum_{i=1}^{M} \sigma_i^2(\boldsymbol{A})$. Compared with (12), one can find that the weighting process only leaves the smallest singular values of the sidelobe coefficient matrix $\boldsymbol{A}$. Thus, the sidelobe power leakage is decreased.

4 Beamforming design examples

4.1 Transmit beamforming with minimum sidelobe power leakage

A transmit beamformer usually concentrates its transmission energy in the desired directions while suppressing the sidelobe power leakage as much as possible because the sidelobe power leakage would introduce co-channel interference to the other receivers in the system. If the DOA of the SOI is available at the transmitter, we can use the proposed approach to minimize the sidelobe power leakage by setting the power leakage weighting vector as follows:

$$g(\theta) = \begin{cases} 0 & \text{for} \quad \theta \in \Theta_M, \\ 1 & \text{for} \quad \theta \in \Theta_S, \end{cases} \tag{13}$$

where Θ_M denotes the directions of the mainlobe range, whereas its complementary set, Θ_S, represents the directions of the sidelobes. In practical, Θ_M can be extended to compensate the drawback of DOA estimation error. θ denotes the discrete angle sample points. Figure 1 shows the beam patterns of a uniform linear array (ULA) equipped with $M = 17$ antennas. The mainlobe width is set as $10°, 20°$, and $30°$. From this figure, one can find that the sidelobe power leakage decreases as the mainlobe width increase, i.e., a larger Θ_M results in lower sidelobe power leakage.

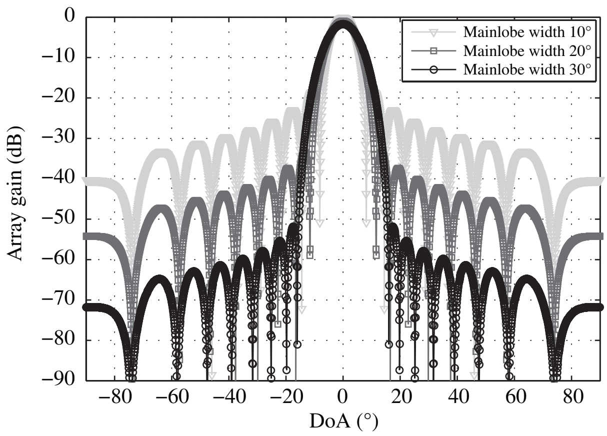

Figure 1 The beam pattern of a ULA with various mainlobe widths.

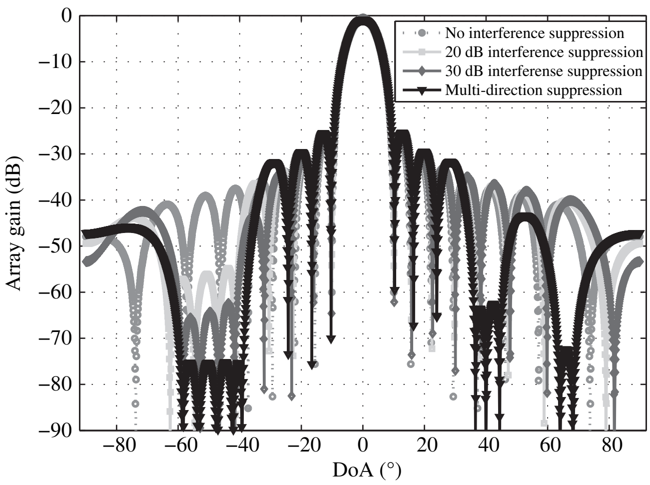

Figure 2 The beam pattern of a ULA with various interference suppression levels.

4.2 Transmit beamforming with interference rejection

If the DOA of strong interference is known at the receiver or the transmitter wants to eliminate sidelobe interference in a direction, the beamformer should decrease the receiving or transmitting beam gain in that direction. To suppress the interference in a given direction range, we set

$$g(\theta) = \begin{cases} 0 & \text{for} \quad \theta \in \Theta_M, \\ 1 & \text{for} \quad \theta \in \Theta_S, \\ \rho & \text{for} \quad \theta \in \Theta_I, \end{cases} \tag{14}$$

where $\rho > 1$ is the weighted factor of the given direction set, Θ_I, denoting the amplitude suppression level. A larger value of ρ would result in a more severe interference suppression, e.g., $\rho = 10$ would result in a 100 times of interference power suppression on the given angel range compared with $\rho = 1$. After multiplying the steering vector and the sidelobe leakage amplitude coefficient matrix $\boldsymbol{A}$, we can obtain the optimal weighting vector by processing the algorithm in Section 3. Figure 2 shows the beam pattern using different suppression factors. In this scenario, a ULA equipped with $M = 17$ antennas is adopted. The mainlobe range is $\Theta_M = [-8^\circ, 8^\circ]$. First, a single-user case is demonstrated. We consider the interference within the region $\Theta_I = [-60^\circ, -40^\circ]$ with suppression factor of ρ =20 dB and 30 dB. In addition, for a multiuser scenario, in which interference from multiple sources is received at the antenna array, we assume that the interference is located in the direction range $\Theta_I = [-60^\circ, -40^\circ]$, $[36^\circ, 46^\circ]$ and $[63^\circ, 67^\circ]$ with objective suppression of 40 dB, 25 dB, and 30 dB.

From Figure 2, one can find that the antenna array gain within the interference region can be suppressed

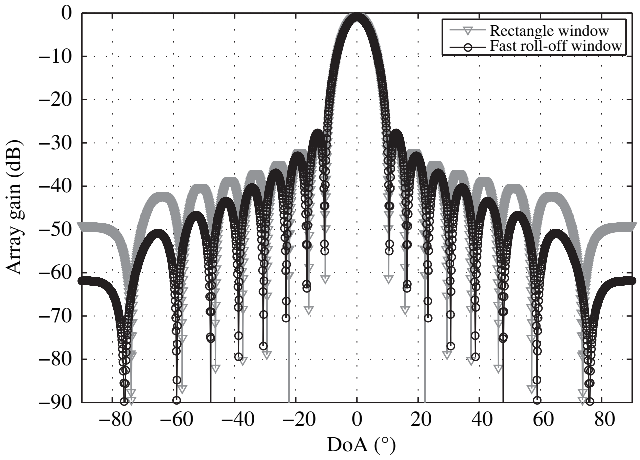

Figure 3 A windowing beam pattern that has faster roll-off in the sidelobe.

to different levels according to the design goal. The proposed beamformer is also able to suppress interference from multiple directions with various suppression levels. This provides a possible application in multi-user scenarios, e.g., to solve the inter-cell interference problem in a cellular system, or to mitigate the cross-tier interference in heterogeneous networks. In practice, the adjustment of Θ_I provides good robustness to the mismatch. For example, by extending the range of Θ_I to be greater than the DOA estimation error of the interference, we can acquire robust beamforming against the estimation error within a certain range.

4.3 Beam pattern shaping

The proposed method also provides a means for shaping the beam pattern by adjusting the window function. For example, the above-mentioned minimum sidelobe power leakage beamformer, known as a rectangular window, provides a sudden decrease, but the sidelobes roll off rather slowly. However, in some other implementations, a fast sidelobe roll-off may be required. In this case, a bell-shaped windowing function is more appropriate. For example, for a ULA with $M = 17$ antennas and assuming the mainlobe region is $\Theta_M = [-8°, 8°]$, the fast roll-off windowing function is designed as

$$g(\theta) = \begin{cases} 0, & \text{for} \quad \theta \in \Theta_M, \\ \exp(0.02 \times |\theta|), & \text{for} \quad \theta \in \Theta_S. \end{cases} \tag{15}$$

This windowing function, different from rectangular window in (13), exponentially expands as $|\theta|$ increases. Thus, the suppression on the directions further from the mainlobe is larger. The beam patterns are shown in Figure 3. One can see that the first two sidelobes are higher using the exponential window function, but the sidelobes fall off faster than the rectangular window.

In practice, one may select windowing functions according to the requirements for the sidelobe roll-off rate and power level of the first sidelobe. Generally speaking, the sidelobes will fall off faster if the windowing function decreases more smoothly.

5 Simulation results

The performance of the proposed beamformer is studied with simulations and compared with other beamformers in this section. A ULA with $M = 17$ omnidirectional half-wavelength-spaced antennas is adopted. Through the simulations, it is assumed that there is only one desired source and one interference source with various DOA mismatches. The signal-to-noise ratio (SNR) is defined as the ratio of the SOI power of a single antenna to its noise power, whereas the signal-to-interference ratio (SIR) is the ratio of the received SOI power to the interference power of a single antenna as well. The noise variance is assumed

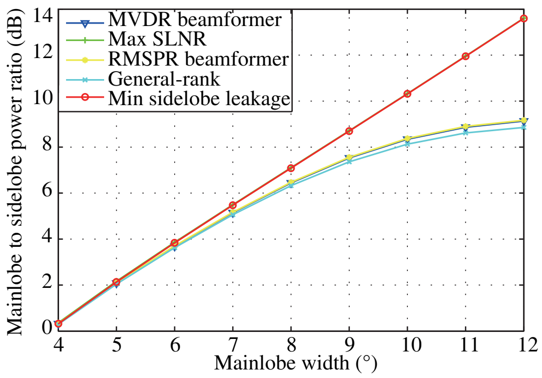

Figure 4 (Color online) Mainlobe-to-sidelobe power ratio for various mainlobe widths.

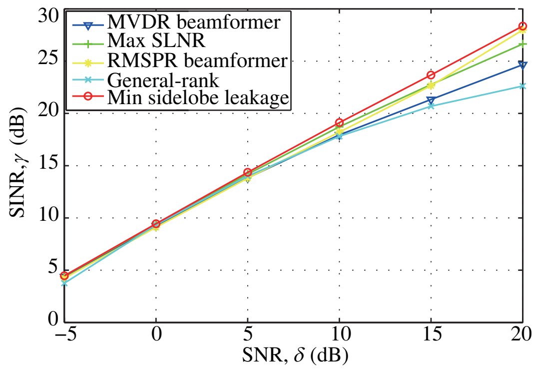

Figure 5 (Color online) Output SINR versus the SNR.

to be equal for the antennas. In the simulations, we concentrate on the effect of the SOI and interference DOA mismatch.

The desired signal and interference are assumed to be locally and incoherently scattered [17, 30–32] with a uniform angular power density characteristic. The angular spread is in the diffusion range of the signals. In the following examples, we consider five beamformers that are able to suppress interference, including: (1) an MVDR beamformer [7], (2) a general-rank beamformer [17], (3) a maximum SLNR beamformer [25], (4) an RMSPR beamformer [26], and (5) the proposed minimum sidelobe leakage beamformer.

In Figure 4, the mainlobe-to-sidelobe power ratio versus the mainlobe width is presented. In this simulation, the mainlobe is defined as the SOI range, whereas the rest is defined as the sidelobe. The arrival angle of the SOI is 0° with an angle spread 8°. We assume the interference with an interference-to-noise power ratio (INR) equal to 30 dB that uniformly arrives in the direction $[46^\circ, 54^\circ]$, according to the angular spreading constraint above. The SNR is set as 10 dB. The parameters for the general-rank beamformer are $\epsilon = 5$ and $\tau = 0.5$ for a nearly optimal result, whereas the parameter for the RMSPR scheme is $\alpha = 5.1$ according to the simulation in [26]. From this figure, we can see that all five beamformers have similar mainlobe-to-sidelobe power ratios when the mainlobe width is narrow. Along with expansion of the mainlobe, the proposed beamformer as well as the minimum SLNR beamformer exhibit good performance for sidelobe power restraint. The minimizing sidelobe power criterion enables the two beamformers to have a strong capability of sidelobe suppression.

In the second simulation, we assume that the actual arrival angles of the SOI and interference are 0° and 50°, respectively, and the DOAs of the two signals have a mismatch scaling of 3°. The SOI uniformly arrives within the range $[-4^\circ, 4^\circ]$, whereas the antenna array receives interference within $[46^\circ, 54^\circ]$ with the INR equal to 30 dB. The other parameters are the same as in the previous simulation. The SINRs are compared for SNRs ranging from -5 dB to 20 dB.

From Figure 5, we can see that all of the SINRs increase as the SNR increases. All beamformers have similar performance in the low-SNR range, whereas the proposed beamformer, maximum SLNR beamformer, and RMSPR scheme are similarly outstanding at high SNRs. This is because these three beamformers are all designed for the minimum sidelobe power; the lower array gain in the sidelobe range makes them robust to the interference mismatch as well as the SOI mismatch. The suppression factor of the proposed beamformer is very flexible to adapt to the possible mismatch, i.e., when there is a mismatch, a moderate suppression factor is adopted within the estimated interference region. Thus, the array gain of other side lobes, especially the neighboring side lobes of the interference region, would not raise much. As a result, the received interference power is lower than the proposed beamformer using a severe suppression factor, as well as the maximum SLNR beamformer and RMSPR beamformer.

The next simulation shows the performance of the mismatch robustness of the beamformers. Here, we assume that the desired signal and interference have uniform angular spread power densities and arrive at the antenna array by the central angles 0° and 60°, respectively. The angular spreads are

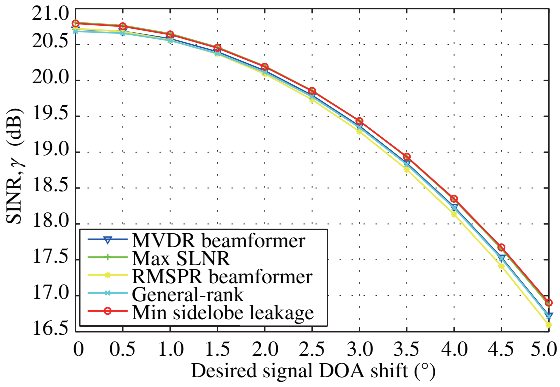

Figure 6 (Color online) Output SINR versus the DOA mismatch of the desired signal.

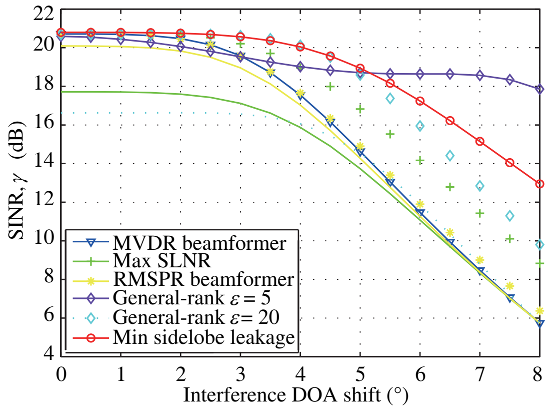

Figure 7 (Color online) Output SINR versus the DOA mismatch of the interference.

set to be the same for the desired signal and interference and equal to 8°. The SNR is 10 dB. In this scenario, we assume that the central angular has a mismatch, but the angular spread remains. Figure 6 shows the received SINR versus the central-angle shift. We can see that the SINR decreases as the DOA accuracy deteriorates. Although the proposed beamformer performance does not intend to perform mainlobe shaping, it still has a good array gain in the mainlobe scope. This enhances the ability of the DOA mismatch of the desired signal.

Next, we will show the performance when the interference DOA has a mismatch. In this scenario, we assume that the DOA of the desired signal has no mismatch, but the central angle of the received interference has DOA skewing. Figure 7 presents the output SINR of the beamformers according to various DOA excursions. The SNR is also 10 dB, and the INR is 30 dB. The desired signal is assumed to arrive at the antenna array uniformly within the scope $[-4^\circ, 4^\circ]$, whereas the actual central angle of the interference is 60° with an 8° angular spread. The parameters for the general-rank beamformer are $\epsilon = 5$ and $\epsilon = 20$, respectively, with $\tau = 0.5$ for a different result. From this figure, we can see that all beamformers provide a good SINR when the interference mismatch is small, but the SINRs decrease as the mismatch increases. The proposed beamformer provides a relatively good SINR when the interference mismatch increases. We can also see that a proper parameter choice would tremendously improve the interference suppression capability of the general-rank beamformer in either highly mismatched or smaller mismatch scenarios, despite the performance loss in the complementary sets.

6 Conclusion

In this paper, a new beamforming criterion that minimizes the weighted sidelobe power leakage while maintaining the norm of the weight vector at unity is proposed. This approach evaluates the weight for the sidelobe scope and minimizes the weighted sidelobe power leakage of the beam pattern through an SVD method. Thus, this beamforming scheme can suppress the gain in the desired directions to an arbitrary level. The simulation results show that this beamformer is flexible for multiple purposes by adjusting the weight function, e.g., to achieve energy-efficient interference suppression or sidelobe-gain shaping goals. The numerical results also show that this beamformer provides good robustness to the DOA mismatch of the desired signal or interference when it is applied as a receiving beamformer.

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Conflict of interest The authors declare that they have no conflict of interest.

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