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A Physarum-inspired approach to supply chain network design

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Abstract A supply chain is a system which moves products from a supplier to customers, which plays a very important role in all economic activities. This paper proposes a novel algorithm for a supply chain network design inspired by biological principles of nutrients' distribution in protoplasmic networks of slime mould Physarum polycephalum. The algorithm handles supply networks where capacity investments and product flows are decision variables, and the networks are required to satisfy product demands. Two features of the slime mould are adopted in our algorithm. The first is the continuity of flux during the iterative process, which is used in real-time updating of the costs associated with the supply links. The second feature is adaptivity. The supply chain can converge to an equilibrium state when costs are changed. Numerical examples are provided to illustrate the practicality and flexibility of the proposed method algorithm.

Keywords supply chain design, Physarum, capacity investments, network optimization, adaptivity

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1 Introduction

A supply chain is a network of suppliers, manufacturers, storage houses, and distribution centers organized to acquire raw materials, convert these raw materials to finished products, and distribute these products to customers [1–4]. With the globalization of market economics, for many companies, especially for high tech companies, the customers are located all over the world and the components are also distributed in many places ranging from Taiwan to South Africa. To design an efficient supply chain network the enterprises must identify optimal capacities associated with various supply activities and the optimal production quantities, storage volumes as well as the shipments. They also must take into consideration the cost related with each activity. The costs, including shipment expenses, and shrinking resources of manufacturing bases, vary from day to day. From a practical standpoint, it is important to consider these

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factors so that the sum of strategic, tactical, and operational costs can be minimized. This requires a systematic approach for the design of a supply chain network based on system-wide insights.

During the past decades, the topic of supply chain network design has received a great deal of attention [5–13]. In 1998, Beamon [14] presented an integrated supply chain network design model formulated as a multi-commodity mixed integer program and treated the capacity associated with each link as a known parameter. Two years later, Sabri and Beamon [15] developed another approach to optimize the strategic and operational planning in the supply chains design problem using a multi-objective function. However, in their model, the cost associated with each link was a linear function and thus the model did not capture the dynamic aspects of the networks, which are prone to congestions. Handfield and Nichols [16] also employed discrete variables in the formulation of the supply chain network model and this model was faced with the same problem mentioned above. Recently, Nagurney [17] presented a framework for supply chain network design and redesign at minimal total cost subjecting to the demand satisfaction from a system-optimization perspective. They employed Lagrange multipliers to deal with the constraint associated with the links, this made the model very complicated and required expensive computation. Meanwhile, some studies focus on supply chain with the consideration of inherent uncertainty in the network. As fuzzy set theory and evidence theory have been utilized to model the uncertainty efficiently [18–21], some researchers have applied them into the supply chain [22–24].

Computer scientists and engineers are often looking into behaviour, mechanics, and physiology of biological systems to uncover novel principles of distributed sensing, information processing and decision making that could be adopted in the development of future and emergent computing paradigms, architectures and implementations. One of the most popular nowadays living computing substrates is a slime mould Physarum polycephalum.

Plasmodium is a vegetative stage of a cellular slime mould P. polycephalum, a single cell with many nuclei, which feeds on microscopic particles [25]. When foraging for its food the plasmodium propagates towards sources of food, surrounds them, secretes enzymes and digests the food; it may form a congregation of protoplasm covering the food source. When several sources of nutrients are scattered in the plasmodium's range, the plasmodium forms a network of protoplasmic tubes connecting the masses of protoplasm at the food sources.

Laboratory experiments and theoretical studies have shown that the slime mould can solve many graph theoretical problems, such as finding the shortest path [26–33], shortest path tree problem [34], network formulation and simulation [35–37], influential nodes identification [38], connecting different arrays of food sources in an efficient manner [39–42], and network design [43–48].

Physarum can be considered as a parallel computing substrate with distributed sensing, parallel information processing and concurrent decision making. When the slime mould colonizes several sources of nutrients, it dynamically updates the thickness of its protoplasmic tubes, depending on how much nutrition is left in any particular source and proximity of the source of repellents, gradients of humidity and illumination [49, 50]. This dynamic updating of the protoplasmic networks inspired us to employ the principle of Physarum foraging behaviour to solve the supply chain network design problem aiming to minimize the total costs of operating the flows and the investment cost. In the Physarum-inspired algorithm, we consider link capacities as design variables and use continuous functions to represent the costs of the links. Based on the system-optimization technique developed for supply chain network integration [51,52], we abstract the economic activities associated with a firm as a network. We make full use of two features of Physarum: a continuity of the flux during the iterative process and the protoplasmic network adaptivity, or reconfiguration.

The paper is structured as follows. In Section 2, we introduce the supply chain network design model and our latest research related to P. polycephalum. In Section 3, we propose an approach to the supply chain network design problem based on the Physarum model. In Section 4, numerical examples are used to illustrate the flow of the proposed method and the methods's efficiency. We summarize our conclusions and provide suggestions for further studies in Section 5.

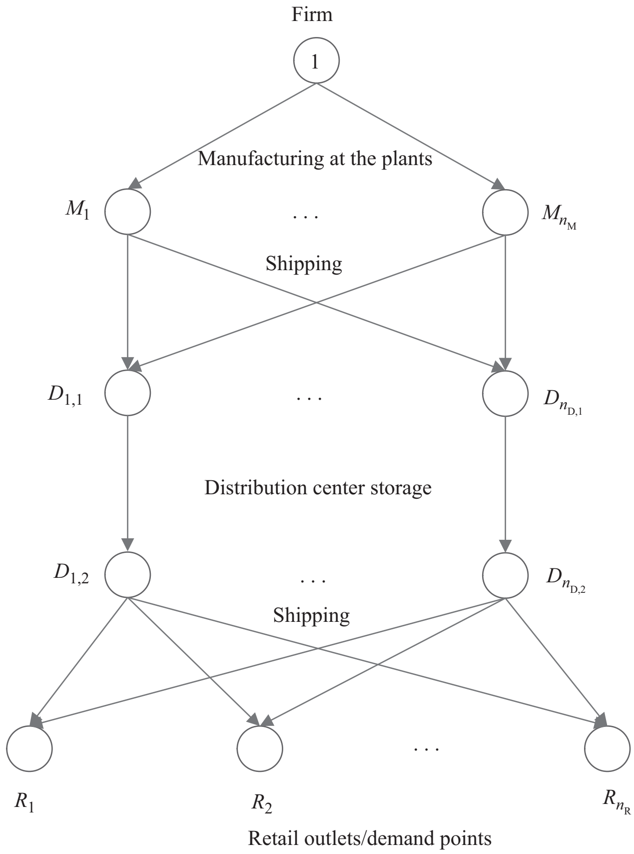

Figure 1 The supply chain network topology.

2 Preliminaries

This section introduces the supply chain network design model and the Physarum model.

2.1 Supply chain network design model [17]

Consider the supply chain network shown in Figure 1. A firm corresponding to node 1 aims to deliver the goods or products to the bottom level nodes corresponding to the retail outlets. The links connecting the source node with the destination nodes represent the activities of production, storage and transportation of good or services. Different network topologies correspond to different supply chain network problems. In this paper, we assume that there exists only one path linking node 1 with each destination node, which can ensure that the demand at each retail outlet can be satisfied.

As shown in Figure 1, the firm takes into consideration n_{M} manufacturers, n_{D} distribution centers when n_{R} retailers with demands $d_{R_1}, d_{R_2}, \ldots, d_{R_{n_{\mathrm{R}}}}$ must be served. Node 1 in the first layer is linked with the possible n_{M} manufacturers, represented as $M_1, M_2, \ldots, M_{n_{\mathrm{M}}}$. These edges in the manufacturing level are associated with the possible distribution center nodes, denoted as $D_{1,1}, D_{2,1}, \ldots, D_{n_{\mathrm{D}},1}$. These links indicate possible shipments between the manufacturers and the distribution centers. The links connecting $D_{1,1}, D_{2,1}, \ldots, D_{n_{\mathrm{D}},1}$ with $D_{1,2}, D_{2,2}, \ldots, D_{n_{\mathrm{D}},2}$ reflect the possible storage options. The links between $D_{1,2}, D_{2,2}, \ldots, D_{n_{\mathrm{D}},2}$ and $R_1, R_1, \ldots, R_{n_{\mathrm{R}}}$ denote the possible shipment pathways connecting the storage centers with the retail outlets.

Let a supply chain network be represented by a graph $G(N, L)$, where N is a set of nodes and L is a set of links. Each link in the network is associated with a cost function and the cost reflects the total cost of all the specific activities in the supply chain network, such as the transport of the product, and the delivery of the product. The cost related to link a is denoted as $\widehat{c}_a$. A path p connecting node 1 with a retail node shown in Figure 1 denotes the entire set of activities related with manufacturing the products, storing them and transporting them, etc. Assume w_k denotes the set of source and destination nodes $(1, R_k)$ and P_{w_k} represents the set of alternative associated possible supply chain network processes joining $(1, R_k)$. Then P is the set of all paths joining $(1, R_k)$ while x_p denotes the flow of the product on path p. Then the following equation must be satisfied:

$$\sum_{p\in P_{w_k}} x_p = d_{w_k}, \quad k = 1, \ldots, n_{\mathrm{R}}. \tag{1}$$

Let f_a represent the flow on link a; then the following flow conservation condition must be met:

$$f_a = \sum_{p\in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{2}$$

where δ_{ap} is a binary variable, $\delta_{ap} = 1$ if link a is included in the path p. Otherwise, $\delta_{ap} = 0$. Eq. (2) means that the inflow must be equal to the outflow on link a.

These flows can be grouped into the vector f. The flow on each link must be a nonnegative number, i.e. the following equation must be satisfied:

$$x_p \geqslant 0, \quad \forall p \in P. \tag{3}$$

Suppose the maximum capacity on link a is expressed by $u_a, \forall a \in L$. It is required that the actual flow on link a cannot exceed the maximum capacity on this link,

$$\begin{aligned} &f_a \leqslant u_a, \quad \forall a \in L,\\ &0 \leqslant u_a, \quad \forall a \in L. \end{aligned} \tag{4}$$

The total cost on each link, for simplicity, is represented as a function of the flow of the product on all the links [51, 53–55],

$$\widehat{c}_a = \widehat{c}_a\left(f\right), \quad \forall a \in L. \tag{5}$$

The total investment cost of adding capacity u_a on link a can be expressed as

$$\widehat{\pi}_a = \widehat{\pi}_a\left(u_a\right), \quad \forall a \in L. \tag{6}$$

In summary, the supply chain network design optimization problem is to satisfy the demand of each retail outlet and minimize the total cost, including the total cost of operating the various links and the capacity investments,

$$\min \sum_{a\in L} \widehat{c}_a\left(f\right) + \sum_{a\in L} \widehat{\pi}_a\left(u_a\right) \quad \text{s.t. constraints (1)–(4).} \tag{7}$$

2.2 Physarum polycephalum

Physarum polycephalum is a large, single-celled amoeboid organism forming a dynamic tubular network connecting the discovered food sources during foraging. The mechanism of tube formation can be described as follows. Tubes thicken in a given direction when shuttle streaming of the protoplasm persists in that direction for a certain time. There is a positive feedback between flux and tube thickness, as the conductance of the sol is greater in a thicker channel. With this mechanism in mind, a mathematical model illustrating shortest path search has been constructed [56].

Suppose the shape of the network formed by the Physarum is represented by a graph, in which a plasmodial tube refers to an edge of the graph and a junction between tubes refers to a node. Two special nodes labeled as N_1, N_2 act as the starting node and ending node respectively. The other nodes are labeled as N_3, N_4, N_5, N_6, etc. The edge between nodes N_i and N_j is M_{ij}. The parameter Q_{ij} denotes the flux through tube M_{ij} from node N_i to N_j. Assume that the flow along the tube is approximated by Poiseuille flow. Then the flux Q_{ij} can be expressed as

$$Q_{ij} = \frac{D_{ij}}{L_{ij}}(p_i - p_j), \tag{8}$$

where p_i is a pressure at a node N_i, D_{ij} is a conductivity of a tube M_{ij}, and L_{ij} is its length.

By assuming that the inflow and outflow must be balanced, we have

$$\sum Q_{ij} = 0 \quad (j \neq 1, 2). \tag{9}$$

For the source node N_1 and the sink node N_2 the following holds:

$$\sum_i Q_{i1} + I_0 = 0, \tag{10}$$

$$\sum_i Q_{i2} - I_0 = 0, \tag{11}$$

where I_0 is the flux flowing from the source node and I_0 is a constant value here.

In order to describe such an adaptation of tubular thickness we assume that the conductivity D_{ij} changes over time according to the flux Q_{ij}. An evolution of $D_{ij}(t)$ can be described by the following equation:

$$\frac{\mathrm{d}}{\mathrm{d}t} D_{ij} = f(|Q_{ij}|) - \gamma D_{ij}, \tag{12}$$

where γ is a decay rate of the tube. The equation implies that conductivity becomes zero if there is no flux along the edge. The conductivity increases with the flux. The f is monotonically increasing continuous function satisfying $f(0) = 0$.

Then the network Poisson equation for pressure can be obtained from Eqs. (8)–(11) as follows:

$$\sum_i \frac{D_{ij}}{L_{ij}}(p_i - p_j) = \begin{cases} -1, & \text{for } i = 1, \\ +1, & \text{for } j = 2, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

By setting p_2=0 as a basic pressure level, all p_i can be determined by solving Eq. (13) and Q_{ij} can also be obtained.

In this paper, $f(Q) = |Q|$ is used because $f\left(|Q_{ij}|\right) = |Q|, \gamma = 1$, the Physarum can always converge to the shortest path regardless of whether the distribution of conductivities in the initial state is random or biased [56]. With the flux calculated, the conductivity can be derived, where Eq. (14) is used instead of Eq. (12), adopting the functional form $f(Q) = |Q|$,

$$\frac{D_{ij}^{n+1} - D_{ij}^{n}}{\delta t} = |Q| - D_{ij}^{n+1}. \tag{14}$$

3 Proposed method

In this section, we employ the Physarum model to solve the supply chain network design problem. Generally speaking, there are two sub-problems to address:

- In the shortest path finding model, there is only one source and one destination in the network while there are more than one retailers in the supply chain network design problem.
- The Physarum model should be modified to satisfy the capacity constraint on each link.

3.1 One-source multi-sink Physarum model

In the original Physarum model [56], there is only one source node and one ending node. In the supply chain network design problem, as shown in Figure 1, there are R_{n_R} retail outlets. From left to right, from top to bottom, we can number the nodes shown in Figure 1. As a result, the following equation is formulated to replace Eq. (13):

$$\sum_i \frac{D_{ij}}{L_{ij}}\left(p_i - p_j\right) = \begin{cases} -\sum\limits_{i=1}^{n_\mathrm{R}} d_{R_i}, & j = 1, \\ +d_j, & j = R_1, R_2, \ldots, R_{n_\mathrm{R}}, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

where $j=1$ means that $\sum_{i=1}^{n_{\rm R}} d_{R_i}$ units of goods are distributed from the firm to the other manufacturing facilities, $j=R_{n_{\rm R}}$ denotes the $n_{\rm R}$ retail outlet needs $d_{R_{n_{\rm R}}}$ units of goods.

In the original Physarum model, the length associated with each link is fixed. In the supply chain network design problem, the cost on each link, let it be a production link, a shipment link, or a storage link, consists of the operation cost of the flow on each link $\widehat{c}_a(f)$ and the investment cost $\widehat{\pi}_a(u_a)$. Ideally speaking, the flow on each link should be equal to its capacity. If its capacity is bigger than the actual flow, this will generate additional cost. From this point of view, when the flow on each link is equal to the capacity, it not only can meet the requirement of flow, but also decrease the total cost. Assuming there is flow f_{ij} passing through the link (i,j), we take the following measure to convert the two costs into one:

$$L_{ij}=\hat{c}_{ij}\left(f_{ij}\right)+\hat{\pi}_{ij}\left(f_{ij}\right). \tag{16}$$

In the Physarum model, it is necessary for us to initialize the related parameters, including the link length L, and the conductivity D at each node. In the supply chain network design model, the capacity and the flow are unknown design variables. If we do not know the specific flow on each link, we cannot determine its cost, which further leads to the initialization failure of the Physarum algorithm. To prevent the initialization failure, we initialize the length on each link as a very small value ranging from 0.01 to 0.0001.

3.2 System optimization in supply chain network

In the Physarum model, we express the cost function using Eq. (16). Based on the Physarum model, all the flow tries to pass through the shortest path. As both the operating cost and the investment cost are a function of flow, the overall cost on the shortest path will increase accordingly. Given this, the flow will shift to other shorter paths from the second. As time goes, the system reaches an equilibrium state called user equilibrium (UE) in which no flow can improve the cost by unilaterally shifting to another route. In other words, all the paths have the same cost.

However, in the supply chain network, it is a system optimum (SO) problem with the objective to minimize the overall cost of the network. For the purpose of addressing this problem using the Physarum model, we must find an alternative way to transform the SO problem into the corresponding UE problem. Based on the method proposed in Refs. [57, 58], we are able to achieve this objective as

$$\widetilde{t}_a\left(x_a\right)=t_a\left(x_a\right)+x_a\frac{\mathrm{d}t_a\left(x_a\right)}{\mathrm{d}x_a},\ \forall a\in L, \tag{17}$$

where x_a represents the flow on link a, $t_a(x_a)$ is the cost function for per unit of flow on link a while $\widetilde{t}_a\left(x_a\right)$ denotes the transformed cost function in the corresponding UE problem.

Since both $\widehat{c}_{ij}\left(f_{ij}\right)$ and $\widehat{\pi}_{ij}\left(f_{ij}\right)$ denote the cost when the flow is f_{ij}, the following equation is built to represent the cost per unit of flow:

$$L_{ij}=\frac{\widehat{c}_{ij}\left(f_{ij}\right)+\widehat{\pi}_{ij}\left(f_{ij}\right)}{f_{ij}}. \tag{18}$$

By combining Eqs. (17) and (18), we have

$$L_{ij}=\frac{\widehat{c}_{ij}\left(f_{ij}\right)+\widehat{\pi}_{ij}\left(f_{ij}\right)}{f_{ij}}+f_{ij}\frac{\mathrm{d}\left(\frac{\widehat{c}_{ij}(f_{ij})+\widehat{\pi}_{ij}(f_{ij})}{f_{ij}}\right)}{\mathrm{d}f_{ij}}. \tag{19}$$

By updating the cost on each link according to the function shown in Eq. (19), we successfully convert the SO problem into the corresponding UE problem. Then the Physarum algorithm is used to find the optimal solution to the UE problem.

3.3 General flow of Physarum algorithm

The main flow of Physarum model is presented in Algorithm 1. The firm and the retail outlets are treated as the starting node and the ending nodes, respectively.

First of all, the conductivity of each tube (D_{ij}) is initialized as a random number between 0 and 1 and other variables are assigned as 0, including the flux through each tube (Q_{ij}), pressure at each node (p_i).

Secondly, we obtain the pressure associated with each node using Eq. (15). Besides, the flux passing through each link and the conductivity in the next iteration can be recorded. Thirdly, we update the cost on each link based on Eq. (19).

There are several possible stopping criteria of Algorithm 1, such as the maximum number of iterations is arrived, conductivity of each tube converges to 0 or 1, flux through each tube remains unchanged, etc. The algorithm described in the present paper halts when $\sum_{i=1}^{N}\sum_{j=1}^{N}\left|D_{ij}^{n}-D_{ij}^{n-1}\right| \leqslant \delta$, where δ is a threshold value, D_{ij}^{n} and D_{ij}^{n-1} represent the conductivity on link (i,j) in the n^{th} and $(n-1)^{\text{th}}$ iteration, respectively.

3.4 Discussion

The Physarum-inspired solution provides new insight for designing an optimal supply chain network. In the algorithm, we need to solve the linear equations shown in Eq. (15) and it is the most time consuming part. So far, the upper bound of computational effort for solving a linear equation with n unknown variables is $O(n^3)$.

Another difficulty is how to choose an appropriate value for σ in the termination condition. In this paper, when $\sum_{i=1}^{N}\sum_{j=1}^{N}\left|D_{ij}^{n}-D_{ij}^{n-1}\right| \leqslant \delta$, the program terminates. An appropriate value for δ is a challenging problem. If σ is too big, we might stop the program before it converges to the optimal solution. On the contrary, if δ is too small, we need to perform much more iterations, which make the computation time consuming. In this paper, we choose the value for δ by trial and error. It is not feasible at the moment to look for an analytical solution because it depends on the property of the cost functions and the topology of the network.

4 Numerical examples

In this section we demonstrate efficiency of the algorithm with three numerical examples. The supply

Algorithm 1 Physarum-inspired model for the optimal supply chain network design (L,1,N,R)

// N is the size of the network;
// L_{ij} is the link connecting node i with node j;
// 1 is the starting node while R is the set of retail outlets;
$D_{ij} \leftarrow (0,1]$ $(\forall i,j=1,2,\ldots,N)$;
$Q_{ij} \leftarrow 0$ $(\forall i,j=1,2,\ldots,N)$;
$p_i \leftarrow 0$ $(\forall i=1,2,\ldots,N)$;
$L_{ij} \leftarrow 0.001$ $(\forall i,j=1,2,\ldots,N)$;
repeat
Calculate the pressure associated with each node according to Eq. (15)

$$\sum_i \frac{D_{ij}}{L_{ij}}(p_i-p_j)=\begin{cases}-\sum_{i=1}^{n_{\mathrm{R}}} d_{R_i}, & j=1,\\ +d_j, & j=R_1,R_2,\ldots,R_{n_{\mathrm{R}}}\\ 0, & \text{otherwise.}\end{cases}$$

$Q_{ij} \leftarrow D_{ij}\times(p_i-p_j)/L_{ij}$ // Using Eq. (8);
$D_{ij} \leftarrow Q_{ij}+D_{ij}$ // Using Eq. (14)
Update the cost of each link;

$$L_{ij}=\frac{\widehat{c}_{ij}(f_{ij})+\widehat{\pi}_{ij}(f_{ij})}{f_{ij}}+f_{ij}\frac{\mathrm{d}\left(\frac{\widehat{c}_{ij}(f_{ij})+\widehat{\pi}_{ij}(f_{ij})}{f_{ij}}\right)}{\mathrm{d}f_{ij}}$$

until a termination criterion is met

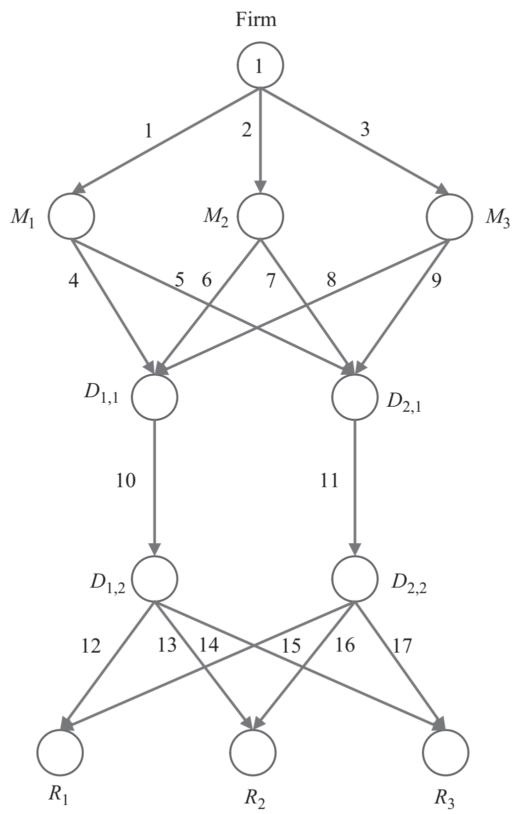

Figure 2 The supply chain network topology for all the examples [17].

chain network topology for all the examples is shown in Figure 2, and δ is 0.001. In addition, we initialize the link length as 0.001.

Example 1. In this example, the demands in each retail outlet are $d_{R_1} = 45, d_{R_2} = 35, d_{R_3} = 5$, respectively. The cost of the flow on each link $\widehat{c}_a(f)$ and the investment cost $\widehat{\pi}_a(u_a)$ are shown in Table 1; the costs are continuous-value functions.

Based on the proposed method, Figure 3 illustrates the flux variation during the iterative process. The flux on each link gets stable with the increase of iterations. The flux on link 14 corresponding to the shipment link connecting the first distribution center with the retailer R_3 gradually decreases to 0 and it should be removed from the supply chain network design. Besides, link 17's flux increases to 5 step by step. Table 1 represents the solution. This solution corresponds to the optimal supply chain network design topology as shown in Figure 4. As can be noted from [17], the method proposed by Nagurney requires 497 iterations to reach the convergence. However, in the proposed method, after about 25 iterations, the results become very stable.

The minimal cost, using Eq. (7), is 16125.65, which is consistent with that of [17]. It took the Physarum algorithm 0.029 s to solve the above supply chain network design problem.

Example 2. The data in example 2 has the same total cost as in example 1 except that we use a linear term to express the total cost associated with the first distribution center (for specific details, please see data for link 10 in Table 2).

Figure 5 shows the changing trend of the flux associated with each link during the iterative process. The complete solution for this problem is shown in Table 2. The flow on link 14 now has positive capacity and positive product flow in contrast to the data shown in Table 1. As a matter of fact, the flow of all the links from the first distribution center to the retail outlets has increased, compared to the values in example 1. Namely, the capacity and product flow on links 12, 13, and 14 are larger than that of example 1. On the contrary, the product flow on the links connecting the second distribution center with the retail outlets has decreased. For example, the solution values on links 15, 16, 17 in example 2 are less

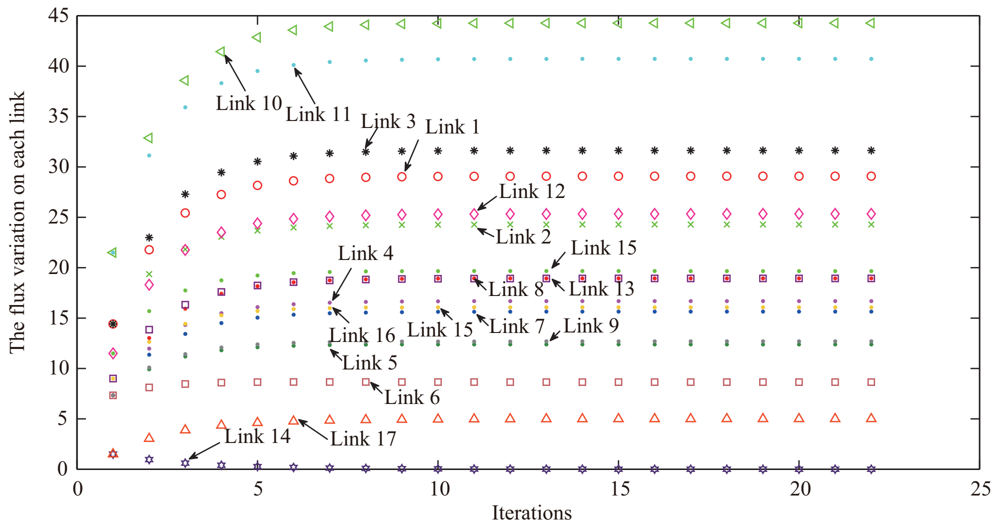

Figure 3 The flux variation associated with each link during the iterative process in example 1.

Table 1 Total cost functions and solution in example 1 adopted from [17]

Link a	$\widehat{c}_a(f)$	$\widehat{\pi}_a(u_a)$	f_a^*	u_a^*
1	$f_1^2+2f_1$	$0.5u_1^2+u_1$	29.08	29.08
2	$0.5f_2^2+f_2$	$2.5u_2^2+u_2$	24.29	24.29
3	$0.5f_3^2+f_3$	$u_3^2+2u_3$	31.63	31.63
4	$1.5f_4^2+2f_4$	$u_4^2+u_4$	16.68	16.68
5	$f_5^2+3f_5$	$2.5u_5^2+2u_5$	12.40	12.40
6	$f_6^2+2f_6$	$0.5u_6^2+u_6$	8.65	8.65
7	$0.5f_7^2+2f_7$	$0.5u_7^2+u_7$	15.64	15.64
8	$0.5f_8^2+2f_8$	$1.5u_8^2+u_8$	18.94	18.94
9	$f_9^2+5f_9$	$2u_9^2+3u_9$	12.69	12.69
10	$0.5f_{10}^2+2f_{10}$	$u_{10}^2+5u_{10}$	44.28	44.28
11	$f_{11}^2+f_{11}$	$0.5u_{11}^2+3u_{11}$	40.72	40.72
12	$0.5f_{12}^2+2f_{12}$	$0.5u_{12}^2+u_{12}$	25.34	25.34
13	$0.5f_{13}^2+5f_{13}$	$0.5u_{13}^2+u_{13}$	18.94	18.94
14	$f_{14}^2+7f_{14}$	$2u_{14}^2+5u_{14}$	0.00	0.00
15	$f_{15}^2+2f_{15}$	$0.5u_{15}^2+u_{15}$	19.66	19.66
16	$0.5f_{16}^2+3f_{16}$	$u_{16}^2+u_{16}$	16.06	16.06
17	$0.5f_{17}^2+2f_{17}$	$0.5u_{17}^2+u_{17}$	5.00	5.00

when compared with that of example 1. Based on the solution shown in Table 2, the final optimal supply chain network is shown in Figure 4. The computation time for this example is 0.049 s.

Example 3. Example 3 has the same data as example 2 except that we use linear terms to replace nonlinear functions representing the total costs associated with the capacity investments on the first and second manufacturing plants. For instance, the cost associated with the capacity investments on the first link is represented by u_1 instead of $0.5u_1^2+u_1$.

Figure 6 shows the flux variation associated with each link during the iterative process. The solution for this problem is given in Table 3. The minimum cost is 10726.48, which is in accordance with that of [17]. It can be noted that once the cost on each link changes, the proposed method can adaptively allocate the flow and the capacity investments. The proposed algorithm solved the above problem in 0.046 s.

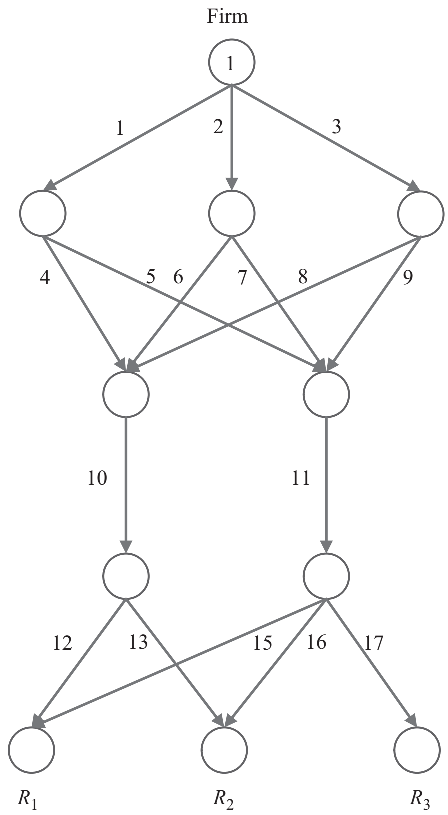

Figure 4 Optimal supply chain network topology in example 1.

Table 2 Total cost functions and solution for example 2. Adopted from [17]

Link a	$\widehat{c}_a(f)$	$\widehat{\pi}_a\,(u_a)$	f_a^*	u_a^*
1	$f_1^2+2f_1$	$0.5u_1^2+u_1$	29.28	29.28
2	$0.5f_2^2+f_2$	$2.5u_2^2+u_2$	23.78	23.78
3	$0.5f_3^2+f_3$	$u_3^2+2u_3$	31.93	31.93
4	$1.5f_4^2+2f_4$	$u_4^2+u_4$	19.01	19.01
5	$f_5^2+3f_5$	$2.5u_5^2+2u_5$	10.28	10.28
6	$f_6^2+2f_6$	$0.5u_6^2+u_6$	13.73	13.73
7	$0.5f_7^2+2f_7$	$0.5u_7^2+u_7$	10.05	10.05
8	$0.5f_8^2+2f_8$	$1.5u_8^2+u_8$	21.77	21.77
9	$f_9^2+5f_9$	$2u_9^2+3u_9$	10.17	10.17
10	$0.5f_{10}^2+2f_{10}$	$5u_{10}$	54.50	54.50
11	$f_{11}^2+f_{11}$	$0.5u_{11}^2+3u_{11}$	30.50	30.50
12	$0.5f_{12}^2+2f_{12}$	$0.5u_{12}^2+u_{12}$	29.58	29.58
13	$0.5f_{13}^2+5f_{13}$	$0.5u_{13}^2+u_{13}$	23.18	23.18
14	$f_{14}^2+7f_{14}$	$2u_{14}^2+5u_{14}$	1.74	1.74
15	$f_{15}^2+2f_{15}$	$0.5u_{15}^2+u_{15}$	15.42	15.42
16	$0.5f_{16}^2+3f_{16}$	$u_{16}^2+u_{16}$	11.82	11.82
17	$0.5f_{17}^2+2f_{17}$	$0.5u_{17}^2+u_{17}$	3.26	3.26

5 Concluding remarks

This paper solved the supply chain network design problem using a bio-inspired algorithm. We proposed a model for the supply chain network design allowing for the determination of the optimal levels of capacity and product flows in the supply activities, including manufacturing, distribution and storage, and subject to the satisfaction of retail outlets. By employing the principles of protoplasmic network

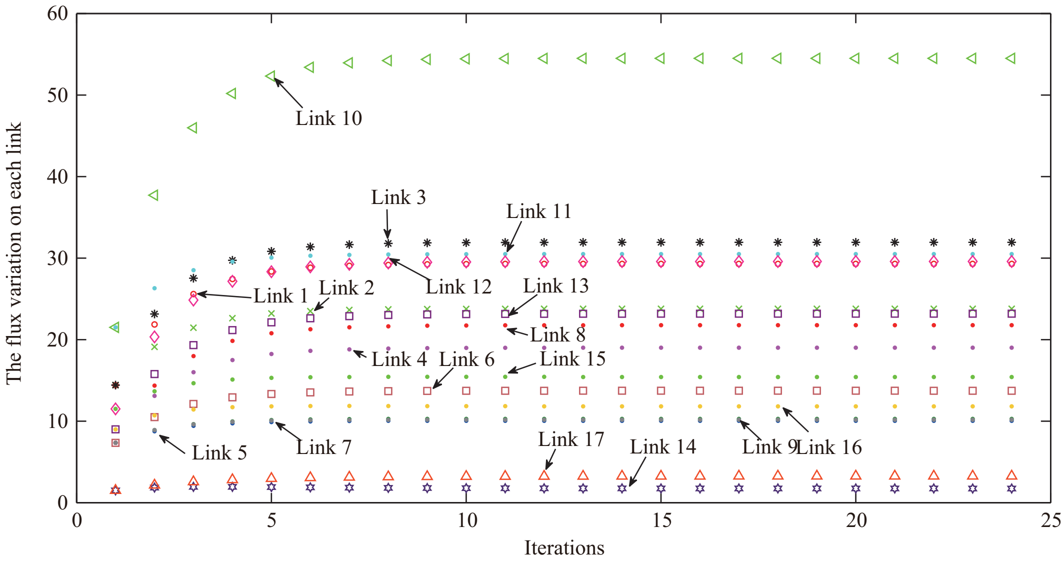

Figure 5 Flux variation associated with each link during the iterative process in example 2.

Table 3 Total cost functions and solution for example 3 adopted from [17]

Link a	$\widehat{c}_a(f)$	$\widehat{\pi}_a\,(u_a)$	f_a^*	u_a^*
1	$f_1^2+2f_1$	u_1	20.91	20.91
2	$0.5f_2^2+f_2$	u_2	45.18	45.18
3	$0.5f_3^2+f_3$	$u_3^2+2u_3$	18.91	18.91
4	$1.5f_4^2+2f_4$	$u_4^2+u_4$	14.74	14.74
5	$f_5^2+3f_5$	$2.5u_5^2+2u_5$	6.16	6.16
6	$f_6^2+2f_6$	$0.5u_6^2+u_6$	23.79	23.79
7	$0.5f_7^2+2f_7$	$0.5u_7^2+u_7$	21.39	21.39
8	$0.5f_8^2+2f_8$	$1.5u_8^2+u_8$	14.79	14.79
9	$f_9^2+5f_9$	$2u_9^2+3u_9$	4.21	4.21
10	$0.5f_{10}^2+2f_{10}$	$5u_{10}$	53.23	53.23
11	$f_{11}^2+f_{11}$	$0.5u_{11}^2+3u_{11}$	31.77	31.77
12	$0.5f_{12}^2+2f_{12}$	$0.5u_{12}^2+u_{12}$	29.10	29.10
13	$0.5f_{13}^2+5f_{13}$	$0.5u_{13}^2+u_{13}$	22.70	22.70
14	$f_{14}^2+7f_{14}$	$2u_{14}^2+5u_{14}$	1.44	1.44
15	$f_{15}^2+2f_{15}$	$0.5u_{15}^2+u_{15}$	15.90	15.90
16	$0.5f_{16}^2+3f_{16}$	$u_{16}^2+u_{16}$	12.30	12.30
17	$0.5f_{17}^2+2f_{17}$	$0.5u_{17}^2+u_{17}$	3.56	3.56

growth and dynamic reconfiguration used by slime mould Physarum polycephalum model, we solve the supply chain network design problem. The cost associated with the capacity investments and the product flows are represented by either linear or continuous functions.

Our model is superior when compared with the previously published slime mould models characterizing its foraging activity and the circulation of chemotaxis. The model of active zones of Physarum in terms of light-sensitive sub-excitable chemical medium was proposed in [59]. The model was designed solely to demonstrate that wave-fragments in reaction-diffusion media and active growing zones of Physarum behave similarly. The propagating wave-fragments do not leave any traces. Therefore, to reconstruct a spanning tree developed by the Oregonator model of Physarum, one must analyse snapshots of the model's space-time behaviour. This is time consuming. Cellular automaton models of the slime mould's foraging behaviour were proposed in [60, 61]. Cellular automata are efficient in term of spatial and temporal complexity: the number of cell/node state is finite, typically less than a dozen, and size of a lattice usually does not exceed 500×500. Therefore, the automaton models allow for fast prototyping

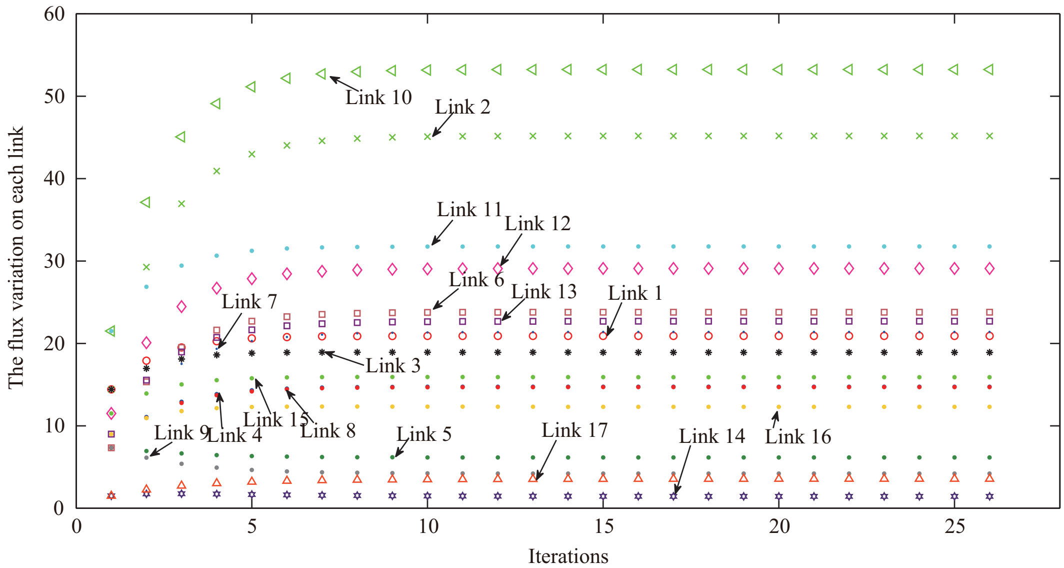

Figure 6 The flux variation associated with each link during the iterative process in example 3.

of the various patterns of foraging behaviour. However, due to the small number of cell/node states, it might be unaffordable if not impossible to implement traffic networks or supply chains in the automaton models. Increasing the number of states in automaton could make the model more realistic, however, fine-granular parallelism with a large number of node states leads to increased time consumption during modelling. The time consumption could be reduced when cellular automata models of Physarum are implemented in hardware [62–64]. We did not consider cellular automata as truly alternative models because they usually focus on the morphology of the patterns and their qualitative dynamics rather than quantitative transitions that are characteristic for supply chains.

Further research can focus on the following directions. First, extension of the method to the design of supply chain network under complicated environment needs to be investigated, such as with customer demands and supply chain network redesign. Second, application of this method to other fields may be investigated, such as the transportation network, mobile networks, and telecommunication networks. Third, the proposed method can be further enriched with approaches adopted in swarm-based modelling of the slime mould [65–67].

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Conflict of interest The authors declare that they have no conflict of interest.

References

1 Liu Z, Nagurney A. Supply chain networks with global outsourcing and quick-response production under demand and cost uncertainty. Annals Oper Res, 2013, 208: 251–289

2 Zhang W, Xu D. Integrating the logistics network design with order quantity determination under uncertain customer demands. Expert Syst Appl, 2014, 41: 168–175

3 Yu M, Nagurney A. Competitive food supply chain networks with application to fresh produce. Eur J Oper Res, 2013, 224: 273–282

4 Hu Z, Du X. Lifetime cost optimization with time-dependent reliability. Eng Optim, 2014, 46: 1389–1410

5 Ma H, Suo C. A model for designing multiple products logistics networks. Int J Phys Distrib & Log Manag, 2006, 36: 127–135

6 Zhu X, Yao Q. Logistics system design for biomass-to-bioenergy industry with multiple types of feedstocks. Bioresource

Tech, 2011, 102: 10936–10945
7 Santoso T, Ahmed S, Goetschalckx M, et al. A stochastic programming approach for supply chain network design under uncertainty. Eur J Oper Res, 2005, 167: 96–115
8 Zhou G, Min H, Gen M. The balanced allocation of customers to multiple distribution centers in the supply chain network: a genetic algorithm approach. Comput Ind Eng, 2002, 43: 251–261
9 Trkman P, McCormack K. Supply chain risk in turbulent environments–a conceptual model for managing supply chain network risk. Int J Prod Econ, 2009, 119: 247–258
10 Altiparmak F, Gen M, Lin L, et al. A steady-state genetic algorithm for multi-product supply chain network design. Comput Ind Eng, 2009, 56: 521–537
11 Ahmadi J A, Azad N. Incorporating location, routing and inventory decisions in supply chain network design. Transport Res Part E: Log Transport Rev, 2010, 46: 582–597
12 Nagurney A. Supply chain network design under profit maximization and oligopolistic competition. Transport Res Part E: Log Transport Rev, 2010, 46: 281–294
13 Bilgen B. Application of fuzzy mathematical programming approach to the production allocation and distribution supply chain network problem. Expert Syst Appl, 2010, 37: 4488–4495
14 Beamon B M. Supply chain design and analysis: models and methods. Int J Prod Econ, 1998, 55: 281–294
15 Sabri E H, Beamon B M. A multi-objective approach to simultaneous strategic and operational planning in supply chain design. Omega, 2000, 28: 581–598
16 Handfield R B, Nichols E L. Supply Chain Redesign: Transforming Supply Chains into Integrated Value Systems. Upper Saddle River: FT Press, 2002
17 Nagurney A. Optimal supply chain network design and redesign at minimal total cost and with demand satisfaction. Int J Prod Econ, 2010, 128: 200–208
18 Jiang W, Yang Y, Luo Y, et al. Determining basic probability assignment based on the improved similarity measures of generalized fuzzy numbers. Int J Comput Commun Control, 2015, 10: 333–347
19 Deng Y. Generalized evidence theory. Appl Intell, 2015, 43: 530–543
20 Deng Y, Mahadevan S, Zhou D. Vulnerability assessment of physical protection systems: a bio-inspired approach. Int J Unconv Comput, 2015, 3–4: 227–243
21 Jiang W, Luo Y, Qin X, et al. An improved method to rank generalized fuzzy numbers with different left heights and right heights. J Intell Fuzzy Syst, 2015, 28: 2343–2355
22 Deng X, Hu Y, Deng Y, et al. Supplier selection using AHP methodology extended by D numbers. Expert Syst Appl, 2014, 41: 156–167
23 Deng Y, Chan F T. A new fuzzy dempster MCDM method and its application in supplier selection. Expert Syst Appl, 2011, 38: 9854–9861
24 Deng Y, Chan F T, Wu Y, et al. A new linguistic MCDM method based on multiple-criterion data fusion. Expert Syst Appl, 2011, 38: 6985–6993
25 Stephenson S L, Stempen H, Hall I. Myxomycetes: a Handbook of Slime Molds. Portland: Timber Press, 1994
26 Nakagaki T, Yamada H, Tóth Á. Intelligence: Maze-solving by an amoeboid organism. Nature, 2000, 407: 470
27 Zhang X, Zhang Z, Zhang Y, et al. Route selection for emergency logistics management: a bio-inspired algorithm. Saf Sci, 2013, 54: 87–91
28 Zhang X, Zhang Y, Hu Y, et al. An adaptive amoeba algorithm for constrained shortest paths. Expert Syst Appl, 2013, 40: 7607–7616
29 Zhang X, Wang Q, Chan F T S, et al. A Physarum polycephalum optimization algorithm for the bi-objective shortest path problem. Int J Unconv Comput, 2014, 10: 143–162
30 Tero A, Kobayashi R, Nakagaki T. Physarum solver: a biologically inspired method of road-network navigation. Phys A, 2006, 363: 115–119
31 Zhang X, Huang S, Hu Y, et al. Solving 0-1 knapsack problems based on amoeboid organism algorithm. Appl Math Comput, 2013, 219: 9959–9970
32 Zhang X, Wang Q, Adamatzky A, et al. A biologically inspired optimization algorithm for solving fuzzy shortest path problems with mixed fuzzy arc lengths. J Optimiz Theory Appl, in press. doi: 10.1007/s10957-014-0542-6
33 Zhang Y, Zhang Z, Deng Y, et al. A biologically inspired solution for fuzzy shortest path problems. Appl Soft Comput, 2013, 13: 2356–2363
34 Zhang X, Liu Q, Hu Y, et al. An adaptive amoeba algorithm for shortest path tree computation in dynamic graphs. arXiv: 1311.0460. 2013
35 Gunji YP, Shirakawa T, Niizato T, et al. An adaptive and robust biological network based on the vacant-particle transportation model. J Theor Biol, 2011, 272: 187–200
36 Shirakawa T, Gunji, Y P. Computation of Voronoi diagram and collision-free path using the plasmodium of physarum polycephalum. Int J Unconv Comput, 2010, 6: 79–88
37 Shirakawa T, Gunji Y P. Emergence of morphological order in the network formation of Physarum polycephalum. Biophys Chem, 2007, 128: 253–260
38 Gao C, Lan X, Zhang X, et al. A bio-inspired methodology of identifying influential nodes in complex networks. PloS one, 2013, 8: e66732
39 Nakagaki T, Iima M, Ueda T, et al. Minimum-risk path finding by an adaptive amoebal network. Phys Rev Lett, 2007, 99: 068104
40 Adamatzky A. Route 20, autobahn 7, and slime mold: approximating the longest roads in USA and Germany with

slime mold on 3-D terrains. IEEE Trans Cybernetics, 2014, 44: 126–136
41 Tero A, Yumiki K, Kobayashi R, et al. Flow-network adaptation in Physarum amoebae. Theory Biosci, 2008, 127: 89–94
42 Jones J, Adamatzky A. Computation of the travelling salesman problem by a shrinking blob. Natural Comput, 2014, 13: 1–16
43 Tero A, Takagi S, Saigusa T, et al. Rules for biologically inspired adaptive network design. Science, 2010, 327: 439–442
44 Adamatzky A, Alonso-Sanz R. Rebuilding Iberian motorways with slime mould. Biosyst, 2011, 105: 89–100
45 Adamatzky A. Bioevaluation of World Transport Networks. Singapore: World Scientific, 2012
46 Adamatzky A, Martínez G J, Chapa-Vergara S V, et al. Approximating Mexican highways with slime mould. Natural Comput, 2011, 10: 1195–1214
47 Gao C, Yan C, Zhang Z, et al. An amoeboid algorithm for solving linear transportation problem. Phys A, 2014, 398: 179–186
48 Adamatzky A, Martinez G J. Bio-imitation of Mexican migration routes to the USA with slime mould on 3D terrains. J Bionic Eng, 2013, 10: 242–250
49 Adamatzky A. Physarum Machines: Computers from Slime Mould. Singapore: World Scientific, 2010
50 Adamatzky A, Schubert T. Slime mold microfluidic logic gates. Mater Today, 2014, 17: 86–91
51 Nagurney A. A system-optimization perspective for supply chain network integration: the horizontal merger case. Transport Res Part E: Log Transport Rev, 2009, 45: 1–15
52 Nagurney A, Woolley T, Qiang Q. Multi-product supply chain horizontal network integration: models, theory, and computational results. Int Trans Oper Res, 2010, 17: 333–349
53 Nagurney A. Supply Chain Network Economics: Dynamics of Prices, Flows and Profits. Cheltenham: Edward Elgar Publishing, 2006
54 Nagurney A, Dong J, Zhang D, et al. A supply chain network equilibrium model. Transport Res Part E: Log Transport Rev, 2002, 38: 281–303
55 Nagurney A, Woolley T. Environmental and cost synergy in supply chain network integration in mergers and acquisitions. In: Multiple Criteria Decision Making for Sustainable Energy and Transportation Systems. Berlin: Springer, 2010. 57–78
56 Tero A, Kobayashi R, Nakagaki T. A mathematical model for adaptive transport network in path finding by true slime mold. J Theor Biol, 2007, 244: 553–564
57 Bell M G, Lida Y. Transportation Network Analysis. Hoboken: John Wiley & Sons, 1997
58 Si BF, Gao ZY. Modeling Network Flow and System Optimization for Traffic and Transportation System (in Chinese). Beijing: China Communications Press, 2013
59 Adamatzky A. If BZ medium did spanning trees these would be the same trees as Physarum built. Phys Lett A, 2009, 373: 952–956
60 Gunji Y P, Shirakawa T, Niizato T, et al. Minimal model of a cell connecting amoebic motion and adaptive transport networks. J Theor Biol, 2008, 253: 659–667
61 Gunji Y P, Shirakawa T, Niizato T, et al. An adaptive and robust biological network based on the vacant-particle transportation model. J Theor Biol, 2011, 272: 187–200
62 Tsompanas M A I, Sirakoulis G C. Modeling and hardware implementation of an amoeba-like cellular automaton. Bioinspir Biomim, 2012, 7: 036013
63 Tsompanas M A I, Sirakoulis G C, Adamatzky A. Evolving transport networks with cellular automata models inspired by slime mould. IEEE Trans Cybern, in press. doi: 10.1109/TCYB.2014.2361731
64 Kalogeiton V S, Papadopoulos D P, Sirakoulis G C. Hey Physarum! Can you perform SLAM? Int J Unconv Comput, 2014, 10: 271–293
65 Adamatzky A, Jones J. Road planning with slime mould: if Physarum built motorways it would route M6/M74 through Newcastle. Int J of Bifurcat Chaos, 2010, 20: 3065–3084
66 Chakravarthy H, Proch P B, Rajan R, et al. Bio inspired approach as a problem solving technique. Netw Complex Syst, 2012, 2: 14–22
67 Liu Y, Zhang Z, Gao C, et al. A physarum network evolution model based on IBTM. In: Advances in Swarm Intelligence. Berlin: Springer, 2013. 19–26