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Characteristic model based adaptive controller design and analysis for a class of SISO systems

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Abstract The design of an adaptive controller and stability analysis of the corresponding closed loop system are discussed for a class of SISO systems based on the characteristic model method. The obtained characteristic model is a second-order slow time-varying linear system with a compress mapping function for the system modeling error. The pole placement method is used to design the controller, and sufficient conditions for the stability of the closed loop system are obtained based on the robust control theory of slow time-varying systems with perturbations. The effectiveness of the proposed method is illustrated by two numerical examples.

Keywords characteristic model, modeling error, pole placement, perturbation analysis, sampling system

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1 Introduction

Traditional adaptive control algorithms are hardly competent for complex systems, especially in the presence of nonlinear uncertainties, due to the prolonged time taken from the plant state change to the model parameter variation and finally to the controller setting adjustment. In this regard adaptive control based on the characteristic model is a practically effective method in meeting the challenge. The characteristic model and with its all-coefficient adaptive control theories based on the characteristic model was proposed by Wu in the 1990s [1,2]. This method usually uses a low order linear time-varying model as the characteristic model to depict the original system which may be high order linear time-invariant or nonlinear, and then the adaptive controller is designed. The characteristic modelling theory and methods have been improved greatly in the last 20 years, with more than 400 successful applications categorized into 9 different types of engineering plants of astronautics and other industries [3,4]. For example, in [5] the authors apply the intelligent adaptive control method based on the characteristic model for rendezvous and docking; References [6–10] present a characteristic model-based adaptive control method for the attitude control of hypersonic vehicle and satellites; Reference [11] introduces an adaptive guidance law design based on the characteristic model for reentry vehicles, just to name a few. Achievements have

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also been made in the characteristic modelling for many kinds of original system models, for example, in [12,13], it is claimed that the original linear time-invariant systems are equivalent to the characteristic model with proper parameters in the input-output sense when the sampling time is short; besides nonlinear systems are also studied in [1,14–16]. It is known that the characteristic model can be generally represented by a second order system for fixed point control with location tracking objectives,

$$Y(k+2) = F_1(k)y(k+1) + F_2(k)y(k) + G(k)U(k+1),$$

where $F_i(k), G(k) \in R^{m\times m}, i = 1,2$ are the characteristic parameters reflecting the characteristics of the force and movement relationship between variables respectively [3]. These characteristic variables gradually vary with time and belong to a closed convex set Ds when the sample time is small enough. Ds can be achieved in the process of constructing characteristic model [12–16].

Despite the practical success of the adaptive control based on the characteristic model, the system stability remains a challenging problem, due to mainly two reasons. First, this method applies digital control scheme and is an indirect adaptive control, by which the model mismatch and the truncation error between the continuous and discrete systems are inevitable. Second, the resulting closed-loop system is a complex hybrid system which generally fails the stability theories of discrete-time system based on approximate models [17,18]. Relative studies are popular in the literature. For example, a golden-section adaptive control system based on the characteristic model for SISO (simple input simple output) system is studied in [19–21] where the stability conditions are given in the case of bounded modelling error. A similar study can be seen in [22,23] for MIMO systems. But all these reports do not consider the truncation error or progressively analyze the sampled-data system stability in the closed-loop system. In [15] the authors introduce a method to construct a second order characteristic model with exponentially convergent modelling error for a minimum phase nonlinear original continuous system with relative order 2, and then consider the mismatch and the sampled-data system stability of the closed-loop system, where the controller is designed by the golden-section adaptive method based on the characteristic model. Incidentally, the stability analysis is for exponential convergent modelling error.

Furthermore, the adaptive control based on the characteristic model should not solely consider the all-coefficient adaptive control method. Therefore, we design a controller by using adaptive control with pole placement based on its characteristic model, which can stabilize the original system output for a class of SISO continuous systems. For the corresponding stability analysis, this paper takes the model mismatch as an unmodeled dynamics, which is bounded and can be state and input dependent. Then we take full consideration of the modelling error between the characteristics model and the original system, using perturbation analysis and the slow time-varying linear system stability theorem to argue the complex mixed system stability conditions.

2 Problem description

2.1 System model

Consider the following continuous SISO system,

$$\begin{cases} \dot{x}_1 = f_1(x_1, \boldsymbol{x}_2), \\ \dot{\boldsymbol{x}}_2 = \boldsymbol{f}_2(x_1, \boldsymbol{x}_2) + \boldsymbol{g}(x_1, \boldsymbol{x}_2)u, \\ y = x_1, \end{cases} \tag{1}$$

where $x_1 \in \mathbb{R}$, $\boldsymbol{x}_2 \in \mathbb{R}^{n-1}$; $f_1 \in \mathbb{R}$, $\boldsymbol{f}_2 \in \mathbb{R}^{n-1}$, $\boldsymbol{g} \in \mathbb{R}^{n-1}$ are smooth function and $\boldsymbol{g}(x_1, \boldsymbol{x}_2) > \boldsymbol{0}$. For the system (1), we have the following assumption.

Assumption 1. The system (1) has but one equilibrium point at the origin and

$$\left|\frac{\partial f_1}{\partial x_1}\right| \leqslant M_1, \quad \left\|\frac{\partial f_1}{\partial \boldsymbol{x}_2}\right\| \leqslant M_2, \quad \|\boldsymbol{g}\| \leqslant M_3, \quad w(x_1, \boldsymbol{x}_2) = \frac{\partial f_1}{\partial \boldsymbol{x}_2}\boldsymbol{f}_2(x_1, \boldsymbol{x}_2) \leqslant M.$$

Remark 1. Assumption 1 can be satisfied by numerous systems, for example, most of minimum phase systems. Furthermore, the assumption $w(x_1, \boldsymbol{x}_2) \leqslant M$ is also often seen in literature [24,25], where M is dependent on the system state close to ball X, and is fixed for fixed bound size X.

2.2 Preliminary background

The adaptive control strategy based on the characteristic model applies digital control scheme which is an indirect adaptive control strategy. We give the basics of stability analysis of sampled-data systems as follows.

Rewriting system (1) as

$$\dot{x} = f(x, u),$$

where $x = [x_1, \boldsymbol{x}_2]^{\mathrm{T}}$ and $f(x, u) = [f_1(x_1, \boldsymbol{x}_2), \boldsymbol{f}_2(x_1, \boldsymbol{x}_2) + \boldsymbol{g}(x_1, \boldsymbol{x}_2)u]^{\mathrm{T}}$.

For sampling time h and controller $u_h = u_h(x_k)$, the accurate discrete model is obtained as

$$x_{k+1} = F_h^e(x_k, u_k(x_k)),$$

whose Euler approximation model is

$$x_{k+1} = F_h^{\mathrm{eu}}(x_k, u_k(x_k)).$$

The characteristic model is often obtained based on the Euler approximation model with unmodelling error. In particular, one can rewrite the characteristic model in the following form,

$$x_{k+1} = F_h^{\mathrm{te}}(x_k, u_k(x_k)),$$

and then the above three models can be unified as

$$x_{k+1} = F_h^*(x_k, u_k(x_k)), \tag{2}$$

where $* \in \{e, \mathrm{eu}, \mathrm{te}\}$, and

$$F_h^e(x_k, u_k(x_k)) = F_h^{\mathrm{eu}}(x_k, u_k(x_k)) + R_h(x_k),$$
$$F_h^{\mathrm{eu}}(x_k, u_k(x_k)) = F_h^{\mathrm{te}}(x_k, u_k(x_k)) + \eta_h(k),$$

where $R_h(x_k)$ is the Euler discrete truncation error, and $\eta_h(k)$ is the modelling error.

Definition 1 ([26, 27])**.** The discrete system (2) is asymptotically stable on D in terms of h, denoted by (β, D), if there exist $h^* > 0$, an open region D and a function $\beta \in KL$ independent of h, that the solution of discrete system (2) $x[k, \xi] \in D$ for every $h \in (0, h^*)$ and $\forall x[0] = \xi \in D,\ k \in N^+$ satisfies

$$\|x[k]\| \leqslant \beta(\|\xi\|, kh).$$

Furthermore, the system is globally asymptotically stable if $D = \mathbb{R}^n$, exponentially stable, denoted by (e, D), if $\beta(\|\xi\|, kh) = B\|\xi\| \exp\{-\varrho kh\},\ B \geqslant 1,\ \varrho > 0$, and finally global exponential stable if $D = \mathbb{R}^n$.

Lemma 1 ([26, 27])**.** For given $h^* > 0$ and $h \in (0, h^*)$, system (2) is (β, D) if and only if there exists a Lyapunov function $V_h(k, \xi)$ which is continuous on ξ with $V_h(k, 0) = 0$ and

$$\alpha_1(\|x[k]\|) \leqslant V_h(k, x[k]) \leqslant \alpha_2(\|x[k]\|),$$

$$\triangle V_{k_h} = V_h(k+1, x[k+1]) - V_h(k, x[k]) \leqslant -h\alpha_3(\|x[k]\|),$$

where $\alpha_i,\ i = 1, 2, 3$ are K functions.

Lemma 2 ([26, 27])**.** For given $h^* > 0$ and $h \in (0, h^*)$, system (2) is (e, D) if and only if there exist a Lyapunov function $V_h(k, \xi)$ which is continuous on ξ with $V_h(k, 0) = 0$ and

$$c_1\|x[k]\|^2 \leqslant V_h(k, x[k]) \leqslant c_2\|x[k]\|^2,$$

$$\triangle V_{k_h} = V_h(k+1, x[k+1]) - V_h(k, x[k]) \leqslant -c_3 h\|x[k]\|^2,$$

where c_1, c_2, c_3 are positive constants.

3 Main results

3.1 The characteristic modeling

According to the characteristic model theory [12–15], the characteristic model of system (1) can be built that holds the following statement.

Theorem 1. For system (1) with Assumption 1 and sufficiently short sample time, if the system control objective is fixed point control or location tracking, then the characteristic model based on Euler approximate discretization method can be described as follows,

$$y(k+1)=\alpha_1(k)y(k)+\alpha_2(k)y(k-1)+\beta_0(k)u(k), \tag{3}$$

where

(1) the modelling error $\eta(k)$ between the characteristic model and the original system of Euler approximate discrete system is less than Mh^2.

(2) $\alpha_1(k),\alpha_2(k)$ and $\beta_0(k)$ are the function related to h, and they are slowly time-varying parameters for sufficiently small h.

(3) $\alpha_1,\alpha_2,\beta_0\in Ds$, where

$$Ds=\left\{(\alpha_1,\alpha_2,\beta_0)\middle| |\alpha_1(k)-2|\leqslant M_1h+Mh^2, |\alpha_2(k)+1|\leqslant M_1h+Mh^2, 0<|\beta_0(k)|\leqslant M_2M_3h^2\right\}.$$

Proof. For $\dot{x}_1=f(x_1,\boldsymbol{x}_2)$, we have

$$\ddot{x}_1=\frac{\partial f_1}{x_1}\dot{x}_1+\frac{\partial f_1}{\boldsymbol{x}_2}\dot{\boldsymbol{x}}_2=\frac{\partial f_1}{x_1}\dot{x}_1+w(x_1,\boldsymbol{x}_2)+\frac{\partial f_1}{\boldsymbol{x}_2}\boldsymbol{g}u.$$

Then

$$x_1(k+1)=2x_1(k)-x_1(k-1)+h\frac{\partial f_1}{x_1}(x_1(k)-x_1(k-1))+h^2\frac{\partial f_1}{\boldsymbol{x}_2}\boldsymbol{g}u+h^2w(k).$$

Denote N_{x_1} the upper bound of $|x_1|$ and

$$s_1=\frac{x_1(k)}{x_1(k)^2+x_1(k-1)^2+N_{x_1}},\quad s_2=\frac{x_1(k-1)}{x_1(k)^2+x_1(k-1)^2+N_{x_1}},\quad s_3=\frac{N_{x_1}}{x_1(k)^2+x_1(k-1)^2+N_{x_1}},$$

where $f_s=s_1x_1(k)+s_2x_1(k-1)+s_3=1$. Then

$$\begin{aligned}x_1(k+1)&=2x_1(k)-x_1(k-1)+h\frac{\partial f_1}{x_1}(x_1(k)-x_1(k-1))+h^2\frac{\partial f_1}{\boldsymbol{x}_2}\boldsymbol{g}u+h^2w(k)f_s\\&=\left(2+h\frac{\partial f_1}{x_1}(k)+h^2w(k)s_1\right)x_1(k)+\left(h^2w(k)s_2-1-h\frac{\partial f_1}{x_1}(k)\right)x_1(k-1)\\&\quad+h^2\frac{\partial f_1}{\boldsymbol{x}_2}\boldsymbol{g}u(k)+h^2w(k)s_3.\end{aligned}$$

Rewrite the above equation as follows,

$$x_1(k+1)=\alpha_1(k)x_1(k)+\alpha_2(k)x_1(k-1)+\beta_0(k)u(k)+\eta(k),$$

where

$$\alpha_1(k)=2+h\frac{\partial f_1}{x_1}(k)+h^2w(k)s_1,\ \ \alpha_2(k)=-1-h\frac{\partial f_1}{x_1}(k)+h^2w(k)s_2,\ \ \beta_0(k)=h^2\frac{\partial f_1}{\boldsymbol{x}_2}\boldsymbol{g},\ \ \eta(k)=h^2w(k)s_3.$$

(1) Since $s_3\leqslant 1$,

$$|\eta(k)|\leqslant Mh^2.$$

(2) $\alpha_1(k), \alpha_2(k), \beta_0(k)$ are the functions related to h according to above parameters in the expression, thus

$$\triangle\alpha_1 = |\alpha_1(k+j) - \alpha_1(k)| \leqslant 2M_1h + 2Mh^2, \quad \triangle\alpha_2 = |\alpha_2(k+j) - \alpha_2(k)| \leqslant 2M_1h + 2Mh^2,$$

$$\triangle\beta_0 = |\beta_0(k+j) - \beta_0(k)| \leqslant 2M_2M_3h^2, \quad \forall j \in N.$$

and therefore the characteristic parameters are slowly time-varying if h is considerably small, since M, M_1, M_2 and M_3 are bounded and $|s_i| \leqslant 1,\ i = 1, 2, 3$.

(3) Since $|s_i| < 1, i = 1, 2, 3$ and $\boldsymbol{g} \neq \mathbf{0}$, then Ds can be obtained as

$$Ds = \left\{(\alpha_1, \alpha_2, \beta_0)\Big||\alpha_1(k) - 2| \leqslant M_1h + Mh^2, |\alpha_2(k) + 1| \leqslant M_1h + Mh^2, 0 < |\beta_0(k)| \leqslant M_2M_3h^2\right\}.$$

3.2 Adaptive controller design

Theorem 1 shows that by selecting an appropriate sample time h, the error between the characteristic model and the Euler approximation discrete model of the original system could be made arbitrarily small. Thus the controller is first designed based on characteristic model without considering the error [15].

Rewriting the characteristic model (3) as the second order time-varying identification model

$$y(k+1) = \psi(k)^{\mathrm{T}}\theta(k), \tag{4}$$

where $\psi(k) = [y(k) \quad y(k-1) \quad u(k)]^{\mathrm{T}}$, $\theta(k) = [\alpha_1(k) \quad \alpha_2(k) \quad \beta_0(k)]^{\mathrm{T}}$. The parameter identification method is used as follows,

$$\hat{\theta}(k) = \Pi\left(\hat{\theta}(k-1) + \frac{a\psi(k-1)e(k)}{\lambda + \|\psi(k-1)\|^2}\right), \tag{5}$$

where $\hat{\theta}(k) = [\hat{\alpha}_1(k) \quad \hat{\alpha}_2(k) \quad \hat{\beta}_0(k)]^{\mathrm{T}}$, $e(k) = y(k+1) - \psi(k)^{\mathrm{T}}\hat{\theta}(k)$, Π is the orthogonal projection operator on the bounded closed convex set Ds. $a, \lambda_0 > 0$ are adjustable parameters of the identification and $a \in (0, 1)$.

Denote $\tilde{\theta}(k) = \theta(k) - \hat{\theta}(k)$, then $e(k) = \psi(k)^{\mathrm{T}}\tilde{\theta}(k)$. Since the characteristic parameter belongs to the closed set Ds, for $\forall\theta_1, \theta_2 \in Ds$

$$\|\theta_1 - \theta_2\| \leqslant k_\theta,$$

and clearly $k_\theta = O(h)$. When h is small enough, the characteristic parameters are slowly time-varying, and

$$\sum_{\tau=k_0}^{k-1} \|\theta(\tau+1) - \theta(\tau)\| \leqslant \upsilon_0 + \upsilon_1(k - k_0), \quad k \geqslant k_0 + 1, \tag{6}$$

where υ_0 and υ_1 are small positive constants.

Under the condition of bounded modelling error, it holds that [1]

$$\sum_{\tau=k_0+1}^{k} \|\hat{\theta}(\tau) - \hat{\theta}(\tau-1)\| \leqslant \sqrt{a}k_\theta(k-k_0)^{\frac{1}{2}} + \sqrt{5ak_\theta\upsilon_0}(k-k_0)^{\frac{1}{2}} + \sqrt{5ak_\theta\upsilon_1}(k-k_0), \quad k \geqslant k_0 + 1. \tag{7}$$

In view of the characteristic model (3), design the control law as

$$L(k, z^{-1})u(k) = P(k, z^{-1})(y(k) - y_m(k)), \tag{8}$$

where $L(k, z^{-1})$ and $P(k, z^{-1})$ satisfy Diophantine Equation:

$$L(k, z^{-1})\hat{A} + P(k, z^{-1})\hat{B} = \bar{A}(\hat{\theta}_k, z^{-1}), \tag{9}$$

where $\bar{A}(\hat{\theta}_k, z^{-1})$ is the desired pole, $y_m(k)$ is the tracking signal, $\hat{A}$ and $\hat{B}$ are the estimate result of characteristic parameters, $L(k, z^{-1})$ and $P(k, z^{-1})$ are no more than one order of z^{-1} polynomial. Since $\hat{A}$ and $\hat{B}$ are coprime, then the Diophantine Equation (9) is always solvable. Under the conditions in

Theorem 1, system (1) of characteristic parameters belongs to a bounded closed convex set Ds, that $\bar{A}(\hat{\theta}_k, z^1)$ can be placed as a stable polynomial with no more than third order. Let $g = \frac{\beta_{0max}}{\beta_{0min}}$ and $L(k, z^{-1}) = l_0$ where

$$l_0 = \begin{cases} 1, & g \leqslant 2, \\ \frac{g}{2}, & g > 2. \end{cases}$$

Also take $\bar{A}(\hat{\theta}_k, z^{-1})$ as

$$\bar{A} = l_0 \prod_{i=1}^{2} (1 - m_i z^{-1}),$$

where $0 \leqslant m_i < 1,\ i = 1, 2$ ensures that $\bar{A}$ is a stable polynomial.

3.3 Stability analysis

The following relationship holds for the characteristic model and the accurate discrete model

$$F_h^e(x_k, u_k(x_k)) = F_h^{\mathrm{te}}(x_k, u_k(x_k)) + m(k), \tag{10}$$

where $m(k) = \eta(k) + R_h(x_k)$, $\eta(k)$ is the modeling satisfying $\eta(k) \leqslant \epsilon$, ϵ being any given arbitrarily small real number, and $R_h(x_k)$ the Euler's local truncation error, satisfying the following equality based on compatibility principle [18]:

$$\|R_h(x_k)\| = \|F_h^e(x_k, u_k(x_k)) - F_h^{\mathrm{eu}}(x_k, u_k(x_k))\| \leqslant h\rho(h), \tag{11}$$

where $\rho \in KL$.

Theorem 2. There exist $h^*, \upsilon_1^* > 0$ and a suitable $\bar{A}(\hat{\theta}_k, z^{-1})$, such that for $\upsilon_1 \leqslant \upsilon_1^*$ and the sample time $h \in (0, h^*)$, the closed-loop system of the characteristic model with the adaptive controller (5) and (8) is locally asymptotically stable at the origin.

Proof. Consider the nominal system without $m(k)$ in (10). Substitute the controller to the system (3), then

$$y(k+1) = \left(\alpha_1(k) - \frac{\beta_0(k)(\hat{\alpha}_1(k) - m1 - m2)}{l_0\hat{\beta}_0(k)}\right) y(k) + \left(\alpha_2(k) - \frac{\beta_0(k)(\hat{\alpha}_2(k) + m1m2)}{l_0\hat{\beta}_0(k)}\right) y(k-1). \tag{12}$$

Let

$$W(k) = [y(k+1) \quad y(k)]^{\mathrm{T}}, \quad \bar{A}^*(k) = \begin{bmatrix} \alpha_1(k) - \frac{\beta_0(k)(\hat{\alpha}_1(k) - m1 - m2)}{l_0\hat{\beta}_0(k)} & 1 \\ \alpha_2(k) - \frac{\beta_0(k)(\hat{\alpha}_2(k) + m1m2)}{l_0\hat{\beta}_0(k)} & 0 \end{bmatrix}^{\mathrm{T}}.$$

The closed-loop system can be rewritten as

$$W(k+1) = \bar{A}^*(k) W(k), \tag{13}$$

where

$$|\lambda_j(\bar{A}^*(k))| = |\lambda_j(\bar{A}(k))| < 1, \quad \forall k \in Z^+,\ j = 1, 2.$$

Hence there exists $\sigma_c \in [0, 1)$ such that [1]

$$\max_j |\lambda_j(\bar{A}^*(k))| \leqslant \sigma_\upsilon,$$
$$\|\bar{A}^*(k)^i\| \leqslant c_\upsilon \sigma_\upsilon^i, \quad \forall k \in Z^+,\ i \in Z^+,\ j = 1, 2,$$

where $c_\upsilon > 1, \sigma_\upsilon = \frac{(\sigma_c + 1)}{2}$. Then

$$\begin{aligned} &\|\bar{A}^*(\tau) - \bar{A}^*(\tau - 1)\| \\ &\leqslant |\alpha_1(\tau) - \alpha_1(\tau - 1)| + |\alpha_2(\tau) - \alpha_2(\tau - 1)| + \left| \frac{\beta_0(\tau)(\hat{\alpha}_1(\tau) - m1 - m2)}{l_0\hat{\beta}_0(\tau)} - \right. \end{aligned}$$

$$\left.\frac{\beta_0(\tau-1)(\hat\alpha_1(\tau-1)-m1-m2)}{l_0\hat\beta_0(\tau-1)}\right| + \left|\frac{\beta_0(\tau)(\hat\alpha_2(\tau)-m1m2)}{l_0\hat\beta_0(\tau)} - \frac{\beta_0(\tau-1)(\hat\alpha_2(\tau-1)-m1m2)}{l_0\hat\beta_0(\tau-1)}\right|$$

$$\begin{aligned}
\leqslant & \left|\alpha_1(\tau)-\alpha_1(\tau-1)\right| + \left|\alpha_2(\tau)-\alpha_2(\tau-1)\right| + \left|\frac{\beta_0(\tau-1)\hat\beta_0(\tau)(\hat\alpha_1(\tau-1)-m1-m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)}\right. \\
& - \frac{\beta_0(\tau-1)\hat\beta_0(\tau)(\hat\alpha_1(\tau)-m1-m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)} + \frac{\beta_0(\tau-1)\hat\beta_0(\tau)(\hat\alpha_1(\tau)-m1-m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)} \\
& - \frac{\beta_0(\tau-1)\hat\beta_0(\tau-1)(\hat\alpha_1(\tau)-m1-m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)} + \frac{\beta_0(\tau-1)\hat\beta_0(\tau-1)(\hat\alpha_1(\tau)-m1-m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)} \\
& \left. - \frac{\beta_0(\tau)\hat\beta_0(\tau-1)(\hat\alpha_1(\tau)-m1-m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)}\right| + \left|\frac{\beta_0(\tau-1)\hat\beta_0(\tau)(\hat\alpha_2(\tau-1)-m1m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)}\right. \\
& - \frac{\beta_0(\tau-1)\hat\beta_0(\tau)(\hat\alpha_2(\tau)-m1m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)} + \frac{\beta_0(\tau-1)\hat\beta_0(\tau)(\hat\alpha_2(\tau)-m1m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)} \\
& - \frac{\beta_0(\tau-1)\hat\beta_0(\tau-1)(\hat\alpha_2(\tau)-m1m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)} + \frac{\beta_0(\tau-1)\hat\beta_0(\tau-1)(\hat\alpha_2(\tau)-m1m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)} \\
& \left. - \frac{\beta_0(\tau)\hat\beta_0(\tau-1)(\hat\alpha_2(\tau)-m1m2)}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)}\right| \\
\leqslant & \left|\alpha_1(\tau)-\alpha_1(\tau-1)\right| + \left|\alpha_2(\tau)-\alpha_2(\tau-1)\right| + \left|\frac{\beta_0(\tau-1)(\hat\alpha_1(\tau)-\hat\alpha_1(\tau-1))}{l_0\hat\beta_0(\tau-1)}\right| + \\
& \left|\frac{\beta_0(\tau-1)(\hat\alpha_1(\tau)-m1-m2)(\hat\beta_0(\tau)-\hat\beta_0(\tau-1))}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)}\right| + \left|\frac{(\hat\alpha_1(\tau)-m1-m2)(\beta_0(\tau)-\beta_0(\tau-1))}{l_0\hat\beta_0(\tau)}\right| \\
& + \left|\frac{\beta_0(\tau-1)(\hat\alpha_2(\tau)-\hat\alpha_2(\tau-1))}{l_0\hat\beta_0(\tau-1)}\right| + \left|\frac{\beta_0(\tau-1)(\hat\alpha_2(\tau)-m1m2)(\hat\beta_0(\tau)-\hat\beta_0(\tau-1))}{l_0\hat\beta_0(\tau)\hat\beta_0(\tau-1)}\right| \\
& + \left|\frac{(\hat\alpha_2(\tau)-m1m2)(\beta_0(\tau)-\beta_0(\tau-1))}{l_0\hat\beta_0(\tau)}\right|.
\end{aligned}$$

Since

$$(\alpha_1(k),\alpha_2(k),\beta_0(k)),(\hat\alpha_1(k),\hat\alpha_2(k),\hat\beta_0(k))\in Ds, \quad \frac{\beta_0(\tau-1)}{\hat\beta_0(\tau-1)}\leqslant 2l_0,$$

then

$$\begin{aligned}
\|\bar A^*(\tau)-\bar A^*(\tau-1)\| \leqslant\ & |\alpha_1(\tau)-\alpha_1(\tau-1)|+|\alpha_2(\tau)-\alpha_2(\tau-1)|+b_1|\hat\beta_0(\tau)-\hat\beta_0(\tau-1)| \\
& +b_2|\beta_0(\tau)-\beta_0(\tau-1)|+2|\hat\alpha_1(\tau)-\hat\alpha_1(\tau-1)|+2|\hat\alpha_2(\tau)-\hat\alpha_2(\tau-1)|,
\end{aligned}$$

where

$$b_1=\frac{2|\hat\alpha_1(\tau)-m1-m2|_{\max}+2|\hat\alpha_2(\tau)-m1m2|_{\max}}{\hat\beta_{0\,\min}},$$

$$b_2=\frac{|\hat\alpha_1(\tau)-m1-m2|_{\max}+|\hat\alpha_2(\tau)-m1m2|_{\max}}{l_0\hat\beta_{0\,\min}}.$$

b_1 and b_2 are seen to be bounded for $(\hat\alpha_1(k),\hat\alpha_2(k),\hat\beta_0(k))\in Ds$ and $0<m_i<1$ where $i=1,2$. Thus

$$\|\bar A^*(\tau)-\bar A^*(\tau-1)\|<\max\{b_1,2\}\left\|\hat\theta(\tau)-\hat\theta(\tau-1)\right\|+\max\{b_2,1\}\|\theta(\tau)-\theta(\tau-1)\|. \tag{14}$$

From (6), (7) and (14)

$$\begin{aligned}
\sum_{\tau=k_0+1}^{k}\left\|\bar A^*(\tau)-\bar A^*(\tau-1)\right\| <\ & \max\{b_1,2\}\left[\sqrt{a}k_\theta(k-k_0)^{\frac12}+\sqrt{5ak_\theta\upsilon_0}(k-k_0)^{\frac12}\right. \\
& \left.+\ \sqrt{5ak_\theta\upsilon_1}(k-k_0)\right]+\max\{b_2,1\}[\upsilon_0+\upsilon_1(k-k_0)].
\end{aligned} \tag{15}$$

Therefore, the closed-loop system (13) is locally asymptotically stable for a fixed $\mu_\upsilon \in (\sigma_\upsilon, 1)$, if there exist $\upsilon_1^* > 0$ and a positive integer N_1, such that for $\upsilon \leqslant \upsilon_1^*$ [1,28–30]

$$\max\{b_1, 2\}\sqrt{5k_\theta \upsilon_0} + \max\{b_2, 1\}\upsilon_1 < \left(\frac{\mu_\upsilon}{\sqrt[N_1]{c_\upsilon}} - \sigma_\upsilon\right) \Big/ N_1 c_\upsilon. \tag{16}$$

The sample time h and $\bar{A}(\hat{\theta}_k, z^{-1})$ determine the value of υ_1, b_1 and b_2. Hence an appropriate sample time and $\bar{A}(\hat{\theta}_k, z^{-1})$ can make the closed-loop system (13) to be locally asymptotically stable. This completes the proof.

Theorem 3. If the closed-loop of the characteristic model (12) is locally asymptotically stable at the origin and $c_3 - c_4 \times \gamma h^2 > 0$, where c_3, c_4 and γ are some positive constants, then the following statements hold,

(i) If $\eta(k) = 0$ or $\eta(k)$ is independent exponential convergence to 0, then the accurate discrete system is (e, D).

(ii) If $\eta(k)$ is not exponential convergence but $\lim_{k\to\infty} \eta(k) = 0$, then the accurate discrete system is (β, D_{r_0}).

(iii) If $\lim_{k\to\infty} \eta(k) \neq 0$, then there exist ϵ, when $\eta(k) < \epsilon$ the accurate discrete system is output bound stable.

Proof. For slow time-varying linear systems, asymptotic and exponential stabilities are equivalent. Thus from Theorem 2 it is known that controllers exist to make the characteristic model locally exponentially stable at the origin. Then a Lyapunov function $V_h(k, x(k))$ exists for the solution of characteristic model $x_1(k),\ x_2(k) \in D$, and

$$c_1\|x[k]\|^2 \leqslant V_h(k, x[k]) \leqslant c_2\|x[k]\|^2, \quad \triangle V_{k_h} \leqslant -c_3\|x[k]\|^2,$$

$$|V(k, x_1) - V(k, x_2)| \leqslant c_4\|x_i\|(\|x_1 - x_2\|), \quad \forall i = 1, 2.$$

That is, for every $0 < h < \min(1, h^*)$, the characteristic model is (e, D) stable and the lipschitz constant of the Lyapunov function $L_v = 2c_4\chi$, where χ is the maximum norm for $x_1, x_2 \in D$. Take $V_h(k, x(k))$ as the Lyapunov function of the accurate discrete system with perturbation. Let $x[k; 0, \xi]$ be the accurate system solution at step time k with $x[0] = \xi$, and $y[k; 0, \xi]$ be the characteristic model system solution at step time k with $x[0] = \xi$. Since the solution of system is unique, then $x[k+1; 0, \xi] = x[k+1; k, x[k; 0, \xi]]$. Therefore

$$\begin{aligned}\triangle V_{k_h} &= V_h(k+1, x[k+1; 0, \xi]) - V_h(k, x[k; 0, \xi]) \\ &= V_h(k+1, x[k+1; k, x[k; 0, \xi]]) - V_h(k+1, y[k+1; k, x[k; 0, \xi]]) \\ &\quad + V_h(k+1, y[k+1; k, x[k; 0, \xi]]) - V_h(k, y[k; k, x[k; 0, \xi]]) \\ &\leqslant c_4\|x[k]\|\|m(k)\| - c_3\|x[k]\|^2.\end{aligned}$$

(i) The case of $\eta(k) = 0$. Since the characteristic model is (e, D), then the accurate discrete model can locally be exponentially stable by [15] and the Euler's local truncation error satisfies

$$\|R_h(x_k)\| \leqslant \gamma h^2\|x_k\|,$$

where γ is a positive constant. Then there exist a constant $h^* > 0$, for $h \in (0, h^*)$ it holds that $c_3 - c_4 \times \gamma h^2 > 0$, or

$$\triangle V_{k_h}(x[k; 0, \xi]]) \leqslant -(c_3 - c_4 \times \gamma h^2)\|x[k]\|^2 \leqslant -h(c_3 - c_4 \times \gamma h^2)\|x[k]\|^2.$$

Thus the accurate discrete model is (e, D) by Lemma 2. Similarly, when $\eta(k)$ is independent exponential convergence to 0, it can be shown that the accurate discrete system is (e, D) made stable by augmenting the state equation to be equivalent to $\eta(k) = 0$.

(ii) The case of $R_h(x_k) \leqslant h\rho(h)$, $\lim_{x_k\to 0} R_h(x_k) = 0$. There exists a sample time to make $m(k) = \eta(k) + R_h(x_k) \leqslant \sigma$, and

$$\begin{aligned}\triangle V_{k_h}(x[k;0,\xi]) &\leqslant c_4\|x[k]\|\|m(k)\| - c_3\|x[k]\|^2 \leqslant c_4\|x[k]\|\sigma - c_3\|x[k]\|^2\\ &\leqslant -(1-\theta)c_3\|x[k]\|^2 - \theta c_3\|x[k]\|^2 + c_4\|x[k]\|\sigma\\ &\leqslant -(1-\theta)c_3\|x[k]\|^2, \quad \forall\|x\| \geqslant \frac{\sigma c_4}{\theta c_3}.\end{aligned}$$

Hence a proper h and $D = \{x \in R^2 | \|x\| < r\}$ can be chosen such that $\sigma < \frac{c_3}{c_4}\sqrt{\frac{c_1}{c_2}}\theta r$. For $\forall\|x(0)\| < \sqrt{\frac{c_1}{c_2}}r$, there always exists a bound $k_1 \in N^+$, for $k \leqslant k_1$,

$$\|x(k)\| \leqslant C\|x(0)\|\exp\{-\vartheta kh\},$$

and for $k > k_1$,

$$\|x(k)\| \leqslant \frac{c_4}{c_3}\sqrt{\frac{c_2}{c_1}}\frac{\sigma}{\theta}.$$

Therefore, for $\forall\|x(0)\| < \sqrt{\frac{c_1}{c_2}}r$,

$$\|x(k)\| \leqslant C\|x(0)\|\exp\{-\vartheta kh\} + \frac{c_4}{c_3}\sqrt{\frac{c_2}{c_1}}\frac{\sigma}{\theta}, \tag{17}$$

where $\theta,\ \vartheta,\ C$ are positive constants and $\theta < 1$. Since $\lim_{k\to\infty}\eta(k) = 0$, then $\lim_{k\to\infty}\sigma(k) = 0$, which means that

$$\lim_{k\to\infty}\|x(k)\| = 0.$$

Therefore the accurate discrete model (β, D_{r_0}) is stable.

(iii) The case of $\lim_{k\to\infty}\eta(k) \neq 0$. The result readily follows from (17).

Remark 2. Even if the modeling error $\eta(k)$ is necessary to satisfy the condition of convergence to 0 in the construct characteristic model process, it is still guaranteed to be bounded. The bounded output of the original system can be achieved by choosing appropriate parameters in the adaptive control method based on the characteristic model.

Remark 3. In (9), if $L(k, z^{-1}) = l_0$ and

$$\bar{A}(\hat{\theta}_k, z^{-1}) = l_0(1 - mq_1(k)z^{-1})(1 - mq_2(k)z^{-1}),$$

where $q_i(k),\ i = 1, 2$ are the roots of $\hat{A}$ and $0 < m < 1$ is adjustable parameter such that $\bar{A}(\hat{\theta}_k, z^{-1})$ becomes a stable polynomial, then the stability analysis result of Theorems 2 and 3 still hold, and the golden-section adaptive control law [1] becomes the special case by taking $m = 0.382$.

Remark 4. Since the sample time h determines the characteristic model parameters change rate υ_1 and the value of total modeling error $m(k) = \eta(k) + R_h(x_k)$, and $\bar{A}(\hat{\theta}_k, z^{-1})$ determines the value of b_1 and b_2, it is thus important to choose an appropriate sample time and $\bar{A}(\hat{\theta}_k, z^{-1})$ in our method to guarantee the stability of the closed-loop system. However, determining h or even its upper bound of modelling, is difficult in the adaptive control design based on the characteristic model [1–3,15], due to the lack of the analytically representable function as well as the hybrid system formalism. In the simulations that follow we use the numerical trial-and-error method to choose the sample time after we fix the desired pole of $\bar{A}(\hat{\theta}_k, z^{-1})$. A systematic way of doing this is one of our future objectives.

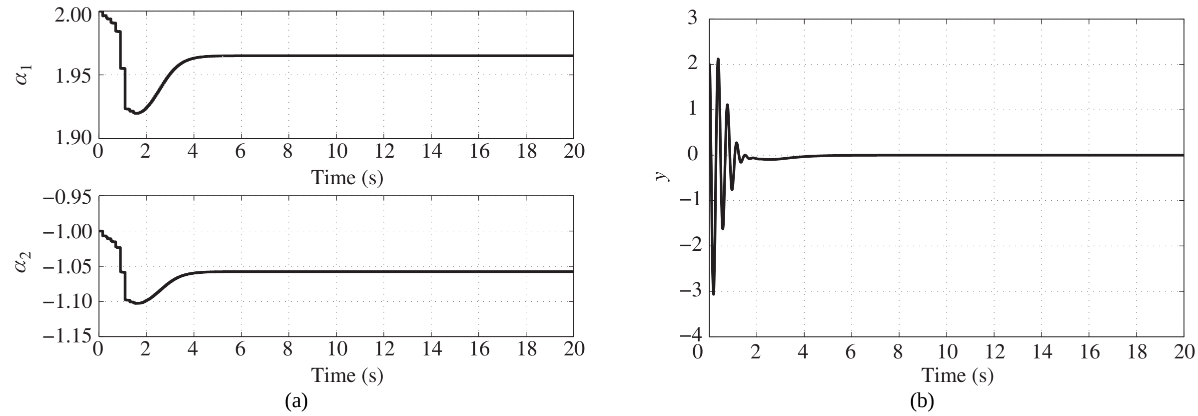

Figure 1 $d(t) = 0$ for the characteristic parameters estimate result (a) and the system output (b).

4 Simulation

Example 1. Consider the following linear system

$$\begin{cases} \dot{\xi} = c_1\xi + c_2x_1, \\ \dot{x}_1 = x_2, \\ \dot{x}_2 = a_1x_1 + a_2x_2 + a_3\xi + d(t) + bu, \\ y = x_1, \end{cases}$$

where $c_1 = -5,\ c_2 = 10,\ a_1 = 3,\ a_2 = 7,\ a_3 = 20,\ b = 8$ and $d(t) = 0,\ 10\sin(1/(t+1))$ and $10\sin(t+1)$ corresponding to three kinds of modelling errors.

Let $\boldsymbol{x}_2 = [\xi; x_2]$, the rewriting the system in the form of (1), we get

$$f_1(x_1, \boldsymbol{x}_2) = x_2, \quad \boldsymbol{g} = [0, b]^{\mathrm{T}}, \quad \boldsymbol{f}_2(x_1, \boldsymbol{x}_2) = \begin{bmatrix} c_1\xi + c_2x_1 \\ a_1x_1 + a_2x_2 + a_3\xi \end{bmatrix}.$$

Then $M_1 = 0,\ M_2 = 1$ and $M_3 = b$. Take $h = 0.05$ and the relevant state closed ball $X = \{\|x\| < 4\}$. Let $M = 130$ and set the initial $x_1 = 2,\ x_2 = 2$ and $\xi = 2$. Hence

$$Ds = \left\{(\alpha_1, \alpha_2, \beta_0)\middle|1.6725 \leqslant \alpha_1 \leqslant 2.3275, \quad -1.3275 \leqslant \alpha_2 \leqslant -0.6725, \quad \beta_0 = 0.02\ \right\}.$$

Let $\bar{A}(\hat{\theta}_k, z^{-1}) = (1 - \frac{1}{2}z^{-1})^2$, then

$$u(k) = -\frac{\hat{\alpha}_1 - 1}{\hat{\beta}_0}y(k) - \frac{\hat{\alpha}_2 + 0.25}{\hat{\beta}_0}y(k-1).$$

(1) $d(t) = 0$. From the simulation results in Figure 1, it is seen that the estimated parameter converges to a fixed number and the continuous system output asymptotically converges to 0.

(2) $d(t) = 10\sin(1/(t+1))$, $\eta(k)$ is not exponentially convergent but satisfies $\lim_{k\to\infty}\eta(k) = 0$. Comparing the simulation results in Figures 2 and 1, we find that the rate of system output convergence is slower, and characteristic parameters converge to a different fixed number. It means that the characteristic parameter is not unique in the characteristic modeling process [15].

(3) $d(t) = 10\sin((t+1))$, $\eta(k) \neq 0$ but is bounded. From the simulation results in Figure 3, it is seen that the system output is bounded stable and the characteristic parameters have more time-varying information when compared with above cases.

In order to show the effects of the configuration pole stability margin on system stability, we simulate $d(t) = 0$ in the example with the same parameters but different $\bar{A}(\hat{\theta}_k, z^{-1})$ as follows

(1) case 1: $\bar{A}(\hat{\theta}_k, z^{-1}) = (1 - \frac{1}{2}z^{-1})^2$.

(2) case 2: $\bar{A}(\hat{\theta}_k, z^{-1}) = (1 - \frac{1}{3}z^{-1})^2$.

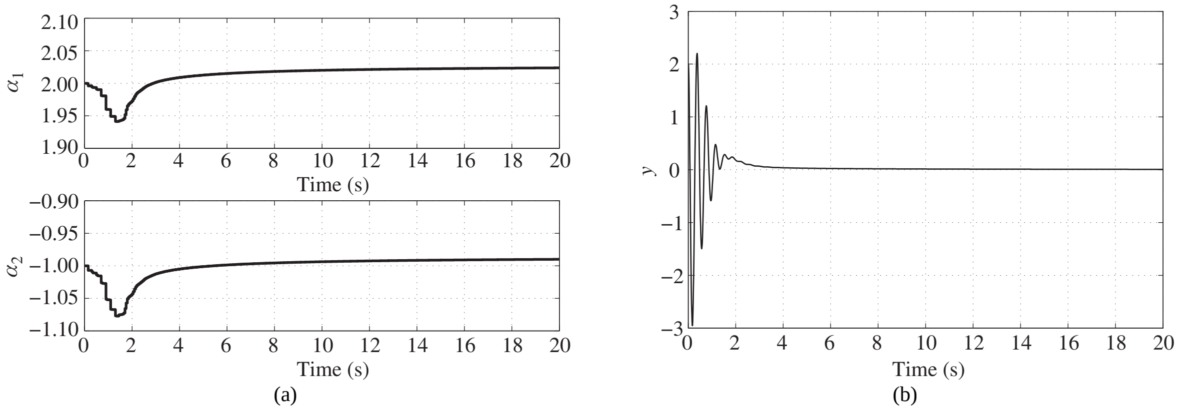

Figure 2 $d(t) = 10\sin(1/(t+1))$ for the characteristic parameters estimate result (a) and the system output (b).

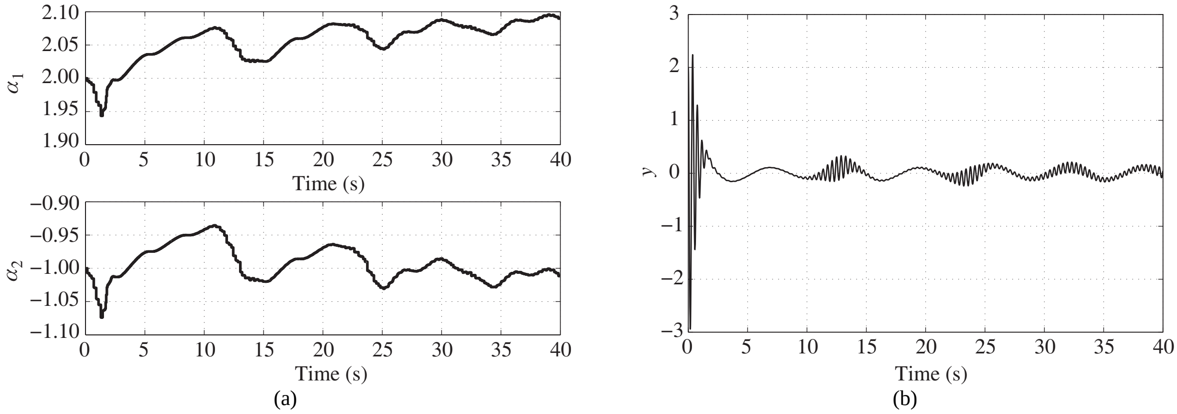

Figure 3 $d(t) = 10\sin(t+1)$ for the characteristic parameters estimate result (a) and the system output (b).

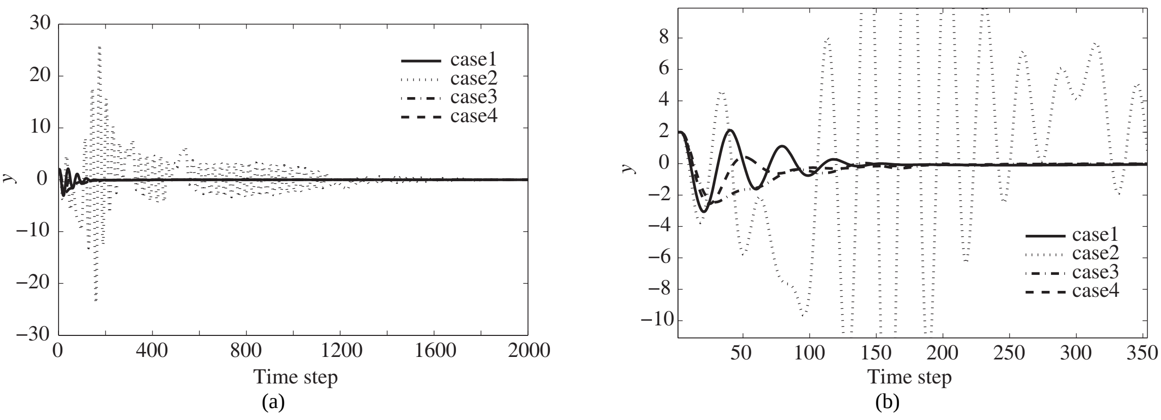

Figure 4 The system output of four different $\bar{A}(\hat{\theta}_k, z^{-1})$ (a) and its enlarged view in [0–350] (b).

(3) case 3: $\bar{A}(\hat{\theta}_k, z^{-1}) = (1 - \frac{2}{3}z^{-1})^2$.

(4) case 4: the golden-section adaptive control law.

The simulation results are shown in Figure 4. All four system outputs converge to 0, but the second case of stability margin leads to larger shock in the system dynamic process, which probably causes the related state to escape from a given state of compact set X, which is unfavourable in practice. The stability margin of the least may lead to the system instability. The golden-section adaptive control law is the best, where the system output dynamic process has the least volatility and the best convergence.

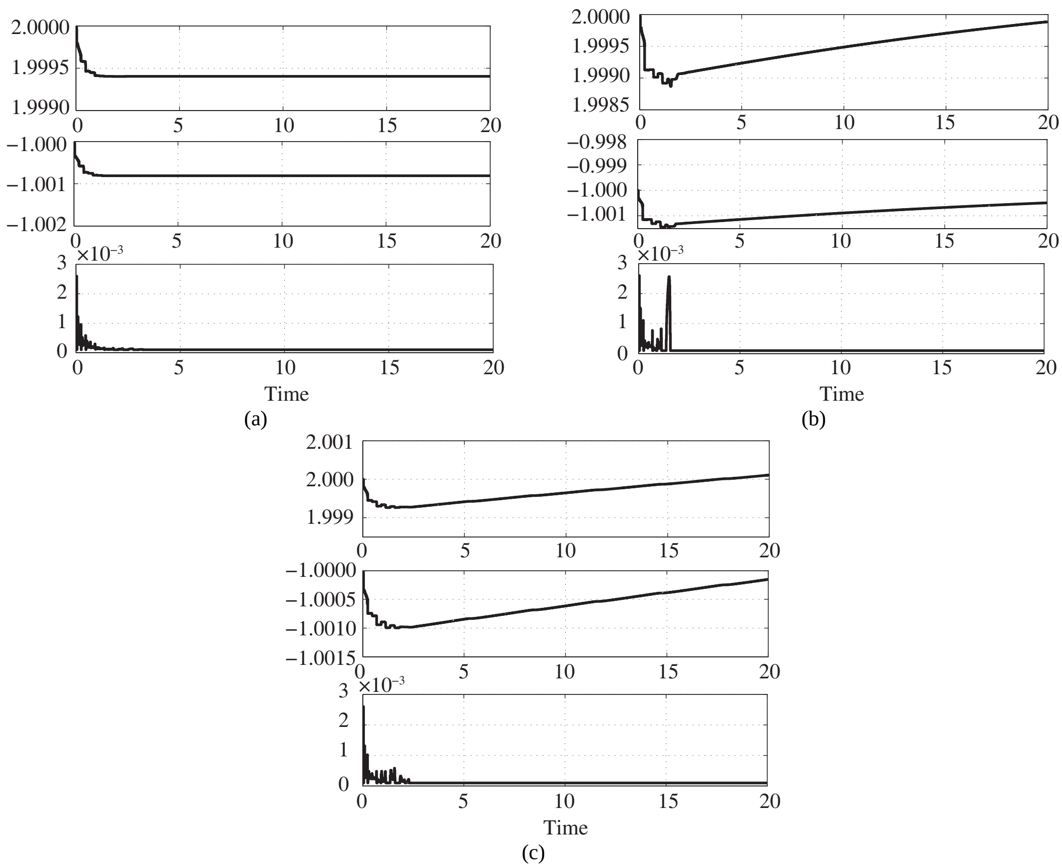

Figure 5 The parameters estimate for $d(t) = 0$ (a), $10\sin(1/(t+1))$ (b) and $10\sin(t+1)$ (c).

Example 2. Consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = 5x_1 + (1 + 4\cos x_1)x_4, \\ \dot{x}_2 = -x_2 + \frac{4x_4x_3^2 \sin x_1}{1+x_4^2}, \\ \dot{x}_3 = x_4 + x_5, \\ \dot{x}_4 = x_3 - 2x_2 - x_2^2\cos(x_1 + x_4) + x_5 + d(t) + (1 + x_2^2)u, \\ \dot{x}_5 = -x_5 + x_3 + x_2, \\ y = x_3, \end{cases}$$

where $d(t) = 0, 10\sin(1/(t+1))$ and $10\sin(t+1)$ correspond to the three kinds of modelling errors.

The system output is related to x_i, $i = 2,3,4,5$, then take $X = \{|x_i| \leqslant 5,\ i = 2,3,4,5\}$, the initial $x = [2,2,2,0,2]^{\mathrm{T}}$ and $M = 500$. Rewriting the system in the form of (1) and $M_1 = 0$, $M_2 = 1$ and $M_3 = 1 + x_2^2 = 26$. Take the sample time $h = 0.01$ and

$$Ds = \big\{(\alpha_1, \alpha_2, \beta_0)\big| 1.95 \leqslant \alpha_1 \leqslant 2.05, \quad -1.05 \leqslant \alpha_2 \leqslant -0.95, \quad 0.0001 \leqslant \beta_0 \leqslant 0.0026 \big\}.$$

Hence $l_0 = 13$. With the desired pole $A = 13(1 - 0.5z^{-1})^2$, the control law is

$$u(k) = -\frac{\hat{\alpha}_1 - 1}{13\hat{\beta}_0}y(k) - \frac{\hat{\alpha}_2 + 0.25}{13\hat{\beta}_0}y(k-1). \tag{18}$$

Figure 5 shows the characteristic parameters estimate result in three kinds of $d(t)$ conditions from the Example 2, which have similar indications as Example 1.

Figure 6 is the simulation output corresponding to the three $d(t)$ conditions. When $d(t) = 0$, the system output converges to 0 at the fastest speed and the dynamic error is the smallest. When $d(t) = 10\sin(t+1)$, the system output error cannot converge to 0 but is bounded, and the error bound is 0.2.

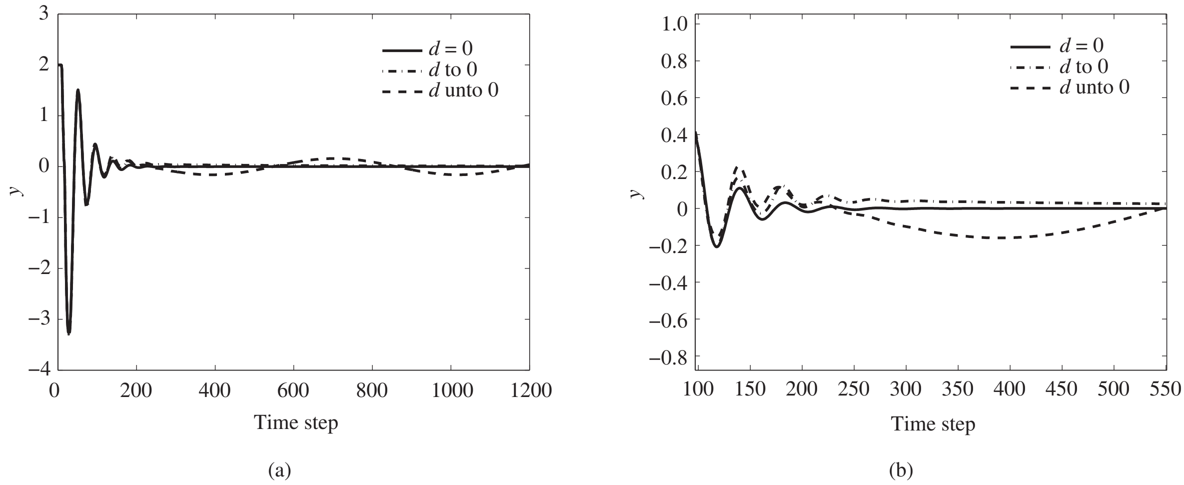

Figure 6 The system output (a) and its enlarge view in [100–550] (b).

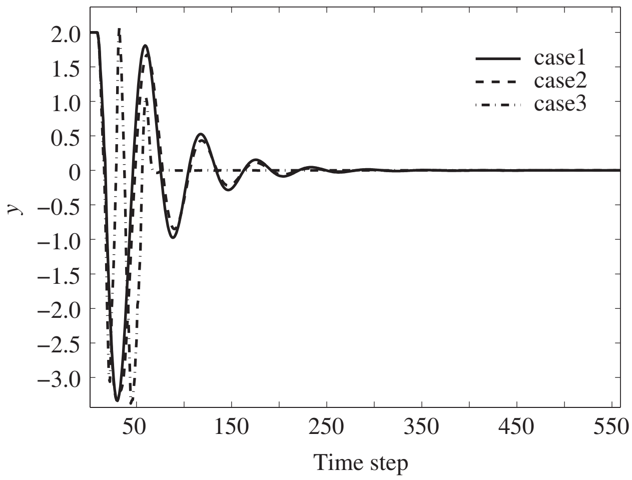

Figure 7 The system output for different cases of controller.

For verification of the expected pole for third order time invariant polynomial and golden-section adaptive controller, we simulated the $d(t) = 0$ case. The results are shown in Figure 7 where

(1) case 1: $\bar{A}(\hat{\theta}_k, z^{-1}) = 13(1 - \frac{1}{2}z^{-1})^3$.

(2) case 2: the golden-section adaptive control law.

(3) case 3: $\bar{A}(\hat{\theta}_k, z^{-1}) = 13(1 - \frac{1}{2}z^{-1})^2$.

The bound of characteristic modelling error is a condition that ensures the stability of the closed-loop system based on the characteristic model with the adaptive controller. The error of characteristic modelling depends on M and the size of the sampling time h. M depends on the given relevant state space. For a larger state set, the closed-loop system stability largely depends on the value of sample time h. We take Example 2 and $d(t) = 0$ for illustration. Here we set the initial $x = [15, 15, 15, 0, 15]^{\mathrm{T}}$ while keeping the other parameters constant. The simulation result in Figure 8(a) shows that the controller fails to stabilize the closed-loop system. Then we decrease the sample time by letting $h = 0.005$, the simulation result in Figure 8(b) shows that the output is convergent to 0.

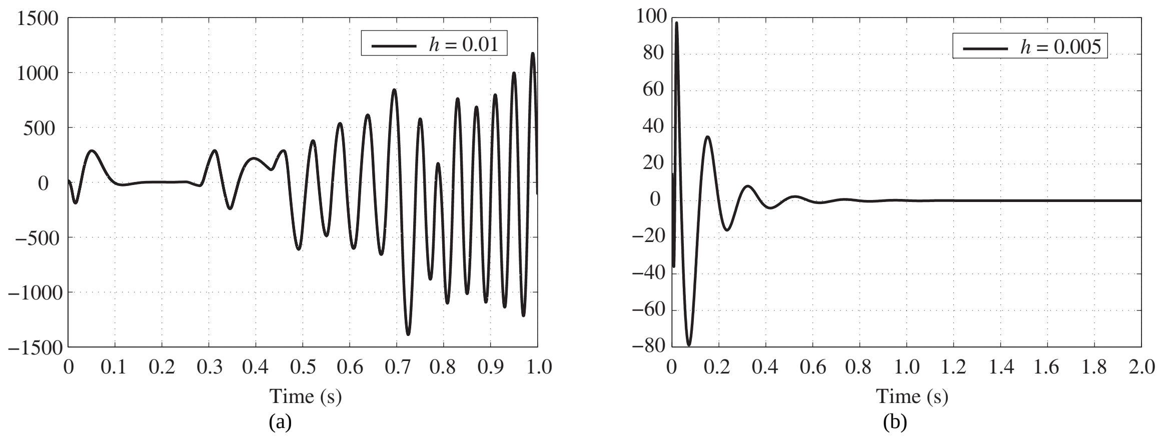

Figure 8 The system output with sample time $h = 0.01$ (a) and $h = 0.005$ (b).

5 Conclusion

The adaptive controller design and stability analysis are investigated for a class of SISO system based on the characteristic model method. The characteristic model is built, the adaptive controller is then designed along with stability conditions. The obtained results are verified by numerical examples.

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Conflict of interest The authors declare that they have no conflict of interest.

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