

Performance analysis for uplink massive MIMO systems with a large and random number of UEs

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Abstract In this paper, we analyze the ergodic achievable rate of an uplink massive multi-user multiple-input multiple-output (MIMO) system with a large and Poisson distributed number of users. In the considered scenario, multiple user equipments (UEs) transmit their information to a base station equipped with a very large number of antennas. New asymptotic expressions for the ergodic achievable rate for large and deterministic number of users are derived for both maximum ratio combining (MRC) detector and zero-forcing (ZF) detector, as well as the ergodic achievable rate for large and random number of users. Simulation results assess the accuracy of these analytical expressions. It is shown that compared with the MRC detector, ZF detector can achieve much higher spectrum efficiency. Also, the results provide a meaningful fact that with different settings the randomness of the number of users will result in different extents of impact on the performance of massive MIMO.

Keywords massive MIMO, large numbers of UEs, maximum ratio combining detector, zero-forcing detector, ergodic achievable rates

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1 Introduction

Massive multiple-input multiple-output (MIMO) is widely accepted as one of the key enabling and most representative technologies for 5G [1,2]. Succeeding the great success of MIMO technologies [3–12], exploiting large spatial degrees of freedom in 5G massive MIMO promises much higher spectrum efficiency than that in 4G [13–15]. In most of the existing work on performance analysis for massive MIMO systems, there are always some mild assumptions on the numbers of user equipments (UEs) and antennas at the base station (BS), for example, the number of users is much smaller than that of the BS antennas [16–26]. In these assumptions, the transmission channels for different users seem to be orthogonal to each other and the interferences from the other users in the cell decrease to zero. Via simple signal processing technology, very high spectrum efficiency can be achieved. Distributed massive MIMO has been investigated in [16], where multiple users communicate with one BS with several large-scale distributed antenna sets. The authors in [17] have derived some valuable expressions for the ergodic achievable rates of the uplink single-cell multi-user massive MIMO with simple linear detectors, while the authors in [22] focus on the multi-cell scenario.

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In some practical scenarios, for example, the concerts of pop stars and the soccer games, there are thousands of people demanding wireless communication services in small areas. Installing a temporal super BS is an economic and effective solution. In this case, the previous mild conditions cannot be always satisfied. Furthermore, each UE never always requires to communicate with BS, which indicates that the number of simultaneously communicating mobile terminals is in nature random and more specifically it follows the Poisson distribution [27]. Thus, considering the large and random number of mobile terminals is very meaningful for system-level performance measures [28,29].

In this paper, we concentrate our attention on the performance analysis for uplink massive MIMO with a large and random number of users, where one BS equipped with M antennas receives the information of \mathcal{K} single-antenna mobile users ($1 \ll \mathcal{K} \leq M$). Different from [17], we focus on the following two fundamental questions:

(1) What is the impact of the large and random number of users on the ergodic achievable rate of massive MIMO?

(2) Can we provide analytical expressions for the ergodic achievable rate of massive MIMO with large and random number of users?

In our work, new asymptotic expressions for the ergodic achievable rates for large and deterministic or large and random number of users are derived for both maximum ratio combining (MRC) detector and zero-forcing (ZF) detector. Our analytical expressions are also substantiated via Monte Carlo simulations. The results indicate that when the number of users is much smaller than the number of BS antennas, the randomness of the number of users can be ignored in massive MIMO. Also, it is shown that when the power or the number of BS antennas is high, the ergodic achievable rate for MRC detector is irrelevant with the transmit power, while that for ZF detector increases linearly with the base-2 logarithm of the power. We note that massive MIMO with ZF detector can achieve high spectrum efficiency compared with the MRC detector.

Notation: Boldface lowercase letters denote vectors, while boldface uppercase letters denote matrices. We use $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ to denote the transpose, conjugate, and conjugate transpose of a matrix or a vector, respectively. The symbol E denotes the statistical expectation operation. The symbol $\|\cdot\|_F$ denotes Frobenius norm of a matrix or a vector. The function $\log_2(\cdot)$ is the base-2 logarithm and $\ln(\cdot)$ is the natural logarithm.

2 System model

In this paper, we investigate an uplink massive MIMO system with one BS equipped with M antennas and multiple users, each equipped with single antenna. The number of users \mathcal{K} is assumed to be a random variable (RV) with a discrete non-negative integer distribution. Since the Poisson distribution is suitable for the practical network with large and random number of users [28,29], it is adopted here. Based on the definition of the Poisson distribution, the probability that \mathcal{K} equals to a deterministic number K is given as

$$\Pr(\mathcal{K} = K) = \frac{\lambda^K}{K!} e^{-\lambda}, \tag{1}$$

where λ is the expectation of \mathcal{K} .

In the considered communication systems, the coverage area of a cell is modeled as a disc and the BS is located in the center of the cell [30]. The mobile users are located uniformly in the cell with $R_0 < D_k < R$, considering the physical distance between the BS and the nearest user. The distribution of the users along the radius of the cell is expressed as [31]

$$f_d(x) = \frac{2}{R^2 - R_0^2} x, \tag{2}$$

where x is the distance between the considered user and the BS. The angle of the user's location (to the horizontal axis) is uniformly distributed on $[0, 2\pi)$.

Symbol $g_{m,k}$ denotes the channel coefficient between the m th antenna of the BS and the k th user. We consider both the small-scale fading and the large-scale fading as follows [13,17]:

$$g_{mk} = h_{mk}\sqrt{\beta_k}, \quad m = 1, 2, \dots, M, \quad k = 1, 2, \dots, K, \quad (3)$$

where h_{mk} is the small-scale fading coefficient, which is modeled as an RV with zero-mean and unit-variance. Without loss of generality, Rayleigh fading is adopted in the simulation, where h_{mk} follows circularly symmetric complex Gaussian (CSCG) distribution, that is, $h_{mk} \sim \mathcal{CN}(0, 1)$. h_{mk} 's are assumed to be mutually independent. β_k models the large-scale fading. We assume that

$$\beta_k = \mu_k/d_k^\nu, \quad (4)$$

where d_k is the distance between m th antenna of the BS and the k th user and ν is the path-loss exponent with typical values ranging from 2 to 6, that is, $2 \leq \nu \leq 6$. Symbol μ_k is modeled as independent and identically distributed (i.i.d.) RV with the known expectation $E[\mu_k]$. Matrix \mathbf{G} is the channel matrix between all K users and all M BS antennas, that is, $[\mathbf{G}]_{mk} = g_{mk}$. Perfect channel state information is assumed at the BS, in other words, the BS knows \mathbf{G} precisely.

The uplink communication is investigated, where the \mathcal{K} users transmit their data in the same time-frequency resource to the BS. Let \mathbf{x} be the $\mathcal{K} \times 1$ signal vector containing the user data, where its k th entry x_k is the information symbol of the k th user. \mathbf{x} is normalized as $E\{\|\mathbf{x}\|_F^2\} = 1$. Let P be the average transmit power of each user. This implies that in this work, we assume that all users have the same transmit power. However, the derived results can be directly extended to the non-equal power case. The $M \times 1$ vector of the received signals at the BS is

$$\mathbf{y} = \sqrt{P}\mathbf{G}\mathbf{x} + \mathbf{n}, \quad (5)$$

where \mathbf{n} is the noise vector, whose entries are assumed to be i.i.d. CSCG RVs with zero-mean and unit-variance, that is, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$.

Since MRC detector and ZF detector have low complexity and achieve good sum-rate performance to other more complicated designs in massive MIMO systems [17], they are used at the receiver, that is,

$$\mathbf{A} = \begin{cases} \mathbf{G}, & \text{for MRC,} \\ \mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}, & \text{for ZF.} \end{cases} \quad (6)$$

Using the linear detector, the received signal is separated into streams by multiplying it with \mathbf{A}^H as follows:

$$\mathbf{r} = \mathbf{A}^H\mathbf{y} = \sqrt{P}\mathbf{A}^H\mathbf{G}\mathbf{x} + \mathbf{A}^H\mathbf{n}. \quad (7)$$

Focusing on the k th element of \mathbf{r} in (7), we have

$$r_k = \sqrt{P}\mathbf{a}_k^H\mathbf{g}_k x_k + \sum_{i=1, i \neq k}^K \sqrt{P}\mathbf{a}_k^H\mathbf{g}_i x_i + \mathbf{a}_k^H\mathbf{n}, \quad (8)$$

where \mathbf{a}_k and \mathbf{g}_k are the k th columns of the matrices \mathbf{A} and \mathbf{G} , respectively.

3 Asymptotic achievable rate analysis

In this section, we derive the expressions of the ergodic achievable rates for deterministic and random number of users in the uplink multi-user MIMO system.

3.1 Deterministic number of users

We first focus on the deterministic number of users and give the following theorem.

Theorem 1. For the uplink massive MIMO system with $M > \mathcal{K} \gg 1$, the ergodic achievable rates of the k th user for $\mathcal{K} = K$ users with the MRC detector and ZF detector, denoted as $R_{k,\text{MRC}}(K)$ and $R_{k,\text{ZF}}(K)$, respectively, have the following asymptotic behavior:

$$R_{k,\text{MRC}}(K) \approx \log_2 \left(1 + \frac{(M-1)P\beta_k}{\frac{2(K-1)P\mathbb{E}[\mu_k]}{(v-2)(R^2-R_0^2)} \left(\frac{1}{R_0^{v-2}} - \frac{1}{R^{v-2}} \right) + 1} \right), \quad (9)$$

$$R_{k,\text{ZF}}(K) \approx \log_2 (1 + (M-K)P\beta_k). \quad (10)$$

Proof. We first focus on the MRC case and have

$$R_{k,\text{MRC}}(K) \approx \log_2 \left(1 + \frac{(M-1)P\beta_k}{P \sum_{i=1, i \neq k}^K \beta_i + 1} \right) = \log_2 \left(1 + \frac{\frac{1}{K-1} (M-1)P\beta_k}{P \left\{ \frac{1}{K-1} \sum_{i=1, i \neq k}^K \beta_i \right\} + \frac{1}{K-1}} \right), \quad (11)$$

where the first step is obtained by [17, Eq. (16)]. The term of $1/(K-1) \sum_{i=1, i \neq k}^K \beta_i$ in the above expression (11) is approximated as

$$\begin{aligned} \frac{1}{K-1} \sum_{i=1, i \neq k}^K \beta_i &\stackrel{\text{a.s.}}{\rightarrow} \mathbb{E}[\beta_i] = \mathbb{E}[\mu_k] \mathbb{E} \left[\frac{1}{d_k^v} \right] \\ &= \mathbb{E}[\mu_k] \times \int_{R_0}^R \frac{1}{x^{v-1}} \frac{2}{R^2 - R_0^2} dx \\ &= \mathbb{E}[\mu_k] \times \frac{2}{(v-2)(R^2 - R_0^2)} \left(\frac{1}{R_0^{v-2}} - \frac{1}{R^{v-2}} \right), \end{aligned} \quad (12)$$

where the first step is obtained by the law of large numbers [32], and the second step is achieved since μ_k and d_k are independent of each other. Substituting (12) into (11), the achievable rate of MRC detector is bounded.

The achievable rate of ZF detector that is bounded using [17, Eq. (20)] and (10) is achieved. Thus, the proof is ended.

We note that the expressions in [17] are claimed to be applied to the $M \gg K \gg 1$ case, that is, the number of the antennas is much larger than the number of users. Meanwhile, the $K \approx M$ case is not examined in [17]. Thus, we focus on a different problem from [17].

However, from (9) and (10), little insight can be obtained on the performance behavior of the uplink massive MIMO with respect to the power and the large number of users. To provide insights from (9) and (10), we provide the following proposition.

Proposition 1. For the uplink massive MIMO system with $M > \mathcal{K} \gg 1$ and $P \gg 1$, the ergodic achievable rates for $\mathcal{K} = K$ users, $R_{k,\text{MRC}}(K)$ and $R_{k,\text{ZF}}(K)$, have the following linear correlation:

$$\begin{cases} R_{k,\text{MRC}}(K) \sim \log_2 \left(\frac{M}{K} \right), \\ R_{k,\text{ZF}}(K) \sim \log_2 (P(M-K)), \end{cases} \quad (13)$$

where $A \sim B$ means A increases linearly with B .

Proof. We first investigate on the MRC case. From (9), we have

$$R_{k,\text{MRC}}(K) \approx \log_2 \left(\frac{\beta_k}{\frac{2\mathbb{E}[\mu_k]}{(v-2)(R^2-R_0^2)} \left(\frac{1}{R_0^{v-2}} - \frac{1}{R^{v-2}} \right)} \cdot \frac{(M-1)}{(K-1)} \right),$$

$$\approx \log_2 \left(\frac{\beta_k}{\frac{2E[\mu_k]}{(v-2)(R^2-R_0^2)} \left(\frac{1}{R_o^{v-2}} - \frac{1}{R^{v-2}} \right)} \right) + \log_2 \left(\frac{M}{K} \right). \quad (14)$$

Since the first item is irrelevant to P , M , and K , the MRC case is proved.

For the ZF case, from (10), we have

$$R_{k,ZF}(K) \approx \log_2(\beta_k) + \log_2(P(M-K)). \quad (15)$$

Then, the ZF case is proved and the proof is ended.

Proposition 1 shows that in uplink massive MIMO with a large number of users and high signal-to-noise ratio (SNR), the achievable rate for the MRC detector is irrelevant to the power P , which indicates that increasing power will not improve the spectrum efficiency for the system with the MRC detector. Based on this fact, Proposition 1 demonstrates that the ZF detector largely outperforms the MRC detector on the achievable rate of the uplink massive MIMO system with a large number of users and high SNR.

3.2 Random number of users

Now, we turn to derive the expressions of the ergodic capacity in uplink massive MIMO for random number of users, which follows the Poisson distribution. We first define the following function as

$$\Pr(\mathcal{K} < K) = \sum_{\mathcal{K}=0}^{K-1} \frac{\lambda^{\mathcal{K}}}{\mathcal{K}!} e^{-\lambda} = \frac{\Gamma(K, \lambda)}{\Gamma(K)}, \quad (16)$$

where the last step is obtained by [33, Eq. (8.352.2)].

Theorem 2. For the uplink massive MIMO system with $M > \lambda \gg 1$, the ergodic achievable rates of the k th user for Poisson distributed number of users with the MRC detector and the ZF detector, denoted as $\mathcal{R}_{k,MRC}(\lambda)$ and $\mathcal{R}_{k,ZF}(\lambda)$, respectively, can be bounded as

$$\mathcal{R}_{k,MRC}(\lambda) \gtrsim \log_2 \left(1 + \frac{(M-1)P\beta_k}{\frac{2(\lambda-1)PE[\mu_k]}{(v-2)(R^2-R_0^2)} \left(\frac{1}{R_o^{v-2}} - \frac{1}{R^{v-2}} \right) + 1} \right), \quad (17)$$

$$\mathcal{R}_{k,ZF}(\lambda) \lesssim \frac{\Gamma(M, \lambda)}{\Gamma(M)} \log_2(1 + (M-\lambda)P\beta_k), \quad (18)$$

where \lesssim represents approximately smaller and \gtrsim presents approximately larger.

Proof. For the MRC detector, from (9), by the convexity of $\log_2(1 + \frac{1}{x})$ and using Jensen's inequality, the following lower bound on the achievable rate is obtained as

$$\mathcal{R}_{k,MRC}(\lambda) \gtrsim \log_2 \left(1 + \frac{(M-1)P\beta_k}{\frac{2PE[\mu_k]}{(v-2)(R^2-R_0^2)} \left(\frac{1}{R_o^{v-2}} - \frac{1}{R^{v-2}} \right) E\{K-1\} + 1} \right). \quad (19)$$

Employing the characteristic of the Poisson distribution $E\{K\} = \lambda$, Eq. (17) is acquired and we gain the achievable rate for MRC.

Next, we turn to derive the achievable rate for the ZF detector. From (10), we have

$$\begin{aligned} \mathcal{R}_{k,ZF}(\lambda) &\approx E\{\log_2(1 + (M-\mathcal{K})P\beta_k) | \mathcal{K}\}, \\ &= E\{\log_2(1 + (M-\mathcal{K})P\beta_k) | \mathcal{K} < M\} + E\{0 | \mathcal{K} \geq M\}, \\ &\stackrel{(a)}{\leq} \{ \log_2(1 + E\{M-\mathcal{K}\}P\beta_k) | \mathcal{K} < M \}, \\ &= \log_2(1 + (M-\lambda)P\beta_k) \Pr(\mathcal{K} < M), \end{aligned} \quad (20)$$

where (a) is derived by the convexity of $\log_2(a - x)$, $a > x$ and using Jensen's inequality. Substituting (16) into (20), Eq. (18) is obtained and the derivation of achievable rate for ZF is finished. Thus, the proof is ended.

We note that (17) is similar to (9), which indicates that the Poisson distribution has little impact on the massive MIMO system for large numbers of users with MRC detector. Since the number of users is large, when K is not close to M , due to the law of large numbers, the fluctuation of the number of users can be ignored. When K is close to M , it is straightforward that the approximation $\frac{a}{b(K+\Delta K)+1} \approx \frac{a}{bK+1}$ still holds, which demonstrates that the influences of the Poisson distribution can be ignored in this scenario.

Comparing (18) with (10), it is shown that when K is close to M , the achievable rate will further decrease due to the randomness of the number of users. When K is close to M , the value of $M - K$ will be small and then even a small fluctuation of K will largely change the value of $M - K$. Thus, the influences of the Poisson distribution should be considered seriously for the ZF detector in this scenario.

Based on Theorem 2, we provide the following proposition to show more insight.

Proposition 2. For the uplink massive MIMO system with $M > \lambda \gg 1$ and $P \gg 1$, the ergodic achievable rates for Poisson distributed number of users, $\mathcal{R}_{k,\text{MRC}}(\lambda)$ and $\mathcal{R}_{k,\text{ZF}}(\lambda)$, have the following approximative linear correlation:

$$\begin{cases} \mathcal{R}_{k,\text{MRC}}(\lambda) \sim \log_2\left(\frac{M}{\lambda}\right), \\ \mathcal{R}_{k,\text{ZF}}(\lambda) \sim \frac{\Gamma(M, \lambda)}{\Gamma(M)} \log_2(P(M - \lambda)). \end{cases} \quad (21)$$

Proof. The proof is similar to the proof of Proposition 1.

It is worth noting that since Proposition 2 is based on Theorem 2, Eq. (21) just shows approximative linear correlations. Proposition 2 demonstrates that with large and Poisson distributed random number of users, the ZF detector largely outperforms the MRC detector on the achievable rate of the uplink massive MIMO system in the high SNR regime.

4 Numerical results

In this section, simulation results are presented to examine the impacts of network parameters on the ergodic achievable rate of the uplink MU-MIMO systems with large and random number of users. It is assumed that the users are located uniformly in a cell with $R = 1000$ and $R_0 = 100$. The transmitted signals suffer from the Rayleigh distributed fast fading, and the lognormal distributed large-scale fading with standard deviation $\sigma_{\text{shadow}} = 10$ dB. The users have the same transmit power, which is set to be $P \times r_{\text{mid}}^v$ where $r_{\text{mid}} = R/2 = 500$ m. So, if a user is located 500 m away from a BS antenna, the average received SNR of the antenna from the user is P . Furthermore, the path loss exponent is set as $v = 3.6$ [29].

Figures 1 and 2 depict the ergodic achievable rate of one user with different number of users K in the case of $M = 100$, and $P = 10$ dB for the case of deterministic number of users and that of random number of users, respectively. We can see from the figures that the numerical curves accurately predict the simulated ones. From Figures 1 and 2, it can be observed that increasing the number of users will largely decrease the achievable rate, since the inter-user interferences will increase with the number of users. For example, Increasing K from 20 to 96 brings about an achievable rate disadvantage of 34% at $M = 100$, $P = 10$ dB with deterministic number of users employing ZF detector. Also, it is evident that the ZF detector largely outperforms the MRC detector on the achievable rate. Furthermore, it is observed that for the MRC detector, the randomness of the number of users has little impact on the achievable rate. While for the ZF detector, when K, λ is close to M , the randomness of the number of users will result in the considerable reduction of the achievable rate.

Figures 3 and 4 plot the ergodic achievable rate of one user with different power P in the case of $M = 100$ and $K, \lambda = 50$ for the case of deterministic number of users and that of random number of

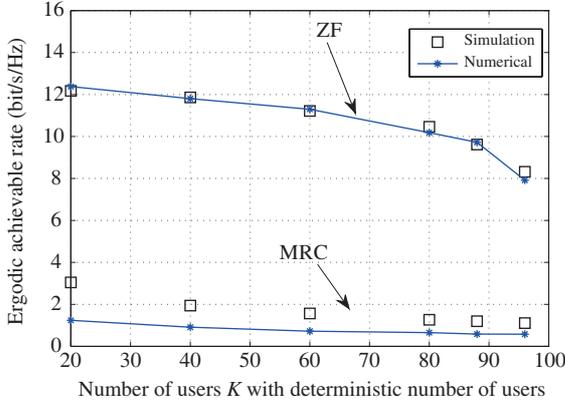


Figure 1 (Color online) The ergodic achievable rate of one user with different number of users K for deterministic number of users in the case of $M = 100$ and $P = 10$ dB.

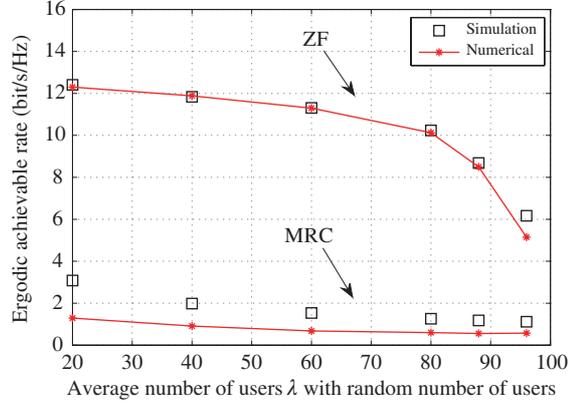


Figure 2 (Color online) The ergodic achievable rate of one user with different average number of users λ for random number of users in the case of $M = 100$ and $P = 10$ dB.

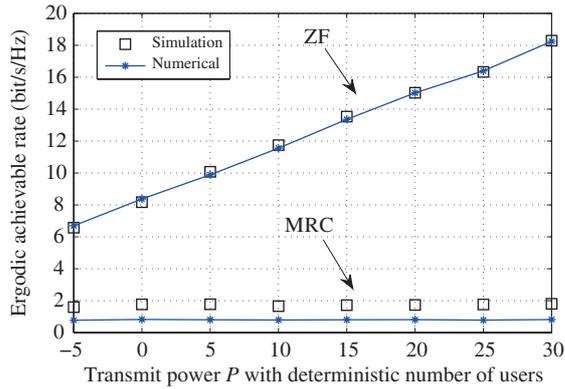


Figure 3 (Color online) The ergodic achievable rate of one user with different power P for deterministic number of users in the case of $M = 100$ and $K = 50$.

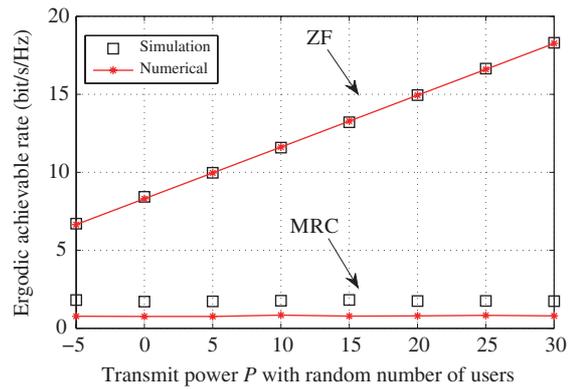


Figure 4 (Color online) The ergodic achievable rate of one user with different power P for random number of users in the case of $M = 100$ and $\lambda = 50$.

users, respectively. It can be concluded that when P is larger than 0 dB, the achievable rate is irrelevant with the transmit power for the MRC detector, while that increases linearly with the base-2 logarithm of the power for the ZF detector. These observations illustrate that the results of Propositions 1 and 2 are valuable even when the transmit power P is 0 dB. The ZF detector reduces the user-interference at the cost of increasing the noise, while the MRC detector will suffer from the user-interference. In traditional massive MIMO, since the number of users is much smaller than the number of BS antennas, the effect of user-interference can be largely reduced according to the law of large numbers and the two detectors have similar performance. However, for the massive MIMO systems with a large number of users, when employing the MRC detector, the effect of user-interference tremendously reduces the system capacity and the noise can be ignored. This indicates that increasing the power will not improve the capacity performance of the system with MRC detector in this situation. Thus, from this point of view, ZF detector will largely outperform the MRC detector on the achievable rate when the transmit power is not low. For example, the ZF detector scenario achieves about 600% higher average rate than the MRC detector at $M = 100$, $K = 50$, $P = 15$ dB with deterministic number of users.

5 Conclusion

In this paper, we have derived new asymptotic expressions for the ergodic achievable rate of a large uplink MIMO system with random and deterministic number of users. In the considered scenario, large and

Poisson distributed number of users communicate to a BS with a large number of antennas. Simulation results were provided to corroborate the accuracy of these analytical expressions. It has been proved that compared with the MRC detector, ZF detector can achieve much higher spectrum efficiency.

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Conflict of interest The authors declare that they have no conflict of interest.

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