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Energy efficient design for multiuser downlink energy and uplink information transfer in 5G

Chunguo LI[1]*, Yanshan LI[2], Kang SONG[1] & Luxi YANG[1]

[1]*Key Laboratory of Underwater Acoustic Signal Processing of Ministry of Education, Southeast University, Nanjing* 210096*, China;*
[2]*ATR National Key Laboratory of Defense Technology, Shenzhen University, Shenzhen* 518060*, China*

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Abstract Simultaneous wireless information and power transfer (SWIPT) is studied in this paper for the wireless powered downlink (DL) and multiuser information uplink (UL) systems. The objective is to maximize the energy efficiency defined as the ratio of the achieved throughput over the energy cost by optimizing the time allocation for the DL and multi-user UL traffics and its goal is to obtain the analytical expression to the optimal time allocation yet the resulting difficulty comes from the sum throughput of the multiuser in UL as well as the corresponding power consumption. To tackle this, the Jensen inequality is applied to approximating the exact expression of the sum throughput for the UL multi-users, leading to an upper-bound of the counterpart. The final closed form is exact in the single-user scenario yet approximate in the multi-user scenario. Numerical simulations verify the tightness of this approximation and the performances of the proposed analytical scheme.

Keywords energy harvest, time allocation, energy efficiency, uplink multiuser, throughput maximization, analytical expression

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1 Introduction

Wireless powered transmission or energy harvest becomes attractive in this literature such as [1–9]. A beamforming scheme is designed in [1] for the energy harvest in multi-input multi-output systems with multiple energy harvest terminal and one wireless information terminal. An algorithm is proposed in [2] for power splitting to serve the simultaneous transfer of both wireless information and energy in multi-input single-output systems. The authors in [3] derive an approximate capacity of the point-to-point energy-harvesting communications without wireless information transfer. The performance limit of simultaneous wireless information and energy transfer is investigated in [4] for multi-input multi-output (MIMO) broadcasting systems with one wireless information terminal and one energy harvesting terminal, where the MIMO advantages are exploited as illustrated in [10,11]. The tradeoff between energy efficiency and spectral efficiency is analyzed in [5] for the point-to-point wireless communications with the transmitter powered by the energy harvesting only. The energy harvest is also studied from the

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* Corresponding author (email: chunguoli@seu.edu.cn)

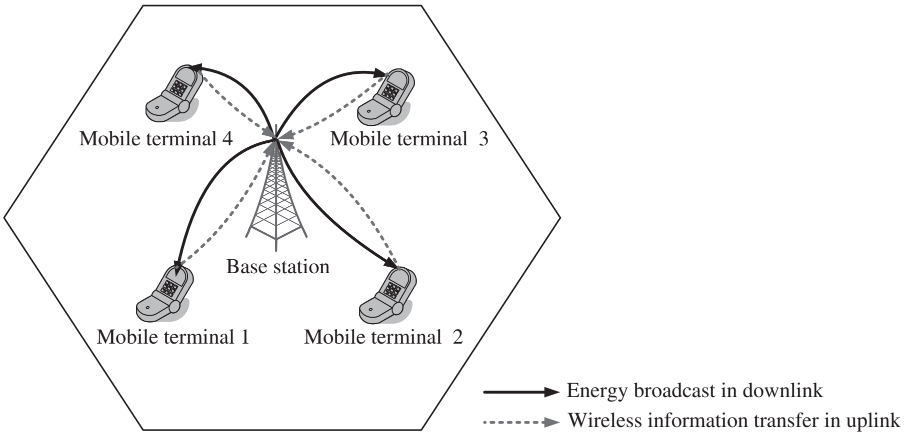

Figure 1 Simultaneous energy broadcast in downlink and wireless information transfer in uplink.

perspective of the outage probability in [6]. All the aforementioned results demonstrate that the wireless energy transfer technique provides a substantial performance gain as compared to the no power supply in wireless communications. It is interesting to note that a throughput maximization scheme is proposed in [7] where several terminals carry out energy harvesting via downlink wireless channel from the base station (BS) and then all the terminals transmit the wireless information to BS via the uplink wireless channel in a TDMA manner. However, the obtained result shown in (10) of [7] is a numerical form since this equation cannot be solved in an analytical expression. Motivated by this observation, we focus on deriving the analytical expression to the time allocation in the simultaneous wireless information and power transfer (SWIPT) system. Moreover, the objective function is fundamentally from the conventional goals such as throughput maximization as in [7], which is the energy efficiency maximization defined as the ratio of the achieved spectral efficiency over the corresponding power consumption that results in great difficulty for the transmission scheme design as addressed in energy efficient communications such as [12–14]. Under the new objective function, it is very difficult to derive the solution in a closed form, which will be tackled in this paper.

In this paper, our goal is to maximize the energy efficiency of both the wireless information and the energy transfer where multiple users collect the energy from BS via the wireless channel and then transmit the wireless information to BS in a time division multiple access (TDMA) manner. Without loss of generality, the time duration is equally allocated to every user for the wireless information transfer in the uplink channels. The solution is obtained in exactly analytical expression for the single-user scenario yet an approximately closed form for any number of users. The numerical simulations verify the negligible gap which results from the approximation in a multi-user scenario.

2 System model

As shown in Figure 1, for any given time duration T, the first part of this duration aT, $(0 < a < 1)$ is dedicated to the wireless energy transfer in downlink (DL) from the base station (BS) to all the users, and the other part of this duration $(1-a)T$ is entirely used for the wireless information transfer from each user in a TDMA manner to BS. As illustrated in Figure 2, with the same category of the quality-of-service, every user is allocated with the equal time duration as $\frac{(1-a)T}{K}$ in uplink (UL) traffic, where K is the number of the users of UL traffic in a TDMA manner. Without loss of generality, the time duration T is set to 1. a is the percentage of the time duration for the DL energy transfer in the total time duration of both DL and UL traffics.

For this setup, the total throughput of the whole system is given by [7]

$$R = \frac{1-a}{K}\sum_{k=1}^{K}\log_2\Big(1+\gamma_k\frac{Ka}{1-a}\Big), \tag{1}$$

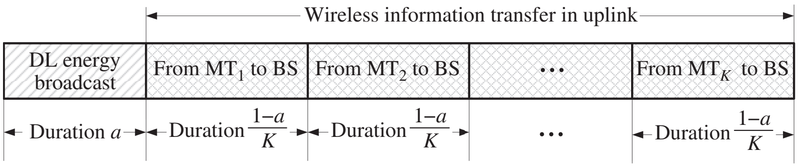

Figure 2 Transmission protocol for energy broadcast in downlink and wireless information access as the TDMA manner in uplink.

where γ_k $(k = 1, 2, \ldots, K)$ is defined as $\gamma_k \triangleq \frac{p_o\xi|h_k|^2|g_k|^2}{\Gamma\sigma_o^2}$, p_o is the transmit power in DL for energy transfer, $\xi \in (0, 1)$ is the energy percent of each user as the receiver in DL for energy harvest, h_k is the wireless channel coefficient from BS to the kth user in DL, g_k is the wireless channel from the kth user to BS in UL, $\Gamma > 1$ is the signal to noise ratio (SNR) gap for the practical modulation and σ_o^2 is the noise covariance for every additive Gaussian white noise (AWGN) channel. The energy received at kth user is given by $(p_o|h_k|^2)\frac{1-a}{K}$ and the total energy across all the users for the wireless information transfer can be calculated as $E_{\text{all}} = \sum_{k=1}^{K}(p_o|h_k|^2)\frac{1-a}{K}$. The energy efficiency as addressed in many systems such as [15,16] of the wireless system is defined as the achievable throughput per second in 1 Hz per Joule, namely the ratio of the throughput over the corresponding energy consumption as

$$\text{EE} \triangleq \frac{R}{E_{\text{all}}} = \frac{\sum_{k=1}^{K}\log_2\left(1 + \gamma_k\frac{Ka}{1-a}\right)}{p_o\sum_{k=1}^{K}|h_k|^2}. \tag{2}$$

We assume that the full channel state information (CSI) is available at the base station, which is required for the optimization of the time allocation over the downlink energy tranfer and the uplink wireless information transfer. It is our future work to study the scenario of non-ideal CSI as in [17,18].

3 Design of energy efficient time allocation for both wireless energy and information transfer

The objective is to maximize the energy efficiency (EE) and guarantee the required quality-of-service (QoS) as

$$\max_{a}\ \text{EE}, \text{ s.t. } R \geqslant r_o, \tag{3}$$

where $r_o > 0$ is the throughput threshold of all the users for satisfactory quality of service (QoS). The problem in (3) can be written equivalently as

$$\min_{a}\ \frac{p_o\sum_{k=1}^{K}|h_k|^2}{\sum_{k=1}^{K}\log_2(1 + \gamma_k\frac{Ka}{1-a})}, \text{ s.t. } \frac{1}{R} \leqslant \frac{1}{r_o}. \tag{4}$$

It is proven from Lemma 3.1 in [7] that the total throughput R is concave with respect to a. Thus, both the cost function and the constraint in (3) are convex with respect to the unknown a. The Lagrangian multiplier approach is applied to (3) to attain the solution. The Lagrangian function is

$$f = \frac{p_o\sum_{k=1}^{K}|h_k|^2}{\sum_{k=1}^{K}\log_2(1 + \gamma_k\frac{Ka}{1-a})} + \lambda\left(\frac{K}{(1-a)\sum_{k=1}^{K}\log_2(1 + \gamma_k\frac{Ka}{1-a})} - \frac{1}{r_o}\right), \tag{5}$$

where $\lambda > 0$ is the Lagrangian multiplier. For KKT conditions, the first-order derivative of f over a is given below,

$$\frac{\partial f}{\partial a} = -\frac{1}{R^2}\left\{R\frac{p_o}{K}\sum_{k=1}^{K}|h_k|^2 + \left[\lambda + (1-a)\frac{p_o}{K}\sum_{k=1}^{K}|h_k|^2\right]\frac{\partial R}{\partial a}\right\}, \tag{6}$$

where $\frac{\partial R}{\partial a}$ is expressed as

$$\frac{\partial R}{\partial a} = \sum_{k=1}^{K}\frac{\gamma_k}{1 - a + \gamma_k Ka} - \frac{1}{K}\sum_{k=1}^{K}\log_2\left(1 + \gamma_k\frac{Ka}{1-a}\right). \tag{7}$$

By substituting (6) and (7) into (5) and setting $\frac{\partial f}{\partial a}$ to zero, we have

$$\frac{1}{K}\sum_{k=1}^{K}\log_2\left(1+\gamma_k\frac{Ka}{1-a}\right)=\left(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+\frac{1}{1-a}\right)\sum_{k=1}^{K}\frac{\gamma_k}{1+\gamma_k\frac{Ka}{1-a}}. \tag{8}$$

The solution to problem (8) is difficult to get in the analytical form because of such a complicated expression. In what follows, we will solve this problem in two cases.

3.1 Number of users $K=1$

In this case, there is only one user for the wireless information transfer in UL traffic. With $K=1$, Eq. (8) is reduced to

$$\log_2\left(1+\gamma_1\frac{a}{1-a}\right)=1+\left(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1}\right)\frac{\gamma_1}{1+\gamma_1\frac{a}{1-a}}. \tag{9}$$

By calculating the exponent arithmetic on both sides of (10), we have

$$\begin{aligned}&\exp\left\{\Big(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1}\Big)\frac{\gamma_1}{1+\gamma_1\frac{a}{1-a}}\ln 2\right\}\cdot\left\{\Big(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1}\Big)\frac{\gamma_1}{1+\gamma_1\frac{a}{1-a}}\ln 2\right\}\\&=\frac{\ln 2}{2}\gamma_1\Big(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1}\Big),\end{aligned} \tag{10}$$

which is in the form of the Lambert function $xe^x=y$ with the solution $x=\mathbb{W}\{y\}$. Here, $\mathbb{W}\{\cdot\}$ is the well-known Lambert function [19,20]. Thus, we have the solution to (10) as

$$1+\gamma_1\frac{a}{1-a}=\frac{\gamma_1(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1})\ln 2}{\mathbb{W}\{\frac{\ln 2}{2}\gamma_1(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1})\}}, \tag{11}$$

which leads to the final solution to (8) as

$$a=\frac{\gamma_1(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1})\ln 2-\mathbb{W}\{\varphi\}}{\gamma_1(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1})\ln 2+(\gamma_1-1)\mathbb{W}\{\varphi\}}, \tag{12}$$

with

$$\varphi\triangleq\frac{\ln 2}{2}\gamma_1\Big(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1}\Big). \tag{13}$$

For the single-user case, the time dedicated to the energy transfer from BS to the user is a as in (12) and that for the wireless information transfer from this user to BS is $1-a$ as

$$1-a=\frac{\gamma_1\mathbb{W}\{\varphi\}}{\gamma_1(\frac{p_o}{\lambda}|h_1|^2+1-\frac{1}{\gamma_1})\ln 2+(\gamma_1-1)\mathbb{W}\{\varphi\}}. \tag{14}$$

With this manner of the time allocation, the energy efficiency is achieved to the optimum for the single-user scenario.

3.2 Number of users $K\geqslant 2$

In this case, there are multiple logrithmic terms in the summation in the left hand side (LHS) and multiple fractional terms in the right hand side (RHS) of (8). It is impossible to obtain the analytical solution to such a complicated problem.

It is observed from (8) that the LHS in (8) is actually the averaged spectral efficiency across all the uplink users for the wireless information transfer. This value is approximated by its upper bound via applying the Jensen inequality to the LHS in (8) as

$$\text{LHS}=\frac{1}{K}\sum_{k=1}^{K}\log_2\left(1+\gamma_k\frac{Ka}{1-a}\right)\leqslant\log_2\left\{\frac{1}{K}\sum_{k=1}^{K}\left(1+\gamma_k\frac{Ka}{1-a}\right)\right\}=\log_2\left(1+\frac{a}{1-a}\sum_{k=1}^{K}\gamma_k\right)\triangleq\tilde{R}. \tag{15}$$

The RHS in (8) involves the summation of multiple fractional terms, which can calculate the averaged value across all these terms as

$$\begin{aligned}\mathrm{RHS} &= \left(\frac{p_o}{\lambda}\sum_{k=1}^{K}|h_k|^2+\frac{K}{1-a}\right)\left(\frac{1}{K}\sum_{k=1}^{K}\frac{\gamma_k}{1+\gamma_k K\frac{a}{1-a}}\right)\\ &\approx \left(\frac{p_o}{\lambda}\sum_{k=1}^{K}|h_k|^2+\frac{K}{1-a}\right)\left\{\frac{\frac{1}{K}\sum_{k=1}^{K}\gamma_k}{\frac{1}{K}\sum_{k=1}^{K}\left(1+\gamma_k K\frac{a}{1-a}\right)}\right\}\\ &= \left(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+\frac{1}{1-a}\right)\left\{\frac{\sum_{k=1}^{K}\gamma_k}{1+\frac{a}{1-a}\sum_{k=1}^{K}\gamma_k}\right\}.\end{aligned} \tag{16}$$

This approximation method that takes the first term of the Taylor's series over the corresponding summation of the multiple fractional forms [21] is widely used in the wireless systems such as [22] for calculating the summation of many fractional terms. It is proven in [23] that this approximation gap approaches to zero in the high regime of the user number K. As verified in numerical simulations, the approximated result characterizes the exact counterpart with a negligible gap.

Substituting (15) and (16) into (8) yields

$$\log_2\left(1+\frac{a}{1-a}\sum_{k=1}^{K}\gamma_k\right)=\left(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+\frac{1}{1-a}\right)\left\{\frac{\sum_{k=1}^{K}\gamma_k}{1+\frac{a}{1-a}\sum_{k=1}^{K}\gamma_k}\right\}. \tag{17}$$

To get the solution to the unkown parameter a in (17), define

$$z \triangleq 1+\frac{a}{1-a}\sum_{k=1}^{K}\gamma_k. \tag{18}$$

By combining (17) and (18), we have

$$\log_2(z)=1+\frac{1}{z}\left\{\Big(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+1\Big)\sum_{k=1}^{K}\gamma_k-1\right\}, \tag{19}$$

where the parameter z is waiting to obtain in the analytical expression. By calculating the exponent computation on both sides of (19) and then multiplying the factor $\big\{\big(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+1\big)\sum_{k=1}^{K}\gamma_k-1\big\}\ln 2$, the desired expression is obtained as

$$\begin{aligned}&\exp\left\{\frac{1}{z}\left\{\left(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+1\right)\sum_{k=1}^{K}\gamma_k-1\right\}\ln 2\right\}\left(\frac{1}{z}\left\{\left(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+1\right)\sum_{k=1}^{K}\gamma_k-1\right\}\ln 2\right)\\ &=\frac{1}{2}\left\{\left(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+1\right)\sum_{k=1}^{K}\gamma_k-1\right\}\ln 2,\end{aligned} \tag{20}$$

leading to the solution as

$$z=\frac{\{(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+1)\sum_{k=1}^{K}\gamma_k-1\}\ln 2}{\mathbb{W}g\{\frac{1}{2}g\{(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+1)\sum_{k=1}^{K}\gamma_k-1g\}\ln 2g\}}. \tag{21}$$

Combining (18) and (21), the analytical expression to the target parameter a is given below

$$a=\frac{2\phi-\mathbb{W}\{\phi\}}{2\phi+\big(\sum_{k=1}^{K}\gamma_k-1\big)\mathbb{W}\{\phi\}}, \tag{22}$$

where

$$\phi \triangleq \frac{1}{2}\left\{\left(\frac{p_o}{K\lambda}\sum_{k=1}^{K}|h_k|^2+1\right)\sum_{k=1}^{K}\gamma_k-1\right\}\ln 2. \tag{23}$$

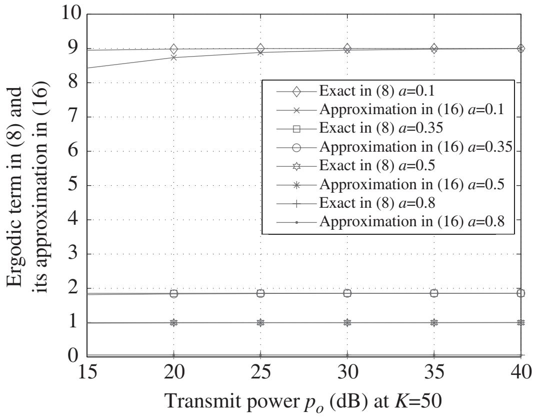

Figure 3 Approximation gap between the exact expression in (8) and its approximation in (16).

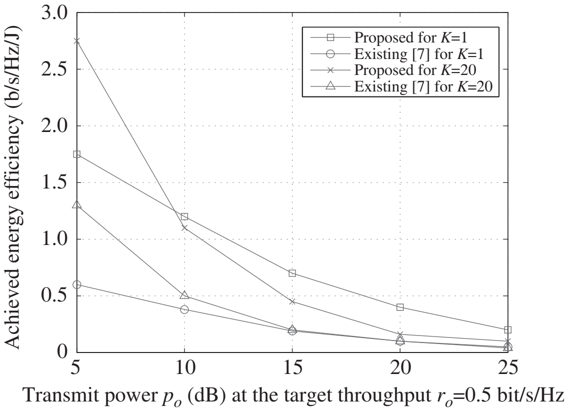

Figure 4 Energy energy comparison achieved by the proposed scheme and the existing throughput-max scheme proposed in [7] vs the increasing transmit power p_o (dB) at the target throughput $r_o = 0.5$ bit/s/Hz.

To this end, both the single-user and multi-user scenario are individually studied for deriving the analytical expression of the time allocation over the dowlink energy transfer and the uplink wireless information transfer. The obtained solution is accurate for the single-user case yet the result for the multi-user case is valid for the approximated problem from applying the Jensen inequality and the approximation of Tayloar series to the original problem. The major differences of our paper from the existing result in [7] lie in two aspects, namely, the new objective function of energy efficiency maximization that is more complicated than the throughput maximization and the final result is in the analytical expression instead of the numerical calculations. It is noted that the number of users served by the base station can be any number since all of them access the service in the manner of TDMA. Also note that the time duration for each uplink of the corresponding user can be different values, which constitutes a more general scenario from the one studied in this paper. It is interesting to research the case with different value settings of the time duration over all the uplinks in the future.

4 Numerical simulations

Numerical simulations are demonstrated to verify the approximation gap from the truncation over the Taylor series as in (16) and to show the effectiveness of the proposed energy-efficient time allocation for the joint wireless transfer of both downlink energy broadcast and uplink wireless multiuser information access. The channel realizations are randomly simulated in 10000 cycles for the Mont Carlo simulation method. The SNR gap for practical modulation is set to $\Gamma = 1$, the energy percentage used as transmit power from the received energy at every user is set to $\xi = 1$, and the noise covariance is set to $\sigma_o^2 = 1$. All the wireless channel coefficients are distributed with $\mathcal{CN}(0, 1)$.

It is seen from Figure 3 that the approximated expression in (16) almost overlaps with the exact expression in (8) for any given value of a and p_o with the maximum gap being 5% at $[p_o = 15, a = 0.1]$, which verifies the effectiveness of this approximation.

Figure 4 demonstrates the energy efficiency over the varying transmit power from BS. Both of the proposed scheme and the existing throughput maximization based scheme [7] are plotted from the perspective of the achieved energy efficiency. In the simulations setting, the required throughput is set to $r_o = 0.5$ bit/s/Hz. The number of user for the wireless information transfer in UL is set to $K = \{1, 20\}$. It is seen from Figure 4 that all the curves decreases with the increase of the transmit power p_o. This is because the increase of the achievable throughput in the numerator of (2) is in the $\log_2(\cdot)$ over p_o yet the growth of the power consumption in the denominator of (2) is in the linear form with respect to p_o. Thus, more power from the BS lower energy efficiency is produced. Independent from the number of users K for the wireless information transfer in UL, the proposed schemes always outperform the existing scheme in [7] from the perspective of the energy efficiency since the existing scheme is to maximize the throughput instead of the energy efficiency. Yet, with the increase of p_o, the gap between the two schemes becomes smaller, where the gain is very slight in the high power regime because more power is not helpful to the higher energy efficiency as long as it enables the required throughput be satisfactory. Thus, the power waste from the perspective of the energy efficiency becomes more serious especially in the high power regime, which makes the gain become more slight.

5 Conclusion

A closed-form solution is supplied in this paper to the time allocation of the multi-user systems with the downlink wireless energy transfer and uplink wireless information transfer. The result is exact in the single-user case yet approximate in the multi-user case which is validated by numerical simulations. The proposed approach achieves a remarkable gain of the energy efficiency as compared to the existing throughput maximization schemes.

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Conflict of interest The authors declare that they have no conflict of interest.

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