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Joint user grouping and resource allocation for uplink virtual MIMO systems

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Abstract MIMO has become a core technology of 5G network to largely improve system throughput. Due to the cost and size of the user equipment (UE), the application of MIMO uplink is limited by the difficulty in practical implementation at the user side. Virtual MIMO has been widely investigated to solve this problem for wireless uplink systems. However, virtual MIMO transmission leads to performance degradation due to the multiuser interference. To obtain good trade-off between the system throughput and transmission performance, we investigate joint user grouping and resource allocation under the consideration of system throughput and average mean squared error (MSE) performance in SC-FDMA uplink systems. Based on linear MIMO detection, we first develop MSE-oriented user grouping criteria for evaluation of transmission performance, then establish dynamic user grouping and optimal resource allocation problems for hard and elastic average MSE constraints. The proposed joint resource allocation algorithm is evaluated in SC-FDMA uplink scenarios and the results show that it achieves maximum system throughput with average MSE guaranteed for the hard MSE constraint algorithms and the alterable trade-off between system throughput and average MSE for the elastic MSE constraint algorithms.

Keywords virtual MIMO, user grouping, average MSE, system throughput, resource allocation

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1 Introduction

5G has become a hot research topic in the field of communication around the world. Compared with 4G, 5G further improves the performance of communication system in throughput, spectral efficiency, delay, connection density and energy consumption etc. In order to meet the requirements of 5G network, Multiple-input multiple-output (MIMO) techniques have been widely used to increase the system capacity and spectrum efficiency (SE) [1]. However, due to the cost and size of the user equipment (UE), the application of MIMO uplink is limited by the difficulty in practical implementation at the user side, especially in a small handset. To deal with this problem, virtual MIMO transmission [2–4] is proposed for the uplink by the usage of cooperative technology [5,6], which assigns two or more users, each deploying single transmit antenna, to the same frequency band and time slot. Compared with a regular MIMO

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system, virtual MIMO can obtain additional multiuser diversity gain by grouping users according to well-designed strategies. Then, how to select partners to form virtual MIMO is a critical issue that directly affects its performance.

A great amount of research has been performed on the criteria of user pairing/grouping for virtual MIMO systems. Most of these proposed criteria are derived from the channel capacity.

To measure the capacity, one approach is treating virtual MIMO as conventional MIMO. In [7], n-user virtual MIMO channel capacity is calculated as [8], and a suboptimal pairing algorithm which select pairing users one by one is proposed. Similarly, in [9], the decision metric which employs instantaneous receive SNR after ML detection is equivalent to MIMO channel capacity. Another approach is to analyze the post-processing SINR for each user after MIMO demultiplex in receiver. In [10], Fan et al. analyze the receive SINR after MMSE equalization and use Shannon capacity as user schedule criterion. Similar method is adopted for uplink virtual MIMO system with ZF/MMSE linear receiver in [11]. In [12], the receive SINR of each user is derived in the case of MMSE-SIC decoder and the sum capacity of paired users is calculated as pairing criterion. Liang et al. [13] present two practical algorithms for selecting a subset of channels in virtual MIMO system, where three criteria such as capacity, BER, and multiplexing gain are studied by converting MIMO channel to a series of independent parallel channels using SVD method.

In order to decrease the computing complexity of pairing algorithm, some research work has been done to simplify the pairing criterion. Based on the idea of reducing interference between two pairing users, Orthogonal Pairing Scheduling (OPS) [14] and Orthogonal Angle (OAPA) [15] are proposed. However, the SNR of paired users are not considered in the criterion which may cause capacity loss. As an improvement, Ref. [14] further presents Determinant Pairing Scheduling (DPS) scheme which is accurate in high SNR. These criteria could extend naturally to the case of more than two users, but it leads to some deviation.

BER or SER is another class of performance metric used for user grouping criteria to justify the virtual MIMO channel quality. Most of these research works are performed with linear MIMO detection such as ZF/MMSE or their expansion of SIC in virtual MIMO systems. In [16], Ruder et al. propose strategies using BER as a grouping optimization criterion, where the BER is evaluated after MMSE linear multiuser equalization. The BER criterion in [13] is presented when BPSK is used for modulation and maximal ratio combining is employed for diversity combination. Although BER or SER is an ideal performance metric for user grouping in data transmission at physical layer, its compute complexity is usually very high because receive signals must be processed after detection and demodulation.

In addition, many research works consider user fairness with grouping criteria. Most of them apply proportional fair idea to partner user scheduling process [9,12,17–19]. In [9], the schedule algorithm chooses first user based on proportional fair (PF) criterion and pairing user to maximize the system throughput. To achieving better fairness among the users, Chen et al. [12] propose double PF(D-PF) algorithm that uses the proportional fairness to decide the first user and choose the pairing user using modified proportional fairness criterion. In [19], Lightweight user grouping algorithm is proposed to solve the fairness problem toward a higher number of users in a virtual MIMO group. To exploit the multiplexing gain and multiuser diversity gain, Wang et al. [18] propose a fairness adjustable pairing criterion using proportional fairness scheduling.

A typical application of user grouping is to combine with frequency resource allocation in SC-FDMA uplink systems. SC-FDMA, also referred to as discrete Fourier transform (DFT) spread orthogonal frequency division multiple access (OFDMA), is currently adopted in the uplink of the 3GPP LTE-A system [20]. The main advantage of SC-FDMA compared to OFDMA significantly lower PAPR, which greatly benefits the mobile terminal in terms of transmit power efficiency.

Different from sub-channel allocation for OFDMA, users can only be assigned multiple sub-channels that are adjacent to each other in SC-FDMA [21,22]. Therefore, it is a very difficult combinatorial problem for subcarrier allocation in virtual MIMO system as the partner user selection should be performed simultaneously.

For the optimization of joint frequency allocation and pairing/grouping, a low complexity solution that

Table 1 Notations

E_s	Average transmit signal power	$\boldsymbol{U}$	Set of uplink users
δ^2	Noise power spectral density	$\boldsymbol{U}_{G_i}$	Set of users in Group G_i
K	Number of total uplink users	$\lvert\boldsymbol{U}_{G_i}\rvert$	Number of users in Group G_i
$N_{\rm RB}$	Number of RBs	$\boldsymbol{G}$	Set of user groups
$N_{\rm sc}^{\rm RB}$	Number of subcarriers in one RB	G_i	ith user group in set $\boldsymbol{G}$
N_r	Number of receive antennas at BS	$\lvert\boldsymbol{G}\rvert$	Number of user groups in $\boldsymbol{G}$
$(\cdot)^{\rm H}$	Hermitian transposition	$\boldsymbol{G}^m$	Set of groups with m users
$\bigotimes$	Kronecker product	$\boldsymbol{B}^m$	m-user grouping matrix
$\lVert\cdot\rVert_F$	Frobenius norm operation	$\boldsymbol{T}$	Resource pattern matrix
$\mathrm{Tr}(\cdot)$	Trace operation	$\boldsymbol{I}_m$	$m \times m$ identity matrix
$\det(\cdot)$	Determinant operation	$\boldsymbol{H}_{i,c}$	Virtual MIMO channel matrix of user group G_i at cth subcarrier
$E\left(\cdot\right)$	Expectation operation	$\boldsymbol{M}$	MSE metric matrix
$X_{i,c}$	Transmitting signal vector of user group G_i at cth subcarrier	$I_{i,j}$	Assignment index indicating ith user group occupying jth RB pattern
$\mathbf{1}_{m\times n}$	$m \times n$ all-one matrix	q_{ij}	Marginal rates of substitution of objective function and f_i and f_j

combined Hungarian algorithm [23,24] and binary switching algorithm is proposed in [16]. Since the same number of resources is allocated to each user pair, it is not optimum for system throughput.

Furthermore, the criteria mentioned above do not give the quantification evaluation of performance. So, the reliable transmission cannot be fully guaranteed when they are used in LTE uplink systems. In this paper, we adopt average MSE of received signals as the evaluation of the user grouping effect. It is shown that the average MSE is closely connected with performance of SER. So, we propose MSE-oriented user grouping criteria to address the quantification evaluation and compute complexity issues of user grouping, and then combine the user grouping criteria with resource allocation for LTE uplink virtual MIMO systems.

In comparison with the existing work, our main contributions are as follows.

- To propose user grouping criteria with quantification evaluation based on average MSE for ZF/MMSE detector. In order to reduce the computational complexity, two forms of suboptimal criteria are derived by the use of the minimum eigenvalue and its estimation.
- To formulate the joint user grouping and resource block (RB) allocation optimization problem based on user group pattern and uplink SC-FDMA resource block pattern. The objective functions of system throughput and average MSE are designed, and then form a multi-objective optimization problem.
- To propose solutions to the multi-objective optimization problem so that a proper trade-off between system throughput and average MSE can be obtained. One solution is for hard MSE constraint which uses constraint method, and another solution is for elastic MSE constraint which uses utility function method.

The rest of this paper is organized as follows. Section 2 gives a brief description of the uplink SC-FDMA multiuser-MIMO system model and presents the optimization object. In Section 3, we first discuss the user grouping problem and develop the MSE oriented criteria, and then propose a joint user grouping and subcarrier allocation algorithm for uplink system. Simulation results are presented in Section 4 and conclusions are drawn in Section 5.

A list of major notations used in this paper are summarized in Table 1.

2 Systenm model

Consider a virtual MIMO uplink system with users and one base station (BS) where the BS and users are equipped with receive antennas and one transmit antenna, respectively, as shown in Figure 1.

In each consecutive subcarrier chunk, the scheduler in BS chooses $|\boldsymbol{U}_{G_i}|$ users among a total of K users to form a virtual MIMO where $|\boldsymbol{U}_{G_i}| \leqslant N_r$. Assume gth user group is scheduled in M_i consecutive

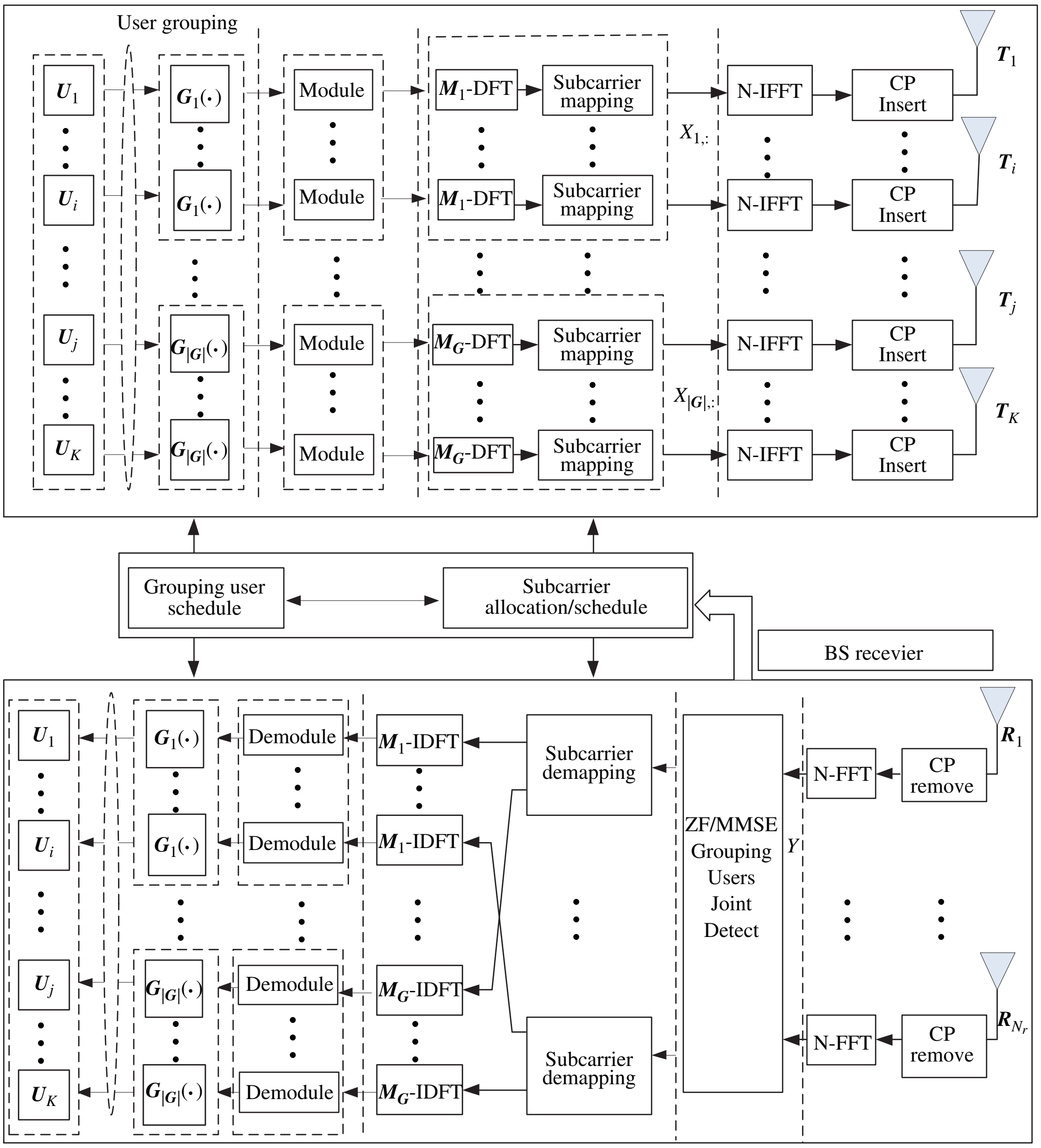

Figure 1 Block diagram of virtual MIMO for LTE Uplink System.

subcarriers with the first index c_i.

Then the received signal vector of user group i at cth subcarrier before MIMO detector can be written as

$$Y_{i,c} = \boldsymbol{H}_{i,c} X_{i,c} + n_{i,c}, \quad c = c_i, c_i+1, \ldots, c_i + M_i - 1, \ M_i = N_{\rm RB}^{i} N_{\rm sc}^{\rm RB}, \tag{1}$$

where $\boldsymbol{H}_{i,c}$ is the $N_r \times |\boldsymbol{U}_{G_i}|$ virtual MIMO channel matrix, $X_{i,c}$ is the $|\boldsymbol{U}_{G_i}| \times 1$ transmitting signal vector, $n_{i,c}$ is the $N_r \times 1$ zero-mean additive white Gaussian noise (AWGN) vector with covariance matrix $\mathrm{E}\{n_{i,c} {n_{i,c}}^{\rm H}\} = \sigma^2 \boldsymbol{I}_{N_r}$.

At the BS, linear detection is utilized. Then, the detection result can be given as

$$\widehat{X_{i,c}} = \boldsymbol{W}_{i,c} Y_{i,c}. \tag{2}$$

For ZF/MMSE detector,

$$\widehat{X_{i,c}}^{\rm ZF} = ({\boldsymbol{H}_{i,c}}^{\rm H} \boldsymbol{H}_{i,c})^{-1} {\boldsymbol{H}_{i,c}}^{\rm H} Y_{i,c}, \tag{3}$$

$$\widehat{X_{i,c}}^{\rm MMSE} = (\sigma^2 \boldsymbol{I}_{N_r} + {\boldsymbol{H}_{i,c}}^{\rm H} \boldsymbol{H}_{i,c})^{-1} {\boldsymbol{H}_{i,c}}^{\rm H} Y_{i,c}. \tag{4}$$

After the subcarrier de-mapping and user de-grouping, the receive data for different users is restored. As the results of ZF detector can be easily extended to MMSE detector, we only consider the ZF detector in this paper.

3 User grouping criteria and resource allocation

In this section, we derive effective user grouping criteria based on ZF/MMSE equalization. Afterwards, we propose the joint user grouping and RB allocation algorithm to optimize the system throughput and MSE performance.

3.1 User grouping criteria

3.1.1 *MSE-oriented user grouping criteria*

For an uplink SC-FDMA system with active users, we write the group set according to the number of users scheduled in one group, that is

$$\boldsymbol{G} = \left\{\boldsymbol{G}^1, \dots, \boldsymbol{G}^m, \dots, \boldsymbol{G}^{N_r}\right\}, \tag{5}$$

where $\boldsymbol{G}^m = \left\{{G^m}_i\right\}, 1 \leqslant m \leqslant N_r$ and ${G^m}_i = \left\{k_1, k_2, \dots, k_m\right\}, 1 \leqslant k_1 < \cdots < k_m \leqslant K$ is ith element in $\boldsymbol{G}^m$.

The index i of ${G^m}_i$, whose derivation is given in appendix, can be expressed as

$$i = \sum_{j=1}^{m-1} A_j + (k_m - k_{m-1}). \tag{6}$$

When grouping user number is two, Eq. (6) is the same as (6) in [10].

For user group $\boldsymbol{G}^m$, we can get $|\boldsymbol{G}^m| = C_K^m$, so the number of user groups is $|\boldsymbol{G}| = \sum_{m=1}^{N_r} |\boldsymbol{G}^m| = \sum_{m=1}^{N_r} \mathrm{C}_K^m$.

Then, with (5) and (6), we can write the index of user group in set $\boldsymbol{G}$ as

$$l = \begin{cases} \sum_{i=1}^{m-1} \mathrm{C}_K^i + \sum_{j=1}^{m-1} A_j + (k_m - k_{m-1}), & m > 1, \\ k_m, & m = 1, \end{cases} \tag{7}$$

i.e., $\boldsymbol{G} = \{G_1, \dots, G_l, \dots, G_{|\boldsymbol{G}|}\}$, and $\boldsymbol{U}_{G_l} = \{k_1, k_2, \dots, k_{|\boldsymbol{U}_{G_l}|}\}$, $1 \leqslant k_1 < \cdots < k_{|\boldsymbol{U}_{G_l}|} \leqslant K$.

For simplicity, we drop the subscript of subcarrier below. Let $\widehat{X_i}$ denote any estimate of X_i from user group G_i. The normalized error covariance, MSE, is defined by

$$\mathrm{MSE}_{\mathrm{G_i}} = \frac{1}{|\boldsymbol{U}_{G_i}|}\mathrm{E}\left\{\left\|\widehat{X_i} - X_i\right\|^2\right\} = \frac{1}{|\boldsymbol{U}_{G_i}|}\mathrm{tr}\left[\mathrm{E}\{(\widehat{X_i} - X_i)(\widehat{X_i} - X_i)^{\mathrm{H}}\}\right]. \tag{8}$$

After linear MIMO detector, Eq. (9) can be written as

$$\mathrm{MSE}_{G_i} = \frac{1}{|\boldsymbol{U}_{G_i}|}\mathrm{E}\left\{\|\boldsymbol{W}_i Y_i - X_i\|^2\right\} = \frac{1}{|\boldsymbol{U}_{G_i}|}\mathrm{tr}\left[\mathrm{E}\Big\{(\boldsymbol{W}_i Y_i - X_i)(\boldsymbol{W}_i Y_i - X_i)^{\mathrm{H}}\Big\}\right]. \tag{9}$$

With (3) and (4), we know that ${\boldsymbol{H}_i}^{\mathrm{H}}\boldsymbol{H}_i$ is an Hermite matrix, according to the property of Hermite matrix, we can get

$$\boldsymbol{P}^{\mathrm{H}}\left({\boldsymbol{H}_i}^{\mathrm{H}}\boldsymbol{H}_i\right)^{-1}\boldsymbol{P} = \Lambda^{-1} = \begin{pmatrix} 1/\lambda_1 & & & \\ & 1/\lambda_2 & & \\ & & \cdots & \\ & & & \lambda_{|U_{G_i}|} \end{pmatrix},$$

where $\boldsymbol{P}$ is unitary matrix, so the trace of $(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)^{-1}$ is $\mathrm{tr}((\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)^{-1}) = (\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \cdots + \frac{1}{\lambda_{|\boldsymbol{U}_{G_i}|}})$, where λ_k, $k = 1, 2, \ldots, |\boldsymbol{U}_{G_i}|$ is the eigenvalue of $\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i$. We obtain

$$\mathrm{MSE}_{G_i}^{\mathrm{ZF}} = \frac{1}{|\boldsymbol{U}_{G_i}|}\sigma^2 \mathrm{tr}\left[(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)^{-1}\right] = \frac{\sigma^2}{|\boldsymbol{U}_{G_i}|}\sum_{k=1}^{|\boldsymbol{U}_{G_i}|}\frac{1}{\lambda_k}. \tag{10}$$

To schedule users in one group as many as possible under guarantees of the MSE threshold, we write the MSE-oriented user grouping criterion as

$$\text{Groups-ZF}(\boldsymbol{U}_{G_i}) = \mathop{\arg\min}_{\boldsymbol{U}_{G_i}}\left\{\mathrm{MSE}_{G_i}^{\mathrm{ZF}}\right\} = \mathop{\arg\min}_{\boldsymbol{U}_{G_i}}\left\{\frac{\sigma^2}{|\boldsymbol{U}_{G_i}|}\sum_{k=1}^{|\boldsymbol{U}_{G_i}|}\frac{1}{\lambda_k}\right\}. \tag{11}$$

When the number of grouping users is two, Eq. (11) can be written as

$$\text{Groups-ZF}(\boldsymbol{U}_{G_i}) = \mathop{\arg\min}_{\boldsymbol{U}_{G_i}}\left\{\sum_{k=1}^{2}\frac{1}{\lambda_k}\right\} = \mathop{\arg\min}_{\boldsymbol{U}_{G_i}}\left(\frac{\mathrm{tr}(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H})}{\det(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H})}\right). \tag{12}$$

which is an inverse relationship to determinant criterion proposed in [14,17].

3.1.2 *Sub-optimal MSE-oriented criteria based minimum eigenvalue*

To reduce the computational complexity, we sort the eigenvalues as $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_{|U_{G_i}|} > 0$, then only consider the minimum eigenvalue $\lambda_{|\boldsymbol{U}_{G_i}|}$ which is the main impact part in grouping criterion(12). Thus, we obtain a sub-optimal grouping criterion as follows

$$\text{S1-Groups-ZF}(\boldsymbol{U}_{G_i}) = \mathop{\arg\min}_{\boldsymbol{U}_{G_i}}\left\{\frac{\sigma^2}{\lambda_{|\boldsymbol{U}_{G_i}|}}\right\}. \tag{13}$$

Eq. (13) is effective for ZF detector based on the assumption that the noise at BS is equal for group users.

3.1.3 *Sub-optimal MSE-oriented criterion based estimation of minimum eigenvalue*

However, to compute the minimum eigenvalue still have high complexity. So an estimation of minimum singular value using lower bound is utilized to overcome this problem.

Denote $\sigma_1 \geqslant \cdots \geqslant \sigma_k \geqslant \cdots \geqslant \sigma_{|\boldsymbol{U}_{G_i}|} > 0$ as singular value of $\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i$, we have [25]

$$\sigma_{|\boldsymbol{U}_{G_i}|}(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i) \geqslant \left(\frac{|\boldsymbol{U}_{G_i}| - 1}{\|(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)\|_F^2}\right)^{\frac{|\boldsymbol{U}_{G_i}|-1}{2}} |\det(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)|. \tag{14}$$

As $\lambda_{|\boldsymbol{U}_{G_i}|} > 0$, we have $\lambda_{|\boldsymbol{U}_{G_i}|} = \sigma_{|\boldsymbol{U}_{G_i}|}(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)$. Using lower bound to estimate the minimum eigenvalue, then

$$\widehat{\lambda}_{|\boldsymbol{U}_{G_i}|} \cong \left(\frac{|\boldsymbol{U}_{G_i}| - 1}{\|(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)\|_F^2}\right)^{\frac{|\boldsymbol{U}_{G_i}|-1}{2}} |\det(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)|. \tag{15}$$

With (13) and (15), we derive another sub-optimal grouping criterion as follows

$$\text{S2-MSE-Groups-ZF}(\boldsymbol{U}_{G_{\mathrm{i}}}) = \mathop{\arg\min}_{\boldsymbol{U}_{G_i}}\left\{\frac{\sigma^2\|(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)\|_F^{|\boldsymbol{U}_{G_i}|-1}}{|\det(\boldsymbol{H}_i{}^{\mathrm{H}}\boldsymbol{H}_i)|(|\boldsymbol{U}_{G_i}| - 1)^{\frac{|\boldsymbol{U}_{G_i}|-1}{2}}}\right\}. \tag{16}$$

3.1.4 *System throughput-oriented user grouping criterion*

We only consider the system throughput over virtual MIMO transmission based on ZF/MMSE equalization, then the system throughput depends on the number of users scheduled per frame. Therefore, the system throughput-oriented user grouping criterion for one RB can be written as

$$\text{TP-Groups}(\boldsymbol{U}_{G_i}) = \mathop{\arg\max}_{\boldsymbol{U}_{G_i}}\left\{|\boldsymbol{U}_{G_i}|\right\}. \tag{17}$$

3.2 Joint user grouping and resource allocation in SC-FDMA

For SC-FDMA in LTE uplink, the adjacent time-frequency RB should be assigned to one user. Assume that N consecutive RBs is available to allocate to users, we can get the allocation pattern number $J=\frac{N_{\mathrm{RB}}(N_{\mathrm{RB}}+1)}{2}+1$, Then the basic resource pattern matrix is expressed as [21]

$$\begin{array}{rcl} \text{pattern} & \begin{array}{cccc}1 & 2 & \cdots & J\end{array} & \\ T_{N_{\mathrm{RB}}\times J} = & \left[\begin{array}{cccc}0 & 1 & \cdots & 1\\ 0 & 0 & \cdots & 1\\ \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & \cdots & 1\end{array}\right] & \begin{array}{c}\mathrm{RB}_1\\ \mathrm{RB}_2\\ \cdots\\ \mathrm{RB}_{N_{rb}}\end{array} \end{array} . \tag{18}$$

Using MSE-oriented Grouping Criteria, we can obtain the MSE metric matrix for all RBs as follows

$$\begin{array}{rcl} \text{group index} & \begin{array}{cccc}1 & 2 & \cdots & |\boldsymbol{G}|\end{array} & \\ \boldsymbol{M}_{N_{\mathrm{RB}}\times|\boldsymbol{G}|} = & \left[\begin{array}{cccc}m_{1,1} & m_{1,2} & \cdots & m_{1,|\boldsymbol{G}|}\\ m_{2,1} & m_{2,2} & \cdots & m_{2,|\boldsymbol{G}|}\\ \cdots & \cdots & \cdots & \cdots\\ m_{N_{rb},1} & m_{N_{rb},2} & \cdots & m_{N_{rb},|\boldsymbol{G}|}\end{array}\right] & \begin{array}{c}\mathrm{RB}_1\\ \mathrm{RB}_2\\ \cdots\\ \mathrm{RB}_{N_{rb}}\end{array} \end{array} , \tag{19}$$

where the elements $\{m_{i,j}\}$ are calculated according to grouping criteria in Subsection 3.1.1–3.1.3.

Then, the average MSE for ith user group at jth resource pattern can be written as

$$\eta_{i,j}=\begin{cases}\frac{\mathrm{sum}(\boldsymbol{M}(:,i).\times\boldsymbol{T}(:,j))}{\mathrm{sum}(\boldsymbol{T}(:,j))}, & \mathrm{sum}\left(\boldsymbol{T}\left(:,j\right)\right)\neq 0,\\ \mathrm{inf}, & \mathrm{sum}\left(\boldsymbol{T}\left(:,j\right)\right)=0.\end{cases} \tag{20}$$

Define $|\boldsymbol{G}|\,J\times 1$ resource allocation vector and $|\boldsymbol{G}|\,J\times 1$ user group average MSE performance vector as

$$\boldsymbol{I}=\left[I_{1,1},\ldots,I_{|\boldsymbol{G}|,1},I_{1,2},\ldots,I_{|\boldsymbol{G}|,2},\ldots,I_{i,j},\ldots,I_{|\boldsymbol{G}|,J}\right]^{\mathrm{T}},$$

$$\boldsymbol{\eta}=\left[\eta_{1,1},\ldots,\eta_{|\boldsymbol{G}|,1},\eta_{1,2},\ldots,\eta_{|\boldsymbol{G}|,2},\ldots,\eta_{i,j},\ldots,\eta_{|\boldsymbol{G}|,J}\right]^{\mathrm{T}},$$

where $I_{i,j}=\{0,1\}\,,\ i=1,\ldots,|\boldsymbol{G}|\,,\ j=1,\ldots,J.$

Then, we write the average MSE vector function of user groups for uplink system as

$$F^{\mathrm{MSE}}\left(\boldsymbol{I}\right)=\left[{f_1}^{\mathrm{MSE}},{f_2}^{\mathrm{MSE}},\ldots,{f_{|\mathbf{G}|J}}^{\mathrm{MSE}}\right]^{\mathrm{T}}=\mathrm{diag}(\boldsymbol{\eta})\boldsymbol{I}. \tag{21}$$

where $f^{\mathrm{MSE}}_{(j-1)*|G|+i}=\eta_{i,j}I_{i,j}$.

On the other hand, we let $\boldsymbol{A}=[\,\mathrm{sum}\boldsymbol{T}(:,1)\ \mathrm{sum}\boldsymbol{T}(:,2)\ \mathrm{sum}\boldsymbol{T}(:,3)\ \cdots\ \mathrm{sum}\boldsymbol{T}(:,J)\,]$, $\boldsymbol{D}=m*\mathrm{ones}(1,|\boldsymbol{G}^m|)$, $\boldsymbol{D}=[\,D_1\ D_2\ \cdots\ D_{N_r}\,]$, $\boldsymbol{B}=[\,B^1\ B^2\ \cdots\ B^m\,]$.

The transmit data stream for virtual MIMO across different resource pattern can be obtained by

$$\boldsymbol{r}=\boldsymbol{A}\otimes\boldsymbol{D}. \tag{22}$$

Then, we have the throughput function for uplink system as

$$f^{\mathrm{TP}}\left(\boldsymbol{I}\right)=\boldsymbol{r}^{\mathrm{T}}\boldsymbol{I}. \tag{23}$$

When both the MSE and throughput oriented objectives are considered, we write the multi-objective optimization problem as

$$\arg\min_{\boldsymbol{I}} F(\boldsymbol{I})=\left(\overline{f_0}^{\mathrm{TP}}(\boldsymbol{I}),F^{\mathrm{MSE}}(\boldsymbol{I})\right) \tag{24a}$$

$$\mathrm{s.t.}\quad \mathrm{AC1}:\ C_1\boldsymbol{I}\leqslant\mathbf{1}_{N_{\mathrm{RB}}\times 1}, \tag{24b}$$

$$\text{AC2}: \; C_2\boldsymbol{I} \leqslant \mathbf{1}_{K\times 1}, \tag{24c}$$

where $\overline{f_0}^{\text{TP}}(\boldsymbol{I}) = N_{\text{RB}}N_r - f^{\text{TP}}(\boldsymbol{I})$, $C_1 = T \otimes \mathbf{1}_{1\times|\boldsymbol{G}|}$, $C_2 = \mathbf{1}_{1\times J} \otimes \boldsymbol{B}$.

The objectives in problem (24) are to maximize the total throughput and minimize the average MSE; AC1 is to ensure that each RB can only be allocated to one user group, AC2 is to ensure that each user can occupy one resource pattern at most. AC2 is similar to lower part of formula (11) in [10], but formula AC1 is different to upper part of formula (11) in [10] because formula (12) in [10] is redundant.

3.3 Trade-off between system throughput and average MSE

Since the objectives in (24) contradict each other, no solution maximizes all the objectives simultaneously. The best tradeoffs between the objectives can be defined in terms of Pareto optimality. The set of all the Pareto optimal points is called the Pareto set (PS) and the set of all the Pareto optimal objective vectors is the Pareto front (PF) [26].

There are several approaches to convert the problem of approximation of the PF into a number of scalar optimization problems. In the following, we adopt two approaches to obtain the different tradeoff results for the objectives of system throughput and average MSE.

3.3.1 *Maximize system throughput with hard MSE constraint*

If a communication system is MSE or SER/BER sensitive, we can consider the MSE object function as constraint, then the problem (24) can be transformed by ε-constraint method as follows

$$\arg\min_{\boldsymbol{I}} \overline{f_0}^{\text{TP}}(\boldsymbol{I}) \tag{25a}$$

$$\text{s.t.} \quad \text{BC1}: \; C_1\boldsymbol{I} \leqslant \mathbf{1}_{N_{\text{RB}}\times 1}, \tag{25b}$$

$$\text{BC2}: \; C_2\boldsymbol{I} \leqslant \mathbf{1}_{K\times 1}, \tag{25c}$$

$$\text{BC3}: \; F^{\text{MSE}}(\boldsymbol{I}) \leqslant \varepsilon_0 \times \mathbf{1}_{|\boldsymbol{G}|J}, \tag{25d}$$

where BC1 and BC2 are the same with AC1 and AC2, and constraint BC3 is to ensure that the average MSE performance is less than the MSE threshold ε_0.

3.3.2 *Maximize system throughput with elastic MSE constraint*

If some deterioration of MSE performance is permitted during the transmission, compared with hard MSE constraint designs, we can transform problem (24) by utility function method as follows

$$\arg\min_{\boldsymbol{I}} U\left(\overline{f_0}^{\text{TP}}(\boldsymbol{I}), F^{\text{MSE}}(\boldsymbol{I})\right) \tag{26a}$$

$$\text{s.t.} \quad \text{CC1}: \; C_1\boldsymbol{I} \leqslant \mathbf{1}_{N_{\text{RB}}\times 1}, \tag{26b}$$

$$\text{CC2}: \; C_2\boldsymbol{I} \leqslant \mathbf{1}_{K\times 1}, \tag{26c}$$

where $U(\cdot)$ is the negative utility function which represents the decision-maker's (DM's) preference information towards objective functions.

To describe the optimal indifference tradeoff between throughput and MSE, we adopt the marginal rates of substitution (MRS) which is defined as follows

$$q_{ij} = \frac{\partial U/\partial \mathrm{f}_j}{\partial U/\partial \mathrm{f}_i} = -\left.\frac{df_i}{df_j}\right|_{dU=0, df_k=0, k\neq i,j} \approx \left|\frac{\Delta f_i}{\Delta f_j}\right|, \tag{27}$$

where

$$f_l = \begin{cases} \overline{f_0}^{\text{TP}}(\boldsymbol{I}), & l = 0, \\ {f_l}^{\text{MSE}}, & l = 1, \ldots, |\boldsymbol{G}|\,J. \end{cases}$$

Without loss of generality, we assume that the MRS of throughput and MSE objective functions for each user group is the same, then

$$\begin{cases} q_{i0} = q_{j0} = R_m, & i \neq j \neq 0, \\ q_{ij} = 1, & i \neq j \neq 0. \end{cases} \tag{28}$$

To simplify the optimal problem, we use weight sum method to define the negative utility function:

$$U\left(\overline{f_0}^{\mathrm{TP}}(\boldsymbol{I}), F^{\mathrm{MSE}}(\boldsymbol{I})\right) = w_0\overline{f_0}^{\mathrm{TP}}(\boldsymbol{I}) + \sum_{l=1}^{|\boldsymbol{G}|J} w_l {f_l}^{\mathrm{MSE}}, \tag{29}$$

where $\sum_{l=0}^{|\boldsymbol{G}|J} w_l = 1$.

From (28) and (29), we have

$$\begin{cases} w_0 = \frac{R_m}{R_m + |\boldsymbol{G}|J}, \\ w_1 = \cdots = w_{|\boldsymbol{G}|J} = \frac{1}{R_m + |\boldsymbol{G}|J}. \end{cases} \tag{30}$$

3.3.3 *Algorithm to the optimization problems*

The optimization problem (25) and (26) are typical binary integer programming problems. So it is suitable to be converted to Office Assignment Problem (OAP) [27], and use a linear programming (LP)-based branch-and-bound (BNB) algorithm to solve the problem.

However, branch-and-bound algorithm is too complex and not practical when user and resource number become large. In order to reduce the algorithm complexity, we convert the optimization problem (25) and (26) to following normalized form

$$\min C^{\mathrm{T}} x \tag{31a}$$

$$\text{s.t.} \quad \boldsymbol{A}x \leqslant b, \tag{31b}$$

$$x_i = \{0, 1\}, \ (b)_i = 1, \tag{31c}$$

where x is the solution variable vector which represents the persons are assigned to the office or not, C is weight vector for assignment, $\boldsymbol{A}$ is equality or inequality constraint matrix, and b is equality or inequality limit vector.

For optimization problem (25), we need to consider the constraint (25d) and determine the user group index i and the resource pattern index j of the elements which are greater than ε_0 in the matrix $F^{\mathrm{MSE}}(\boldsymbol{I})$. Based on index i, j and their relationship with vector index of $F^{\mathrm{MSE}}(\boldsymbol{I})$ in (21), we turn the corresponding columns in the matrix C_1 of constraint (25b), the matrix C_2 of constraint (25c) and the matrix $\boldsymbol{r}$ of (23) to zeros, and then remove constraint (25d) from problem (25).

4 Simulation results

4.1 Overall simulation design

For comparison purpose, the joint resource allocation algorithms using three grouping criteria are implemented in this section. First one adopts criterion (11), denoted as Groups-ZF; second one adopts criterion (13), denoted as S1-Groups-ZF; last one adopt criterion (16), denoted as S2-Groups-ZF. In addition, the resource allocation algorithm using conventional criterion called determinant pairing scheduling (DPS) mentioned in [14] is also implemented in this section which denoted as Algorithm in [14].

To evaluate the performance of the proposed dynamic grouping and joint resource allocation algorithms, we conduct the simulations based on LTE uplink wherein the BS is equipped with four receive antennas and each user has only one antenna. The simulation parameters are listed in Table 2. In addition, we adopt the pedestrian test environment channel A as suggested by ITU-R M.1225 [28]. The simulation setup parameters are listed in Table 2.

Table 2 Simulation setup

Channel parameters	Channel model: ITU Ped-A	Carrier frequency: 2 GHz
	Sampling frequency: 1.92 MHz	Maximum Doppler shift: 10 Hz
Simulation parameters	FFT size: 128	Modulation: 16-QAM
	N^{RB}: 6	$N_{\mathrm{sc}}^{\mathrm{RB}}$: 12
	OFDM symbols per frame: 14	RB configure: 127
	Number of users: 20	MIMO detector: ZF
	UE transmit antenna number: 1	Receive antenna number: 4
	TTI duration: 1 ms	Simulation frames: 1000

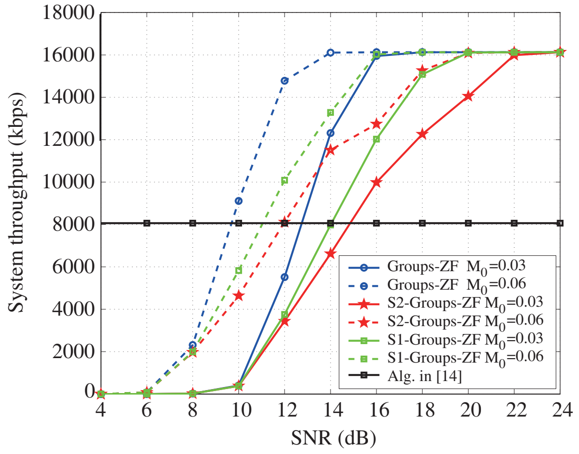

Figure 2 (Color online) System throughput versus SNR with different thresholds for hard MSE constraint algorithm (16-QAM, 1,2,3,4-user dynamic grouping) and algorithm in [14] (16-QAM, 2-user static grouping).

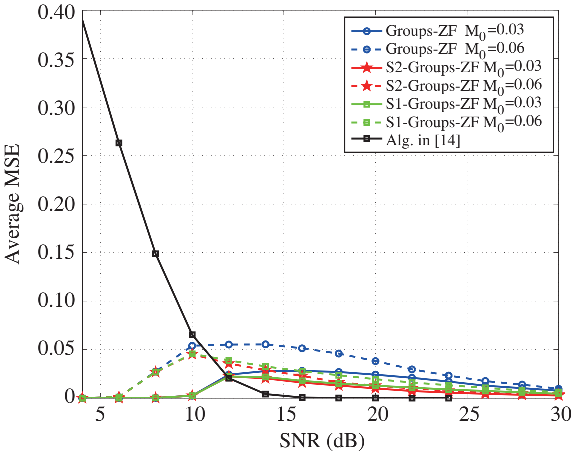

Figure 3 (Color online) Average MSE versus SNR with different thresholds for hard MSE constraint algorithm (16-QAM, 1,2,3,4-user dynamic grouping) and algorithm in [14] (16-QAM, 2-user static grouping).

4.2 Simulation results

4.2.1 *Performance evaluation of algorithm for maximizing system throughput with hard MSE constraint*

The comparison of proposed criteria with DPS criterion is done in this part. Simulations based on different grouping criteria are conducted respectively to show the performance results come from proposed algorithm to hard MSE constraint with different performance threshold.

Figures 2 and 3 illustrate respectively the system throughput and average MSE of proposed user grouping criteria and DPS criterion. The former versus the average SNR with hard MSE constraints of $M_0 = \{0.03, 0.06\}$. The latter is the 2-user static grouping using DPS criterion without MSE constraint.

From Figure 3, we can see that each of the average MSE curves is successfully suppressed below the corresponding threshold except the DPS criterion curve. While the MSE curve of DPS criterion is much higher than other curves when SNR is lower than 10 dB because the DPS criterion does not have MSE constraint. As the SNR increases, the average MSE curves descend which is lower than proposed user grouping criteria because its number of uses in one group is 2 while the proposed user grouping criteria is 3 or 4. In terms of the proposed algorithm, in the region of low SNR, the number of virtual MIMO transmit antennas gradually increase, and this lead to detection performance degradation. Then, the average MSE curves ascend though SNR increases. As the SNR increases, the grouping user number tends to the maximum value and becomes stable. This is described by throughput curves in Figure 2. So, in the region of high SNR, the MSE curves descend due to the SNR increasing. The gap between the MSE curves and threshold mainly results from the variation of virtual MIMO transmit antennas number which leads to the step change of detection performance.

In Figure 2, the throughputs of DPS criterion do not change with SNR while the throughputs of proposed algorithm increase with the SNR and keep steady finally. Conclusion can be obtained that the proposed algorithm is better than DPS criterion in terms of the flexibility of throughput. Furthermore,

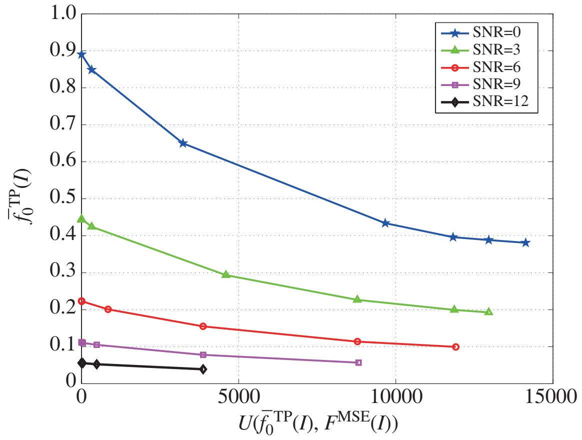

Figure 4 (Color online) Indifference cures of $U(\overline{f_0}^{\mathrm{TP}}(I), F^{\mathrm{MSE}}(I))$ under different SNR (16-QAM, 1,2,3,4-user dynamic grouping).

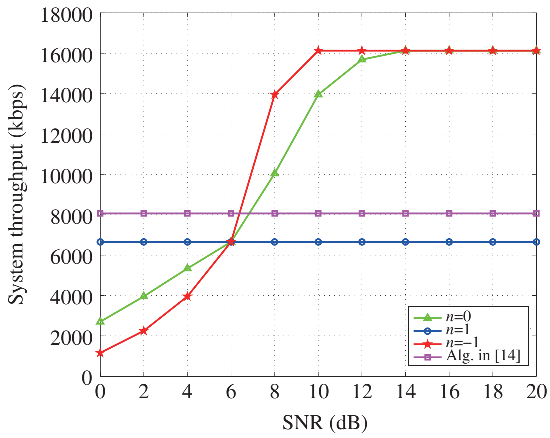

Figure 5 (Color online) System throughput versus SNR with different MRS for elastic MSE constraint algorithm (16-QAM, 1,2,3,4-user dynamic grouping) and algorithm in [14] (16-QAM, 2-user static grouping).

system throughputs of proposed algorithms increase when we set a low MSE threshold. In addition, given a threshold, the throughput curve of Groups-ZF algorithm rises fastest and reaches its saturation value first, followed by the S1-Groups-ZF, and the S2-Groups-ZF is the slowest. The main reason is that the metrics in grouping criteria for Groups-ZF, S1-Groups-ZF and S2-Groups-ZF algorithms are gradually stringent with regard to average MSE.

So, compared with DPS criterion, proposed criteria can achieve a more flexible and greater system throughput under the constraint of MSE threshold. And the proposed algorithms satisfy the hard MSE/SER requirements and provide a way to attain different tradeoff between system throughput and MSE/SER performance by setting different MSE threshold.

4.2.2 *Performance evaluation of algorithm for maximizing system throughput with elastic MSE constraint*

In this section, the simulations for algorithm to optimization problem (26) are conducted and the criterion of S1-Groups-ZF is adopted. For other MSE criteria the simulations will have the similar results.

(1) Indifference surface of negative utility function $U(\cdot)$. If the transmission power is normalized, the indifference surface can be written as

$$U\left(\overline{f_0}^{\mathrm{TP}}(\boldsymbol{I}), F^{\mathrm{MSE}}(\boldsymbol{I})\right)\Big|_{\sigma^2} = \left(w_0\overline{f_0}^{\mathrm{TP}}(\boldsymbol{I}) + \sum_{l=1}^{|\boldsymbol{G}|J} w_l {f_l}^{\mathrm{MSE}}\right)\Bigg|_{\sigma^2} = \text{constant}. \tag{32}$$

Since each one of $\{{f_l}^{\mathrm{MSE}},\ l = 1, \ldots, |\boldsymbol{G}|J\}$ is the same, we only need to study the indifference curve of $\overline{f_0}^{\mathrm{TP}}(\boldsymbol{I})$ and any one of ${f_l}^{\mathrm{MSE}}$. The weighting vector is given by (32) where we set the MRS $R_m = [0.14, 0.07, 0.035, 0.014, 0.008, 0.004, 0.002]$. Figure 4 shows the indifference curves under the conditions of SNR $= \{0, 3, 6, 9, 12\}$ dB. From the figure we can see that the negative utility function is reduced with SNR increasing. Furthermore, given different MRS values we can obtain different tradeoffs between system throughput and average MSE for each indifference curve.

(2) Effect of tradeoff between system throughput and average MSE under variable SNR. To obtain the flexible tradeoff between system throughput and average MSE under variable SNR based on indifference surface, one effective approach is to setup MRS as a function of σ^2.

In this subsection, we demonstrate the effect of different tradeoffs on condition that MRS preset value is the power function of σ^2, i.e., $R_m = R_0(\frac{s^2}{{s_0}^2})^n$. We choose the reference point $R_0 = 0.004$, ${\sigma_0}^2 = 0.2512$ (SNR $= 6$) and let $n = 0, 0.5, 1$. In the meantime, we compare the proposed algorithm with DPS algorithm in terms of system throughput, average MSE.

The system throughput results are shown in Figure 5. From the figure we see that DPS algorithm has higher throughput than the case of $n = 1$ because they all have the fixed number of grouping user and

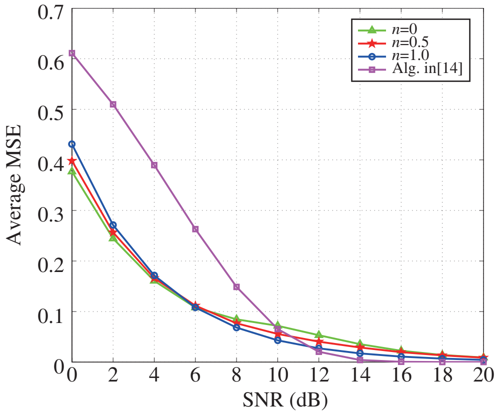

Figure 6 (Color online) Average MSE versus SNR with different MRS for elastic MSE constraint algorithm (16-QAM, 1,2,3,4-user dynamic grouping) and algorithm in [14] (16-QAM, 2-user static grouping).

the number of DPS algorithm is greater than the case of $n = 1$. The case of $n = 0.5$ and $n = 0$ have changeable system throughputs which are greater than DPS algorithms' when SNR is greater than 7 dB. For the proposed algorithm, we see that the throughput remains stable over all SNR in the case of $n = 1$. The main reason is that the MSE-oriented criteria are proportional to σ^2 so that MRS is proportional to σ^2 due to (27) and (28). So, the user grouping results are only dependent on virtual MIMO channel state information (CSI) in this case. This leads to the similar user grouping results for all SNR value, and then the same system throughput. For $n = 0$, the MRS R_m grows faster than it for $n = 0.5$ as SNR increases. So, the system throughput for $n = 0$ increases much larger than that for $n = 0.5$ due to the DM's preference when SNR increases from 0 dB to 14 dB. When SNR is greater than 14 dB, the curves for $n = 0$ become horizontal because it reach the maximum throughput which is dependent on the receive antenna number N_r. In addition, It can be observed that all the curves meet at the point where they have the same MRS R_m.

In Figure 6, we compare the MSE trends of proposed algorithm and DPS algorithm. The MSE curve of DPS scheme is higher than other curves when SNR is lower than 10 dB because the DPS scheme does not have MSE constraints but the proposed algorithm takes part of the MSE constraints into account. As the SNR increases, the average MSE curve descends which is lower than the proposed algorithm because its number of users in one group is 2 while the proposed user grouping criterion is 3 or 4. Compared with the system throughput results above, in terms of the proposed algorithm, the MSE curves show opposite trends for $n = 0, 0.5, 1$. That is, the MSE performance improves fastest for $n = 1$, and then for $n = 0.5$, and last for $n = 0$. The curves meet at the point of SNR = 6 dB. So, the different tradeoffs between system throughput and average MSE are obtained according to DM's preferences.

5 Conclusion

In this paper, we investigate the user grouping in uplink virtual MIMO systems with ZF detection. Through the consideration of both system throughput and the receive signal detection performance, we derive the MSE oriented user grouping criteria and propose joint user grouping and RB allocation algorithms with hard and elastic average MSE constraints. The simulation results demonstrate that compared with the traditional user grouping algorithm called DPS, the proposed algorithm with hard average MSE constraint attains maximum system throughput with guaranteed average MSE and the proposed algorithm with elastic average MSE constraint could achieve the desired tradeoff between system throughput and SER performance according to DM's preference.

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Conflict of interest The authors declare that they have no conflict of interest.

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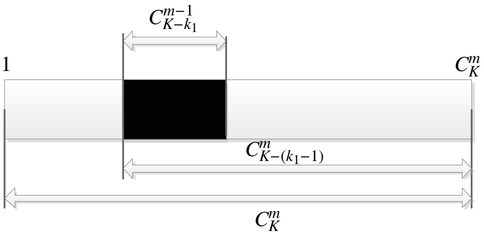

Figure A1 Illustration of the range of i according to k_1.

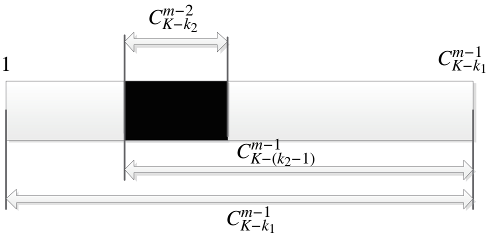

Figure A2 Illustration of the range of i according to k_1 and k_2.

Appendix A Proof of (6)

For $G^m{}_i = (k_1, k_2, \ldots, k_m), 1 \leqslant k_1 < \cdots < k_m \leqslant K$, we have $i \in \left[1, C_K^m\right]$. The number index i is derived sequentially by $k_1, k_2, k_3, \ldots, k_m$.

First, determine range of i according to k_1. As shown in Figure A1, given k_1, the black part demonstrates the range of i. Define $C_K^m - C_{K-(k_1-1)}^m$ as A_1, then $i \in [A_1 + 1, A_1 + C_{K-1}^{m-1}]$, where C_K^m denotes total number range of i and $C_{K-(k_1-1)}^{m-1}$ denotes number range of i when first user number is greater than or equal to k_1.

Second, determine range of i according to k_1 and k_2. As shown in Figure A2, the black part demonstrates the range of i. Define $C_{K-k_1}^{m-1} - C_{K-(k_2-1)}^{m-1}$ as A_2, then $i \in [A_1 + A_2 + 1, A_1 + A_2 + C_{K-k_2}^{m-2}]$, where $C_{K-k_2}^{m-2}$ denotes number range of i when first and second user number are greater than or equal to k_1 and k_2 respectively.

And then, repeat the similar step until reach the condition of k_{m-1}. Correspondingly, if define $C_{K-k_{j-1}}^{m-j+1} - C_{K-(k_j-1)}^{m-j+1}$ as A_j, then $i \in [\sum_{j=1}^{m-1} A_j + 1, \sum_{j=1}^{m-1} A_j + C_{K-k_{m-1}}^1]$, where $k_0 = 0$.

At last step, when given $k_1, k_2, k_3, \ldots, k_m$, we can eventually get

$$i = \sum_{j=1}^{m-1} A_j + (k_m - k_{m-1}).$$