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Power allocation for massive MIMO: impact of power amplifier efficiency

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Abstract With the rapid development of information technology, massive MIMO is becoming attractive for the fifth generation (5G) communication because of its outstanding performance in both spectral efficiency (SE) and energy efficiency (EE). Recently, many algorithms have been proposed to improve the EE while achieving high SE in massive MIMO systems. In previous work, the power amplifier (PA) efficiency is always considered as a constant. However, the PA efficiency changes with the output power in reality. In the practical situation, the simplification which treats the PA efficiency as a constant will not get the EE optimization based on our analysis. In this paper, we propose a more general EE model of massive MIMO systems considering PA efficiency as a variable, and investigate a power allocation algorithm based on zero-forcing (ZF) precoding so that we can guarantee the SE and EE at the same time. Simulation results show the trade-off between EE and SE, demonstrate the distinction with previous work, and imply that relatively higher transmit power will be more energy efficient.

Keywords 5G, massive MIMO, energy efficiency, green communication, power amplifier efficiency

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1 Introduction

It is well known that multiple-input multiple-output (MIMO) technology can provide more degrees of freedom which are beneficial to multiplexing gains and diversity gains. It can be proved that the ergodic capacity of a MIMO fading channel is proportional to the minimum of the numbers of the transmit and receiver antennas [1,2]. MIMO provides significant performance enhancement without additional transmit power and bandwidth resources, and this is why it has been one of the research focuses in the past decade. Since the amount of energy consumption for information and communication technology (ICT) increases rapidly with an explosive growth of service requirement, ICT plays a more and more important part in global warming [3]. Meanwhile, an increasing interest in multi-media services such as video conference and online high definition video stream is leading to a great deal of demand for high data rate communication. Because of the two issues, as one of the main technologies of 5G communication, massive MIMO has been proposed in [4,5] to provide high data rate communication and reduce energy consumption at the same time.

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In [4], the authors investigate the sum capacity of cellular networks assuming that both base station (BS) and users are equipped with unlimited numbers of antennas. In [5], the authors show the high throughputs for both the uplink and the downlink of a time-division duplex multi-cell system which employs BSs equipped with very large numbers of antennas. Compared to single antenna systems, abundant capacity gains and better interference management capabilities are observed for MIMO in [4,5]. The data rate can be improved significantly due to the unlimited numbers of antennas if we grow the numbers of antennas at both the transmitter and the receiver. However, the advantages of massive MIMO do not come for free. They lead to financial problems for service providers because of the rapidly increasing cost for energy consumption in circuitries. Motivated by environmental and economical costs, green communication is a new research direction that aims at designing wireless networks with better coverage and higher energy efficiency (EE) [6].

As EE is becoming a focus for wireless communication design, there are some properties of EE having been studied on massive MIMO in recent years. In [7,8], the authors report that with the base stations (BSs) equipped with very large antenna arrays, transmission can enhance EE significantly in future communication systems. When the number of BS antennas, grows to infinite, the transmit power will reduce proportionally to $1/M$ if the BS obtains perfect channel state information (CSI), and proportionally to $1/\sqrt{M}$ if the CSI is not perfect. The relationship of EE and spectral efficiency (SE) has been studied in [9]. The authors investigate the character of EE-optimal points and conclude that reducing circuit power bring exponential EE increase with linear SE loss. The impact of non-ideal hardware of massive MIMO systems is studied in [10]. It is shown that the impact of impairments in the large scale arrays vanishes asymptotically and interference becomes negligible. In [11], the authors investigate the closed-form expressions not only for the EE-optimal numbers of transmit antennas, but also for the numbers of receiver antennas and the transmit power, and the impact of the propagation environment as well as coefficients in their power consumption model. However, to the best of the authors knowledge, there are few literatures discussed about EE with considering the impact of variable power amplifier (PA) efficiency except [12,13]. In [12,13], the authors propose a PA switching technique to improve EE so that a high EE over a wide range SE can be achieved.

With current technology, because of the nonlinear effect and inefficiency of PA, it is a practical challenge for system design to achieve high SE and EE. In previous work, the PA efficiency is always considered as a constant, e.g., 30%, in the power consumption model for simplicity, but it changes with the output power in reality [14]. The conventional precoding scheme cannot guarantee that the PAs work on their efficient region. If the energy efficiency model consider the PA efficiency as a variable, to get a better EE performance, new precoding scheme, users scheduling and resource allocation method are probable to be investigated. And this may lead to a new research field.

Motivated by the aforementioned observations, we try to improve the EE without switching PA, because PA switching technique may lead to more space problems and financial cost. Based on zero-forcing (ZF) precoding, we formulate the resource allocation problem for energy efficient communication in massive MIMO systems as an optimization problem which aims at improving EE and delivering a required SE.

The remainder of this paper is organized as follows. In Section 2, the system model for massive MIMO including energy consumption analysis is introduced. Section 3 introduces a power allocation algorithm for achieving maximum EE in downlink multi-user MIMO. Simulation results are discussed in Section 4. Section 5 concludes the paper.

2 System model

This paper considers the downlink of MIMO system in a single cell which there is a BS equipped with M antennas communicating with N single antenna users, where $M \gg N$, with perfect CSI at the transmitter side. Let the complex channel gain between the mth BS antenna and the nth user be denoted by $f_{n,m}$. The vector of channel gains from the BS antennas to the nth user is denoted by $\boldsymbol{f}_n = (f_{n,1}, f_{n,2}, \ldots, f_{n,M})$. $\boldsymbol{F} \in \mathbb{C}^{N \times M}$ is the channel gain matrix with $f_{n,m}$ as its (n,m)th entry, and it models independent fast fading, and large-scale fading. The coefficient $f_{n,m}$ can be written as

$$f_{n,m} = \sqrt{\beta_n} h_{n,m}, \tag{1}$$

where $h_{n,m}$ is the fast fading coefficient from the mth antenna to the nth user, and $\sqrt{\beta_n}$ models the large-scale fading of which the value changes very slowly with time. Then we have

$$\boldsymbol{F} = \boldsymbol{D}^{1/2}\boldsymbol{H}, \tag{2}$$

where $[\boldsymbol{H}]_{nm} = h_{n,m}$, $\boldsymbol{D} = \mathrm{diag}(\beta_1, \beta_2, \ldots, \beta_N)$. And we can denote $\boldsymbol{h}_n = (h_{n,1}, h_{n,2}, \ldots, h_{n,M})^{\mathrm{T}}$.

Let x_m denote the complex symbol transmitted from the mth BS antenna. The downlink received symbol of the nth user using ZF precoding can be expressed as

$$y_n = \sum_{m=1}^{M} f_{n,m} x_m + \omega_n, \quad n = 1, 2, \ldots, N, \tag{3}$$

where $\omega_n \sim \mathbb{CN}\left(0, \sigma^2\right)$ is the AWGN noise at the nth receiver. Further, the downlink transmission can be expressed for all users in matrix form as

$$\boldsymbol{y} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{\omega} = \boldsymbol{D}^{1/2}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{\omega}. \tag{4}$$

Consider ZF precoding with the beamforming matrix $\boldsymbol{G} \in \mathbb{C}^{M\times N}$, which is the pseudo-inverse matrix of $\boldsymbol{F}$, with $g_{m,n}$ as its (m,n)th entry. And $\boldsymbol{p} = [p_1, p_1, \ldots, p_N]^{\mathrm{T}}$ denote the power allocated for all users. $\boldsymbol{P} \triangleq \mathrm{diag}\left(\boldsymbol{p}\right)$, then the ZF precoding matrix can be expressed as

$$\boldsymbol{W} = \boldsymbol{G}\boldsymbol{P}^{1/2}. \tag{5}$$

Suppose that the information symbol for all N users is $\boldsymbol{u} = [u_1, \ldots, u_N]^{\mathrm{T}}$. Then the channel model can be expressed as

$$\boldsymbol{y} = \boldsymbol{D}^{1/2}\boldsymbol{H}\boldsymbol{G}\boldsymbol{P}^{1/2}\boldsymbol{u} + \boldsymbol{\omega}, \tag{6}$$

where the information symbol satisfies $\mathrm{E}\left[\boldsymbol{u}\boldsymbol{u}^{\mathrm{H}}\right] = \boldsymbol{I}$.

We can express $\boldsymbol{G}$ as $\boldsymbol{G} = [\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_N]$, where $\boldsymbol{g}_n = [g_{1,n}, g_{2,n}, \ldots, g_{M.n}]^{\mathrm{T}}$. Using the properties of ZF precoding, we can get that

$$\begin{cases} \boldsymbol{h}_j^{\mathrm{T}}\boldsymbol{g}_k \neq 0, & j = k, \\ \boldsymbol{h}_j^{\mathrm{T}}\boldsymbol{g}_k = 0, & j \neq k. \end{cases} \tag{7}$$

So we can give the received signal to noise ratio (SNR) at user n as

$$\gamma_n = \frac{\beta_n \left|\boldsymbol{h}_n^{\mathrm{T}}\boldsymbol{g}_n\right|^2 p_n}{\sigma^2}, \tag{8}$$

and the maximum transmit data rate can be expressed as

$$R_n = \log_2\left(1 + \gamma_n\right) = \log_2\left(1 + \frac{\beta_n \left|\boldsymbol{h}_n^{\mathrm{T}}\boldsymbol{g}_n\right|^2 p_n}{\sigma^2}\right). \tag{9}$$

Furthermore, the transmit power at mth antenna on BS can be expressed as

$$\sum_{n=1}^{N} \left|g_{m,n}\right|^2 p_n. \tag{10}$$

3 Power allocation

In this paper, we define EE as the ratio of data rate over the total power consumption [15]:

$$\eta_{\mathrm{EE}} = \frac{C}{P_t/\rho + P_c} \ \ \text{(bits/Joule)}, \tag{11}$$

where C is the channel capacity, which is equal to the maximum transmit data rate. P_t is the transmit power, and ρ is the PA efficiency, which is treated as a constant in previous work. P_c is the circuit power, which is affected by the number of antennas. In this paper, P_c is modeled as a linear function of number of transmit antennas [16,17] as

$$P_c = MP_0 + P_1, \tag{12}$$

where P_0 and P_1 are constant.

From the transmit data rate in (9) and the transmit power at mth antenna in (10), we can obtain the EE of the downlink multi-user MIMO as

$$\eta_{\mathrm{EE}} = \frac{\sum_{n=1}^{N} \log_2\big(1 + \frac{\beta_n |\boldsymbol{h}_n^{\mathrm{T}} \boldsymbol{g}_n|^2 p_n}{\sigma^2}\big)}{\sum_{m=1}^{M} \frac{\sum_{n=1}^{N} |g_{m,n}|^2 p_n}{\rho(\sum_{n=1}^{N} |g_{m,n}|^2 p_n)} + P_c}, \tag{13}$$

where $\rho\left(\cdot\right)$ is the function of PA efficiency changing with the output power. In other words, we consider the PA efficiency as a variable.

So we can formulate the EE optimization as follows,

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & \eta_{\mathrm{EE}} \\ \text{s.t.} \quad & \textstyle\sum_{n=1}^{N} |g_{m,n}|^2 p_n \leqslant \frac{P}{M}, \qquad m = 1, 2, \ldots, M, \\ & p_n \geqslant 0, \qquad\qquad\qquad\quad\ \ n = 1, 2, \ldots, N, \end{aligned} \tag{14}$$

where P is the maximum transmit power at BS, and P/M is the power constraint per antenna.

Since the PA efficiency changes with the output power, it is almost impossible to judge whether the total EE will increase or not like previous work as the transmit power changes. We assume that ZF precoding is used to give each user an information rate which is not less than a target one, and minimize the total power consumption at the BS for simplicity from the optimization problem (14). Then we can get a new optimization problem as follows,

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \textstyle\sum_{m=1}^{M} \frac{\sum_{n=1}^{N} |g_{m,n}|^2 p_n}{\rho(\sum_{n=1}^{N} |g_{m,n}|^2 p_n)} + P_c \\ \text{s.t.} \quad & \log_2\left(1 + \frac{\beta_n |\boldsymbol{h}_n^{\mathrm{T}} \boldsymbol{g}_n|^2 p_n}{\sigma^2}\right) \geqslant R_n, \qquad n = 1, 2, \ldots, N, \\ & \tfrac{M}{P} \textstyle\sum_{n=1}^{N} |g_{mn}|^2 p_n \leqslant 1, \qquad\qquad\quad m = 1, 2, \ldots, M, \\ & p_n \geqslant 0, \qquad\qquad\qquad\qquad\qquad\ \ n = 1, 2, \ldots, N. \end{aligned} \tag{15}$$

The Lagrange function of problem (15) can be shown as

$$\begin{aligned} L\left(\boldsymbol{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\xi}\right) = & \sum_{m=1}^{M} \frac{\sum_{n=1}^{N} |g_{mn}|^2 p_n}{\rho(\sum_{n=1}^{N} |g_{mn}|^2 p_n)} + P_c + \sum_{n=1}^{N} \lambda_n \left(R_n - \log_2\left(1 + \frac{\beta_n |\boldsymbol{h}_n^{\mathrm{T}} \boldsymbol{g}_n|^2 p_n}{\sigma^2}\right)\right) \\ & + \sum_{m=1}^{M} \mu_m \left(\frac{M}{P} \sum_{n=1}^{N} |g_{mn}|^2 p_n - 1\right) - \sum_{n=1}^{N} \xi_n p_n, \end{aligned} \tag{16}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_N]$, $\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_m]$, $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_N]$ are Lagrange multipliers for the constraints in (15). The Karush-Kuhn-Tucker (KKT) conditions of problem (15) can be expressed as follows,

$$\frac{\partial L\left(\boldsymbol{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\xi}\right)}{\partial p_n} = 0, \tag{17}$$

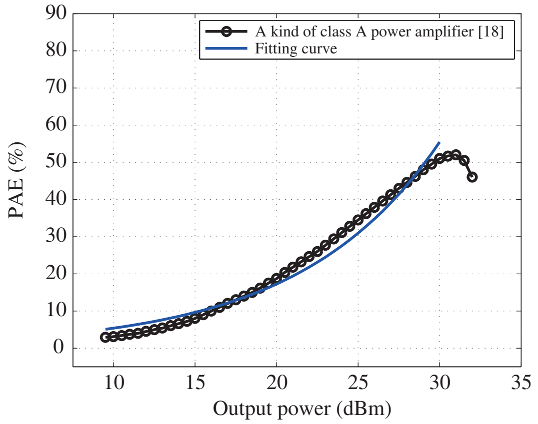

Figure 1 PA efficiency vs. output power and fitting function.

$$\log_2\left(1+\frac{\beta_n|\boldsymbol{h}_n^{\mathrm{T}}\boldsymbol{g}_n|^2 p_n}{\sigma^2}\right) \geqslant R_n, \quad n=1,2,\ldots,N, \tag{18}$$

$$\frac{M}{P}\sum_{n=1}^{N}|g_{mn}|^2 p_n \leqslant 1, \quad m=1,2,\ldots,M, \tag{19}$$

$$\lambda_n\left(R_n-\log_2\left(1+\frac{\beta_n|\boldsymbol{h}_n^{\mathrm{T}}\boldsymbol{g}_n|^2 p_n}{\sigma^2}\right)\right)=0, \tag{20}$$

$$\mu_m\left(\frac{M}{P}\sum_{n=1}^{N}|g_{mn}|^2 p_n-1\right)=0, \tag{21}$$

$$\xi_n p_n=0, \tag{22}$$

$$p_n,\lambda_n,\xi_n \geqslant 0, \quad n=1,2,\ldots,N, \tag{23}$$

$$\mu_m \geqslant 0, \quad m=1,2,\ldots,M. \tag{24}$$

Different from previous work which considered the PA efficiency as a constant, the PA efficiency is treated as a variable, such as Figure 1. In the practical situation, the simplification which treats the PA efficiency as a constant will not get the real EE optimization. So our power allocation scheme will be more practical and this may lead to a new thinkable direction.

$$\begin{aligned}\frac{\partial L(\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\xi})}{\partial p_n} = &\sum_{m=1}^{M}\frac{|g_{mn}|^2\cdot\rho(\sum_{n=1}^{N}|g_{mn}|^2 p_n)-(\sum_{n=1}^{N}|g_{mn}|^2 p_n)\cdot\rho'(\sum_{n=1}^{N}|g_{mn}|^2 p_n)}{\rho^2(\sum_{n=1}^{N}|g_{mn}|^2 p_n)}\\ &-\lambda_n\frac{\beta_n|\boldsymbol{h}_n^{\mathrm{T}}\boldsymbol{g}_n|^2}{\ln 2(\sigma^2+\beta_n|\boldsymbol{h}_n^{\mathrm{T}}\boldsymbol{g}_n|^2 p_n)}+\frac{M}{P}\sum_{m=1}^{M}\mu_m|g_{mn}|^2-\xi_n.\end{aligned} \tag{25}$$

There are different types of power amplifiers which are commonly designated as Classes A–F [19]. In this paper, we mainly talk about the impact of PA efficiency, and hope to propose an algorithm to solve the optimization problem (15) above, so that we just need to choose a type of power amplifier. Since the transmit power reduces to a very low degree in massive MIMO, we can choose Class A power amplifiers. Figure 1. shows one kind of Class A power amplifier. The PA efficiency changes with the output power [18]. The fitting function of the PA efficiency is usually expressed in series form, but according to (25), we can find that (17) will be very complex when we consider the PA efficiency is a variable, even in series model. So we express it with Least Squares in exponential model for simplicity as

$$\rho\left(x\right)=a\cdot\exp(b\cdot x), \tag{26}$$

where x is the output power of the PA in units of dBm.

$$
\begin{aligned}
&\frac{\partial L(\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\xi})}{\partial p_n}\\
&=\sum_{m=1}^{M}\frac{|g_{mn}|^2\cdot a\cdot\exp(b\cdot 10\log_{10}(\sum_{n=1}^{N}|g_{mn}|^2p_n))}{a^2\exp[2\cdot b\cdot 10\log_{10}(\sum_{n=1}^{N}|g_{mn}|^2p_n)]}\\
&\quad-\lambda_n\frac{|\boldsymbol{h}_n\boldsymbol{g}_n|^2}{\ln 2(\sigma^2+|\boldsymbol{h}_n\boldsymbol{g}_n|^2p_n)}+\frac{M}{P}\sum_{m=1}^{M}\mu_m|g_{mn}|^2\\
&\quad-\sum_{m=1}^{M}\frac{(\sum_{n=1}^{N}|g_{mn}|^2p_n)\cdot ab\cdot\exp(b\cdot 10\log_{10}(\sum_{n=1}^{N}|g_{mn}|^2p_n))\cdot\frac{10|g_{mn}|^2}{\ln 10(\sum_{n=1}^{N}|g_{mn}|^2p_n)}}{a^2\exp[2\cdot b\cdot 10\log_{10}(\sum_{n=1}^{N}|g_{mn}|^2p_n)]}-\xi_n\\
&=\sum_{m=1}^{M}\frac{|g_{mn}|^2(1-\frac{10b}{\ln 10})}{a\cdot\exp(b\cdot 10\log_{10}(\sum_{n=1}^{N}|g_{mn}|^2p_n))}-\lambda_n\frac{1}{\ln 2(\sigma_n^2+p_n)}+\frac{M}{P}\sum_{m=1}^{M}\mu_m|g_{mn}|^2-\xi_n=0. \qquad (27)
\end{aligned}
$$

Substituting (3) into (25), we can simplify the (17) in (27) to find there are nonlinear equations to be solved so that there is no close form solution for this problem. Meanwhile, if we use constant envelope ZF precoding [20], we can find that the reduced form of Hessian matrix of (15) as follows,

$$
\begin{bmatrix}
\sum_{m=1}^{M}|g_{m1}|^4 & \sum_{m=1}^{M}|g_{m1}|^2|g_{m2}|^2 & \cdots & \sum_{m=1}^{M}|g_{m1}|^2|g_{mN}|^2\\
\sum_{m=1}^{M}|g_{m2}|^2|g_{m1}|^2 & \sum_{m=1}^{M}|g_{m2}|^4 & \cdots & \sum_{m=1}^{M}|g_{m2}|^2|g_{mN}|^2\\
\vdots & \vdots & \ddots & \vdots\\
\sum_{m=1}^{M}|g_{mN}|^2|g_{m1}|^2 & \sum_{m=1}^{M}|g_{mN}|^2|g_{m2}|^2 & \cdots & \sum_{m=1}^{M}|g_{mN}|^4
\end{bmatrix}\cdot C, \qquad (28)
$$

where C is a constant which multiplies each element in the matrix.

$$
\begin{aligned}
\boldsymbol{G}^{\mathrm{H}}\boldsymbol{G}&=\begin{bmatrix}
\sum_{m=1}^{M}|g_{m1}|^2 & \sum_{m=1}^{M}g_{m1}^{\mathrm{H}}g_{m2} & \cdots & \sum_{m=1}^{M}g_{m1}^{\mathrm{H}}g_{mN}\\
\sum_{m=1}^{M}g_{m2}^{\mathrm{H}}g_{m1} & \sum_{m=1}^{M}|g_{m2}|^2 & \cdots & \sum_{m=1}^{M}g_{m2}^{\mathrm{H}}g_{mN}\\
\vdots & \vdots & \ddots & \vdots\\
\sum_{m=1}^{M}g_{mN}^{\mathrm{H}}g_{m1} & \sum_{m=1}^{M}g_{mN}^{\mathrm{H}}g_{m2} & \cdots & \sum_{m=1}^{M}|g_{mN}|^2
\end{bmatrix}\\
&=\left(\boldsymbol{F}\boldsymbol{F}^{\mathrm{H}}\right)^{-1}=\left(\boldsymbol{D}^{1/2}\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}\left(\boldsymbol{D}^{1/2}\right)^{\mathrm{H}}\right)^{-1}\\
&=\frac{1}{M}\left(\boldsymbol{D}^{1/2}\frac{\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}}{M}\left(\boldsymbol{D}^{1/2}\right)^{\mathrm{H}}\right)^{-1}\underset{\text{Result on Very Long Radom Vectors}}{\overset{M\to\infty}{\approx}}\frac{1}{M}\left(\boldsymbol{D}^{1/2}\boldsymbol{I}\left(\boldsymbol{D}^{1/2}\right)^{\mathrm{H}}\right)^{-1}\\
&=\frac{1}{M}\boldsymbol{D}^{-1}. \qquad (29)
\end{aligned}
$$

From the theory of Random Very Long Vectors, the column vectors of the propagation matrix are asymptotically orthogonal in massive MIMO system, so that $\boldsymbol{G}^{\mathrm{H}}\boldsymbol{G}$ is approaching to a diagonal matrix in (29). It is easy to get the conclusion that the order of $\sum_{m=1}^{M}g_{mi}^{\mathrm{H}}g_{mj}$, $i,j=1,2,\ldots,N$ is not more than 1 as M goes to infinite. When and only when $i=j$, the order of $\sum_{m=1}^{M}g_{mi}^{\mathrm{H}}g_{mj}$ is 1, because the order of M is 1. Comparing the elements in (28) with those in (29), we can get

$$
\sum_{m=1}^{M}|g_{mi}|^2|g_{mj}|^2\leqslant\left(\sum_{m=1}^{M}\left|g_{mi}^{\mathrm{H}}g_{mj}\right|\right)^2,\quad i,j=1,2,\ldots,N. \qquad (30)
$$

It means that there always exists a constant A related to M, which can guarantee that the Hessian matrix in (28) is approaching to a diagonal matrix as M goes large, and this means the Hessian matrix is strictly diagonally dominant. What is more, the Hessian matrix is a real symmetric matrix whose elements are positive. Because of the two characteristics above, strictly diagonally dominant and real symmetric, we

Table 1 Simulation parameters

Parameter	Value
Number of antennas at BS, M	256
Maximum number of users, N	20
Noise power, σ^2	-104 dBm
Cell radius, R	250 m
Minimum distance from the BS to users	35 m
Path loss model (dB)	$128.1 + 37.6\log_{10}d$ km
Maximum power per antenna at BS	30 dBm
Circuit power parameters, P_0	0.01 W, 0.1 W, 1 W
Circuit power constant, P_1	20 W
Constant of exponential model, a	0.01673
Constant of exponential model, b	0.1167

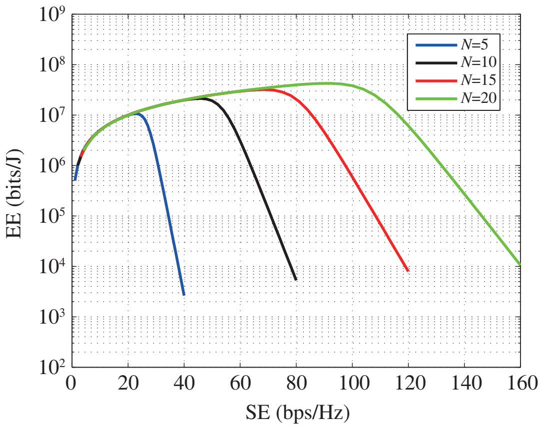

Figure 2 EE vs. SE with different number of users.

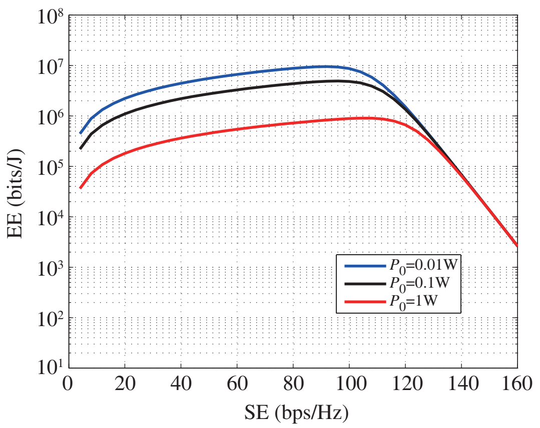

Figure 3 EE vs. SE with different values of P_0.

can say the Hessian matrix is positive semidefinite in a massive MIMO system, which means the problem (15) is a convex one, and it is easy to use iteration algorithm to solve the transmit power and the optimal EE [21].

4 Numerical results

For the numerical results, the related system parameters used in the simulation are listed in Table 1, which refer to the EARTH project [22] and 3GPP [23].

Figure 2 shows the EEs achieved by solving the problem in Section 3 versus the SE with different numbers of users communicating with BS in the massive MIMO system. It can be observed in Figure 2 that the optimal curves of the EE-SE region offer a global perspective on the EE-SE tradeoff. We can find the maximum points on the EE-SE curves related to different users, beyond which the EE cannot be increased any more, no matter how much energy is used in addition. The more users there are, the more degrees of freedom massive MIMO can provide, the more SE and optimal EE can be achieved. Using these results, we can use as less energy as we can to communicate with as more users as possible while satisfying target SE.

Figure 3 shows the EE-SE relationships with different values of P_0. We can also find that the optimal curves show the EE-SE tradeoff. It can be observed that when the circuit power increases, the optimal EE will decrease while the corresponding SE increases. What is more, we can notice that the slopes of the three curves become the same when the SE goes to infinite since it is determined by the number of

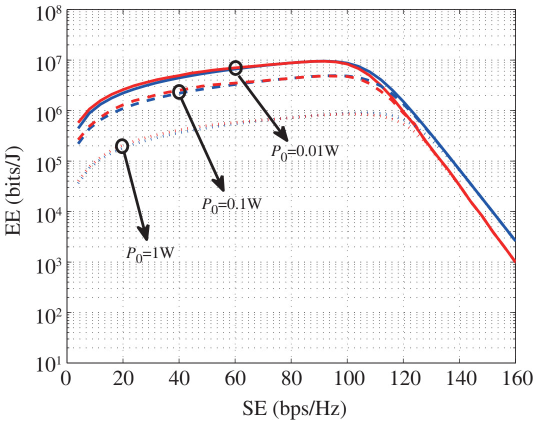

Figure 4 The impact of PA efficiency on EE.

antennas at the user side [5]. That is to say, however, the P_0 changes, the optimal curves of EE-SE will be the same as SE increases to infinite.

Furthermore, Figure 4 shows the impact of PA efficiency on the EE in our massive MIMO system. The PA efficiency in red curves is treated as a constant like previous work, e.g., 30%, while it changes with output power in blue curves. In the low SE region, the transmit power needed is usually very small, so that the PA efficiency in reality is much more less than 30%, such as Figure 1. The circuit power dominates the energy consumption while the transmit power contributes little, so that the impact of PA efficiency on the EE almost could be regardless, which in Figure 3 is that the red curves are very close to the blue ones. Although the red curves are above the blue ones, the differences between them are very little. The larger the P_0 is, the less the differences exist.

On the other hand, in the high SE region, the transmit power needed usually becomes large and dominates the energy consumption. The PA efficiency becomes larger than 30%, such as Figure 1. The blue curves go above the red ones in turn. What is more, it is obvious to find the differences between the red lines and blue ones. However, the slopes of the red curves and blue curves become the same respectively when the SE goes to infinite just as we stated previously. And in this paper, we can further draw a conclusion based on [5] that the slopes are also affected by the PA efficiency. In other words, no matter what the P_0 is, the differences between red curves and blue ones will be the same respectively as SE goes to infinite, which means that it will make big sense to consider about the impact of PA efficiency on EE in massive MIMO systems especially in high SE region. And we can also conduct a similar discussion for Figure 2 to find the impact of PA efficiency on EE in massive MIMO systems with different users.

5 Conclusion

In this paper, we have investigated the EE-SE relation in downlink massive MIMO system. Different from previous work, we consider the PA efficiency as a variable changing with output power, and talk about the impact of the PA efficiency on SE and EE in massive MIMO system. A problem to maximize the EE of system which considers the PA efficiency as a variable is proposed. We prove that the Hessian matrix of the optimization problem is positive semidefinite so that it is a convex optimization problem. Simulation results show EE-SE tradeoff and describe the difference between the two systems which consider the PA efficiency as a constant or variable. In the low SE region, the impact of PA efficiency on EE can be regardless because the transmit power is so small that even the PA efficiencies are different in the two scenarios, almost making no sense here. While in the high SE region, the differences become obvious since the transmit power dominates the energy consumption, so the impact of PA efficiency on EE of systems should be taken seriously. The slope of EE-SE curve is determined by the number of antennas at the user side and the PA efficiency. What is more, relatively more users, lower circuit power, and higher transmit power will make systems more energy efficient.

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Conflict of interest The authors declare that they have no conflict of interest.

References

1 Tse D, Viswanath P. Fundamentals of Wireless Communication. Cambridge: Cambridge University Press, 2005
2 Lau V K N, Kwok Y K. Channel-Adaptation Technologies and Cross-Layer Design for Wireless Systems with Multiple Antennas—Theory and Applications. Hoboken: John Wiley & Sons, Inc., 2005
3 Fettweis G P, Zimmermann E. ICT energy consumption—trends and challenges. In: Proceedings of 11th International Symposium on Wireless Personal Multimedia Communications, Lapland, 2008
4 Aktas D, Bacha M, Evans J, et al. Scaling results on the sum capacity of cellular networks with MIMO links. IEEE Trans Inform Theory, 2006, 52: 3264–3274
5 Mazetta T L. Noncooperative cellular wireless with unlimited numbers of base station antennas. IEEE Trans Wirel Commun, 2010, 9: 3590–3600
6 Chen Y, Zhang S, Xu S, et al. Fundamental trade-offs on green wireless networks. IEEE Commun Mag, 2011, 49: 30–37
7 Rusek F, Persson D, Lau B K, et al. Scaling up MIMO: opportunities and challenges with very large arrays. IEEE Signal Process Mag, 2013, 30: 40–60
8 Ngo H Q, Larsson E G, Marzetta T L. Energy and spectral efficiency of very large multiuser MIMO systems. IEEE Trans Commun, 2013, 61: 1436–1449
9 Xu Z, Han S, Pan Z, et al. EE-SE relationship for large-scale antenna systems. In: Proceedings of IEEE International Conference on Communications Workshops, Sydney, 2014. 38–42
10 Bjornson E, Hoydis J, Kountouris M, et al. Massive MIMO systems with non-ideal hardwares: energy efficiency, estimation, and capacity limits. IEEE Trans Inform Theory, 2014, 60: 7112–7139
11 Bjornson E, Sanguinetti L, Hoydis J, et al. Designing multi-user MIMO for energy efficiency: When is massive MIMO the answer? In: Proceedings of IEEE Wireless Communicatongs and Networking Conference, Istanbul, 2014. 242–247
12 Joung J, Ho C K, Sun S. Power amplifier switching (PAS) for energy efficient systems. IEEE Wirel Commun Lett, 2013, 2: 14–17
13 Joung J, Ho C K, Sun S. Spectral efficiency and energy efficiency of OFDM systems: impact of power amplifiers and countermeasures. IEEE J Sel Area Commun, 2014, 32: 208–220
14 Raab F H, Asbeck P, Cripps S, et al. Power amplifiers and transmitters for RF and microwave. IEEE Trans Microwave Theory, 2002, 50: 814–826
15 Li G Y, Xu Z, Xiong C, Yang C, et al. Energy-efficient wireless communications: tutorial, survey, and open issues. IEEE Wirel Commun Mag, 2011, 18: 28–35
16 Kim H S, Daneshrad B. Energy-constrained link adaptation for MIMO OFDM wireless communication systems. IEEE Trans Wirel Commun, 2010, 9: 2820–2832
17 Xu Z, Yang C, Li G Y, et al. Energy-efficient configuration of spatial and frequency resources in MIMO-OFDMA systems. IEEE Trans Commun, 2013, 61: 564–575
18 Hussaini A s, Elfergani I T E, Rodriguez J, et al. Efficient multi-stage load modulation radio frequency power amplifier for green radio frequency front end. IET Sci Meas Technol, 2012, 6: 117–124
19 Krauss H L, Bostian C W, Raab F H. Solid State Radio Engineering. Hoboken: John Wiley & Sons, Inc., 1980
20 Mohammed S K, Larsson E G. Per-antenna constant envelope precoding for large multi-user MIMO systems. IEEE Trans Commun, 2013, 61: 1059–1071
21 Boyd S P, Vandenberghe L. Convex Optimization. Cambridge: Cambridge University Press, 2004
22 Imran M A, Katranaras E, Auer G, et al. Energy efficiency analysis of the reference systems, areas of improvements and target breakdown. Technical Report INFSO-ICT-247733. 2011
23 3GPP. Coordinated multi-point operation for LTE physical layer aspects. TR 36.819 v0.0.1. 3GPP Release 11. 2011. http://www.3gpp.org/