Analysis of the Secrecy Rate

We assume \( p_j \) is the probability that \( f_j \) is selected as a signal-band while \( 1 - p_j \) for interference-band. From (1) and (4), when \( f_j \) is a signal-band, the rate to Bob with a Gaussian input can be written as

\[
R_{B,j} = \log_2(1 + \frac{P_j \mathbf{h}_j \mathbf{o}_H \mathbf{o}_j \mathbf{h}_j^H}{J\sigma_n^2}) \text{ when } f_{\text{sig}}(j) = 1. \tag{S1}
\]

Since each pair of adjacent bands have half overlap, for Eve, the signal-band is interfered by half power of the two adjacent bands. Recalling (2) and (4), when \( f_{\text{sig}}(j) = 1 \), the signal to interference and noise ratio (SINR) of Eve in the \( j \)th frequency band can be described as

\[
\gamma_{E,j} = \frac{P_A \mathbf{G}_j \mathbf{o}_H \mathbf{o}_j \mathbf{G}_j^H}{J} + \frac{P_j \mathbf{G}_j \mathbf{o}_H \mathbf{o}_j \mathbf{G}_j^H}{2J} + \frac{P_j \mathbf{G}_j \mathbf{o}_H \mathbf{o}_j \mathbf{G}_j^H}{2J} \right)^{-1} \tag{S2}
\]

The leakage rate to Eve is essentially the average mutual information between Alice and Eve. As we know, Eve does not know whether \( f_j \) is a signal-band. However, he knows the selected probability \( p_j \). Since the leakage rate is an average value, the leakage rate to Eve can be described as

\[
R_{E,j} = p_j \log_2 \left[ |\mathbf{I}_{N_x} + \gamma_{E,j}| \right] \text{ when } f_{\text{sig}}(j) = 1. \tag{S3}
\]

From (S1) - (S3), the secrecy rate (SR) of the \( j \)th sub-band can be computed as

\[
R_{s,j} = \begin{cases} 
0 & \text{ when } f_{\text{sig}}(j) = 0 \\
\log_2(1 + \frac{P_j \mathbf{h}_j \mathbf{o}_H \mathbf{o}_j \mathbf{h}_j^H}{J\sigma_n^2}) - p_j \log_2 \left[ |\mathbf{I}_{N_x} + \gamma_{E,j}| \right] & \text{ when } f_{\text{sig}}(j) = 1
\end{cases} \tag{S4}
\]

As Eve’s instantaneous CSI is unknown, we can get the SR expectation of the TFBH scheme with its distribution

\[
R_{s,\text{TFBH}}^{\text{TFBH}} = \mathbb{E} \left[ \sum_{j=1}^{N_j} f_{\text{sig}}(j) \cdot R_{s,j} \right]. \tag{S5}
\]

Then we discuss the scenario with the existence of the genie-aided Eve2 who knows everything and can do any signal processing. Since Eve2 is assumed to be able to find whether \( f_j \) is a signal-band, the leakage rate to Eve2 can be described as

\[
R_{E2,j} = \log_2 \left[ |\mathbf{I}_{N_x} + \gamma_{E,j}| \right] \text{ when } f_{\text{sig}}(j) = 1. \tag{S6}
\]

Then, similar to Eve, the secrecy rate of the \( j \)th sub-band can be computed as

\[
R_{s2,j} = \begin{cases} 
0 & \text{ when } f_{\text{sig}}(j) = 0 \\
\log_2(1 + \frac{P_j \mathbf{h}_j \mathbf{o}_H \mathbf{o}_j \mathbf{h}_j^H}{J\sigma_n^2}) - \log_2 \left[ |\mathbf{I}_{N_x} + \gamma_{E,j}| \right] & \text{ when } f_{\text{sig}}(j) = 1
\end{cases} \tag{S7}
\]

From (S6) and (S7), we can get the SR expectation of the TFBH scheme when Eve2 exists.

\[
R_{s,\text{TFBH}}^{\text{TFBH}} = \mathbb{E} \left[ \sum_{j=1}^{N_j} f_{\text{sig}}(j) \cdot R_{s2,j} \right]. \tag{S8}
\]

Simulations and Numerical Results

We evaluate the performance of the proposed TFBH scheme through some simulations and compare this with that of the traditional AN scheme. All channels of the system are generated to be zero-mean standardized complex Gaussian variables. The received additive noise is assumed to be AWGN with zero-mean and variance 0dB.

In Figure S1 (a), we illustrate the ergodic secrecy rates of the TFBH scheme when Eve exists and compare them with that of the traditional AN method, when the total transmit power of Alice \( P_A \) varies from 10dB to 28dB. We set \( N_s = 4 \) and \( N_f = 2,4,8,10 \), respectively. Bob uses 10 frequency bands for transmission, among which \( k \) bands are randomly selected as signal-bands in the TFBH method, \( \mathbb{E}\{k\}=3 \). Namely, the power efficiency is 30%. To ensure safety and make Eve do not have any prior information on which band will be selected, all the band selection strategies will be coded appropriately so that each sub-band is selected with equal probability. Because the adjacent bands intersect with each other, in the AN method, Bob can only utilizes 5 non-overlapped frequency bands for information transmission. For fair comparison we assign the same power \( P_A \) to these two schemes. Besides, in the traditional AN method, instantaneous CSI of Eve is also assumed to be unknown to Alice. As a result, we do one dimensional search over \( P_s/P_A \) for each \( P_A \) in order to maximize the ergodic secrecy rate \( R_{s,\text{AN}}^{\text{TFBH}} \) in the simulation.
From Figure S1 (a) we can find that, the increase of Eve’s antenna number $N_E$ reduces the secrecy rates of both methods. This is because, with more antennas, Eve’s capability of information interception will be increased. However, as we can see, the impact on these two schemes is different. When $N_E=2$, the secrecy rates of the two schemes both have significant increases as $P_A$ increases. However, for the AN scheme, the secrecy rate improvement becomes more and more insignificant as $N_E$ increases. Especially when $N_E=8$, which means Eve has twice antennas as Alice does, the secrecy rate stays near 0 even if the transmitter has unlimited $P_A$ for the AN scheme. This result has also been proved in [5]. It is because when Eve has enough antennas, it can gather more information and eliminate the artificial noise with CSI.

While for the TFBH scheme, the secrecy rates always have significant increases as $P_A$ increases. For any given $N_E$, the TFBH method has a bigger secrecy rate than the AN method in the same power efficiency, especially when $N_E \geq N_A$. This is because the confusion of the spectrums greatly reduces the leakage rate to Eve.

In Figure S1 (b), we demonstrate the secrecy rate of the system when the genie-aided Eve2 exists. As we have assumed, Eve2 is able to distinguish signals and interferences by some unknown methods. The total transmit power of Alice $P_A$ varies from 10dB to 34dB. From Figure S1 (b) we can see that when the SNR is low, the TFBH scheme is always inferior to the AN scheme. The reason why AN outperforms TFBH is that, the power allocation strategies of the AN method are more flexible. It is easy to decide how much power should be assigned to artificial noise based on the actual situation. However, as $P_A$ increases, the secrecy rate of the TFBH scheme increases dramatically and will outperform the AN scheme. This is because, as Alice do not know the instantaneous CSI of Eve2, the interference capability of the AN scheme may have a large uncertainty. While for the TFBH scheme, since each signal-band of Eve2 is interfered by two adjacent bands with two different artificial noises, the probability of obtaining a poor interference effect is much smaller.