SCIENCE CHINA
Information Sciences

• **RESEARCH PAPER** • January 2016, Vol. 59 010201:1–010201:13
Special Focus on Distributed Control of Nonlinear Multi-Agent Systems and Applications doi: 10.1007/s11432-015-5442-3

Leader-following adaptive consensus of multiple uncertain rigid spacecraft systems

He CAI[1,2] & Jie HUANG[1,2]*

[1] *Shenzhen Research Institute, The Chinese University of Hong Kong, Shenzhen, China;*
[2]*Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong, China*

Received April 12, 2015; accepted July 17, 2015; published online November 23, 2015

Abstract The existing results on the leader-following attitude consensus for multiple rigid spacecraft systems assume that all the parameters of the spacecraft systems are known exactly and the information flow among the followers is bidirectional. In this paper, we remove these two assumptions. First, by introducing a new Lyapunov function, we allow the communication network to be directed. Second, we convert the leader-following consensus problem into an adaptive stabilization problem of a well defined error system. Then, under the standard assumption that the state of the leader system can reach every follower through a directed path, we further show that this stabilization problem is solvable by a distributed adaptive control law. Moreover, we also present the sufficient condition for guaranteeing the convergence of the estimated parameters to the unknown actual parameters.

Keywords adaptive control, attitude consensus, multi-agent system, nonlinear distributed observer, parameter convergence.

Citation Cai H, Huang J. Leader-following adaptive consensus of multiple uncertain rigid spacecraft systems. Sci China Inf Sci, 2016, 59(1): 010201, doi: 10.1007/s11432-015-5442-3

1 Nomenclature

$\otimes$	Kronecker product
$\|\|\cdot\|\|$	Euclidean norm
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathrm{col}(\cdot)$	for $x_i \in \mathbb{R}^{n_i}$, $i = 1, \ldots, m$, $\mathrm{col}(x_1, \ldots, x_m) = (x_1^{\mathrm{T}}, \ldots, x_m^{\mathrm{T}})^{\mathrm{T}}$
$(\cdot)^{\times}$	cross product operator: for $x = \mathrm{col}(x_1, x_2, x_3) \in \mathbb{R}^3$, $x^{\times} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$

* Corresponding author (email: jhuang@mae.cuhk.edu.hk)

$L(\cdot)$	parameter linearization operator: for $x = \mathrm{col}(x_1, x_2, x_3) \in \mathbb{R}^3$, $L(x) = \begin{bmatrix} x_1 & 0 & 0 & 0 & x_3 & x_2 \\ 0 & x_2 & 0 & x_3 & 0 & x_1 \\ 0 & 0 & x_3 & x_2 & x_1 & 0 \end{bmatrix}$
1_n	n-dimensional column vector whose components are all 1
$\mathbb{Q}$	set of all quaternion: $\mathbb{Q} = \{q \mid q = \mathrm{col}(\hat{q}, \bar{q}), \hat{q} \in \mathbb{R}^3, \bar{q} \in \mathbb{R}\}$
$\odot$	quaternion product: for $q_i, q_j \in \mathbb{Q}$, $q_i \odot q_j = \begin{bmatrix} \bar{q}_i \hat{q}_j + \bar{q}_j \hat{q}_i + \hat{q}_i^{\times} \hat{q}_j \\ \bar{q}_i \bar{q}_j - \hat{q}_i^{\mathrm{T}} \hat{q}_j \end{bmatrix}$
q^*	quaternion conjugate: for $q \in \mathbb{Q}$, $q^* = \mathrm{col}(-\hat{q}, \bar{q})$
$\mathbb{Q}_u$	set of all unit quaternion: $\mathbb{Q}_u = \{q \mid q \in \mathbb{Q},\ \bar{q}^2 + \hat{q}^{\mathrm{T}} \hat{q} = 1\}$
q^{-1}	quaternion inverse: for $q \in \mathbb{Q}_u$, $q^{-1} = q^*$
$q(\cdot)$	for $x \in \mathbb{R}^3$, $q(x) : x \mapsto \mathrm{col}(x, 0) \in \mathbb{Q}$
$C(\cdot)$	for $q \in \mathbb{Q}$, $C(q) = (\bar{q}^2 - \hat{q}^{\mathrm{T}} \hat{q}) I_3 + 2\hat{q}\hat{q}^{\mathrm{T}} - 2\bar{q}\hat{q}^{\times}$
$\mathcal{I}$	inertial frame
$\mathcal{B}_i$	body frame of the ith spacecraft
$\mathcal{B}_0$	body frame of the leader
q_i	unit quaternion expression of the relative attitude of $\mathcal{B}_i$ and $\mathcal{I}$
q_0	unit quaternion expression of the relative attitude of $\mathcal{B}_0$ and $\mathcal{I}$
ω_i	angular velocity of $\mathcal{B}_i$ relative to $\mathcal{I}$, expressed in $\mathcal{B}_i$
ω_0	angular velocity of $\mathcal{B}_0$ relative to $\mathcal{I}$, expressed in $\mathcal{B}_0$
J_i	positive definite inertia matrix of the ith spacecraft, expressed in $\mathcal{B}_i$
u_i	control torque of the ith spacecraft, expressed in $\mathcal{B}_i$

2 Introduction

The formation flying of networked spacecraft systems has been extensively studied recently [1–7]. One of the key issues in spacecraft formation flying is to asymptotically align the attitude and angular velocity of all the spacecraft systems to the desired attitude and angular velocity generated by a reference system called the leader system. Such a problem is also called the leader-following consensus of multiple spacecraft systems.

Depending on whether or not the state of the leader is accessible to all the followers, there are roughly two control schemes: decentralized control and distributed control. The former one assumes that the state of the leader is available for all the followers [6], while the latter one only requires that the state of the leader can pass to each of the followers through a path [1, 2, 4]. The results in [1] and [4] have both achieved leader-following consensus for angular velocity. In [1], the consensus for attitude is achieved in a leaderless way in the sense that the attitudes of all the followers will converge to a common trajectory determined by the initial condition. In [4], the result relies on the assumption that the communication graph is of some special structure, such as a tree. More recently, the authors of this paper solved the leader-following consensus problem for multiple rigid spacecraft systems in [2] under the same assumption on the communication graph as in [1]. The result in [2] has two features. First, a marginally stable linear system is introduced to generate the desired angular velocity. This scheme enables the control law to handle a class of reference trajectories. Second, the control law achieves both attitude and angular velocity tracking. It is noted that a key technique developed in [2] is a nonlinear distributed observer for the leader system which will also play an important role in this paper.

Like all previous papers on the leader-following consensus problem of multiple spacecraft systems, the result in [2] assumed that the communication links among the followers are undirected, and all the parameters in the spacecraft system are known precisely. To make the result of [2] more practical, in this paper, we will remove these two assumptions. To remove the first assumption, we introduce a new Lyapunov function, and extend the nonlinear observer introduced in [2] to the case where the communication network is directed. To remove the second assumption, we employ an adaptive control technique to deal with the uncertain parameters in the spacecraft systems. Finally, we address the convergence issue of the estimated unknown parameters to the unknown parameters.

3 Problem formulation

Consider a group of N rigid spacecraft systems with the following motion equations:

$$\dot{q}_i = \frac{1}{2} q_i \odot q(\omega_i), \tag{1a}$$

$$J_i \dot{\omega}_i = -\omega_i^{\times} J_i \omega_i + u_i, \ i = 1, \ldots, N, \tag{1b}$$

where $q_i \in \mathbb{Q}_u$ is the unit quaternion expression of the attitude of the body frame $\mathcal{B}_i$ of the ith spacecraft relative to the inertial frame $\mathcal{I}$; $\omega_i \in \mathbb{R}^3$ is the angular velocity of $\mathcal{B}_i$ relative to $\mathcal{I}$; $J_i \in \mathbb{R}^{3\times 3}$ is the uncertain positive definite inertia matrix; $u_i \in \mathbb{R}^3$ is the control torque. ω_i, J_i and u_i are all expressed in $\mathcal{B}_i$.

Like [2, 8], we assume that the desired attitude q_0 of system (1) is generated by the following system

$$\dot{q}_0 = \frac{1}{2} q_0 \odot q(\omega_0), \tag{2}$$

where $q_0 \in \mathbb{Q}_u$ represents the attitude of the leader frame $\mathcal{B}_0$ relative to the inertial frame $\mathcal{I}$; $\omega_0 \in \mathbb{R}^3$ is the angular velocity of $\mathcal{B}_0$ relative to $\mathcal{I}$, expressed in $\mathcal{B}_0$.

By viewing the system composed of (1) and (2) as a multi-agent system of $(N+1)$ agents with (2) as the leader and the N subsystems of (1) as N followers, we can define a graph[1)] $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ with $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. Here the node 0 is associated with the leader system (2) and the node i, $i = 1, \ldots, N$, is associated with the ith subsystem of the follower system (1). For $i = 0, 1, \ldots, N$, $j = 1, \ldots, N$, $(i, j) \in \bar{\mathcal{E}}$ if and only if u_j can use the full state of agent i for control. Let $\bar{\mathcal{N}}_i$ denote the neighbor set of the node i of $\bar{\mathcal{G}}$. We can further define a subgraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of $\bar{\mathcal{G}}$ where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is obtained from $\bar{\mathcal{E}}$ by removing all the edges between node 0 and the nodes in $\mathcal{V}$.

In terms of $\bar{\mathcal{G}}$, we describe a distributed control law as follows, for $i = 1, \ldots, N$,

$$u_i = k_i(q_i, \omega_i, \varphi_i, \psi_i), \tag{3a}$$

$$\dot{\psi}_i = f_i(q_i, \omega_i, \varphi_i), \tag{3b}$$

$$\dot{\varphi}_i = g_i(\varphi_i, \varphi_j - \varphi_i, j \in \bar{\mathcal{N}}_i), \tag{3c}$$

where k_i, f_i and g_i are smooth functions, and $\varphi_0 = \mathrm{col}(q_0, \omega_0)$.

Define the attitude and angular velocity errors between systems (1) and (2) as follows:

$$\epsilon_i = q_0^{-1} \odot q_i, \tag{4a}$$

$$\hat{\omega}_i = \omega_i - C(\epsilon_i)\omega_0, \tag{4b}$$

where $\epsilon_i \in \mathbb{Q}_u$ is the unit quaternion representation of the relative attitude between $\mathcal{B}_i$ and $\mathcal{B}_0$; $C(\epsilon_i) = (\bar{\epsilon}_i^2 - \hat{\epsilon}_i^{\mathrm{T}}\hat{\epsilon}_i)I_3 + 2\hat{\epsilon}_i\hat{\epsilon}_i^{\mathrm{T}} - 2\bar{\epsilon}_i\hat{\epsilon}_i^{\times}$ is the direction cosine matrix between $\mathcal{B}_i$ and $\mathcal{B}_0$, which will transform any vector expressed in $\mathcal{B}_0$ into $\mathcal{B}_i$; $\hat{\omega}_i$ is the difference between the angular velocities ω_0 and ω_i, expressed in $\mathcal{B}_i$. Then, we have

$$\dot{\epsilon}_i = \frac{1}{2} \epsilon_i \odot q(\hat{\omega}_i), \tag{5a}$$

$$J_i \dot{\hat{\omega}}_i = -\omega_i^{\times} J_i \omega_i + J_i(\hat{\omega}_i^{\times} C(\epsilon_i)\omega_0 - C(\epsilon_i)\dot{\omega}_0) + u_i. \tag{5b}$$

1) See Appendix for a summary of graph.

Remark 1. For $i = 0, 1, \ldots, N$, $||q_i(0)|| = 1$ implies $||q_i(t)|| = 1$ for all $t \geqslant 0$. Also, by Proposition 1 of [9], $\mathcal{B}_i$ and $\mathcal{B}_0$ coincide if and only if $\hat{\epsilon}_i = 0$.

We now state our problem as follows.

Problem 1. Given systems (1), (2) and the graph $\bar{\mathcal{G}}$, design a control law of the form (3) such that, for $i = 1, \ldots, N$,

$$\lim_{t\to\infty} \hat{\epsilon}_i(t) = 0 \text{ and } \lim_{t\to\infty} \hat{\omega}_i(t) = 0,$$

for all $\omega_i(0) \in \mathbb{R}^3$ and all $q_i(0) \in \mathbb{Q}_u$.

Remark 2. The above problem includes two special cases that have been studied in the literature. First, when $N = 1$, the above problem reduces to the problem studied in [10]. Second, when all the subsystems are exactly known and the graph $\mathcal{G}$ is undirected, the above problem reduces to the problem studied by ourselves in [2]. The current problem poses two specific technical difficulties. First, in contrast with [10], we need to employ a distributed observer which leads to a much more complicated closed-loop system than the case of $N = 1$. Second, in contrast with [2], we need to employ adaptive control techniques to handle the parameter uncertainty. As a result, the approaches in [10] and [2] cannot be directly applied to analyze the stability of the closed-loop system. We have to develop a more sophisticated technique to analyze the stability of the closed-loop system. Moreover, we will also apply the recent result in [11] to address the convergence issue of the estimated unknown parameters to the actual unknown parameters.

Some assumptions are listed as follows.

Assumption 1. $\bar{\mathcal{G}}$ contains a spanning tree with the node 0 as the root.

Remark 3. Assumption 1 is a standard assumption in the leader-following consensus problem. Compared with our previous work [2], we no longer require the subgraph $\mathcal{G}$ to be undirected.

Assumption 2. The desired angular velocity $\omega_0(t)$ is generated as follows:

$$\dot{v} = Sv, \tag{6a}$$

$$\omega_0 = Wv, \tag{6b}$$

where $v \in \mathbb{R}^q$, $S \in \mathbb{R}^{q\times q}$, $W \in \mathbb{R}^{3\times q}$, and the system (6a) is marginally stable.

Remark 4. As pointed out in [2], Assumption 2 is made so that the desired angular velocity ω_0 is bounded. Under Assumption 2, the desired angular velocity is allowed to take the following form:

$$\omega_{0i} = r_i + \sum_{j=1}^{n_i} a_{ij} \sin(b_{ij}t + \phi_{ij}),\ i = 1, 2, 3, \tag{7}$$

where $\omega_0 = \mathrm{col}(\omega_{01}, \omega_{02}, \omega_{03})$, n_1, n_2, n_3 are some positive integers, $r_i \in \mathbb{R}$ are unknown constants, $a_{ij} \in \mathbb{R}$ are unknown amplitudes, $\phi_{ij} \in \mathbb{R}$ are unknown initial phases, $b_{ij} \in \mathbb{R}$ are known frequencies.

It is noted that, in contrast with [2], we have introduced an output $\omega_0 = Wv$ for the leader system so that the three components of ω_0 can be different as indicated in (7).

4 Main result

Let $\bar{\mathcal{A}} = [a_{ij}]$ be any weighted adjacency matrix of $\bar{\mathcal{G}}$. For $i = 1, \ldots, N$, let

$$\dot{\eta}_i = \frac{1}{2}\eta_i \odot q(\hat{\xi}_i) + \eta_{di}, \tag{8a}$$

$$\dot{\xi}_i = S\xi_i + \xi_{di}, \tag{8b}$$

where

$$\hat{\xi}_i = W\xi_i, \tag{9a}$$

$$\eta_{di} = \mu_1 \sum_{j=0}^{N} a_{ij}(\eta_j - \eta_i), \tag{9b}$$

$$\xi_{di} = \mu_2 \sum_{j=0}^{N} a_{ij}(\xi_j - \xi_i), \tag{9c}$$

$\eta_0 = q_0$, $\xi_0 = v$, μ_1, μ_2 are some positive real numbers, and, for $i = 1, \ldots, N$, $\eta_i \in \mathbb{Q}$, $\xi_i \in \mathbb{R}^q$.

Lemma 1. Consider (2) and (8). Under Assumptions 1 and 2, for all $\eta_i(0) \in \mathbb{Q}$, $\xi_i(0) \in \mathbb{R}^q$, any $\mu_1, \mu_2 > 0$, for $i = 1, \ldots, N$, $\eta_i(t)$ and $\xi_i(t)$ exist and are bounded for all $t \geqslant 0$ and satisfy

$$\lim_{t\to\infty}(\eta_i(t) - q_0(t)) = 0,\ \lim_{t\to\infty}(\hat{\xi}_i(t) - \omega_0(t)) = 0. \tag{10}$$

Moreover, for $i, j = 0, 1, \ldots, N$, $\int_0^\infty ||\xi_i(\tau) - \xi_j(\tau)||\mathrm{d}\tau$ and $\int_0^\infty ||\eta_i(\tau) - \eta_j(\tau)||\mathrm{d}\tau$ exist and are bounded.

See the Appendix for the detailed proof.

Remark 5. There are two differences between the system (8) and the distributed observer employed in [2]. First, by using the property of the M-matrix as shown in Lemma 4, a new Lyapunov function is designed for the nonlinear distributed observer, and hence, unlike in [2], the subgraph $\mathcal{G}$ in Lemma 1 does not have to be undirected. Second, corresponding to the output $\omega_0 = Wv$ of the leader system, the subsystem (8b) also has an output $\hat{\xi}_i = W\xi_i$. As a result, the input to the subsystem (8a) is $\hat{\xi}_i$ instead of the state ξ_i.

Now define the error signals as follows:

$$e_i = \eta_i^* \odot q_i, \tag{11a}$$

$$\bar{\omega}_i = \omega_i - C(e_i)\hat{\xi}_i + k_{i1}\hat{e}_i, \tag{11b}$$

where $k_{i1} > 0$, $e_i \in \mathbb{Q}$, $\bar{\omega}_i \in \mathbb{R}^3$, $C(e_i) = (\bar{e}_i^2 - \hat{e}_i^{\mathrm{T}}\hat{e}_i)I_3 + 2\hat{e}_i\hat{e}_i^{\mathrm{T}} - 2\bar{e}_i\hat{e}_i^{\times}$, which leads to the following error system by [2]:

$$\dot{e}_i = \frac{1}{2}e_i \odot q(\bar{\omega}_i - k_{i1}\hat{e}_i) + e_{di} \tag{12a}$$

$$J_i\dot{\bar{\omega}}_i = -\omega_i^{\times}J_i\omega_i + J_i\left((\bar{\omega}_i - k_{i1}\hat{e}_i)^{\times}C(e_i)\hat{\xi}_i - C(e_i)WS\xi_i + \frac{1}{2}k_{i1}(\hat{e}_i^{\times} + \bar{e}_iI_3)(\bar{\omega}_i - k_{i1}\hat{e}_i)\right) + \gamma_i + u_i, \tag{12b}$$

where

$$e_{di} = \frac{1}{2}(e_i^{\mathrm{T}}e_i - 1)q(\hat{\xi}_i) \odot e_i + \eta_{di}^* \odot q_i, \tag{13a}$$

$$\gamma_i = -J_i(\varsigma_i\hat{\xi}_i + C(e_i)W\xi_{di} - k_{i1}\hat{e}_{di}), \tag{13b}$$

with

$$\varsigma_i = 2\bar{e}_i\bar{e}_{di}I_3 - 2\hat{e}_i^{\mathrm{T}}\hat{e}_{di}I_3 + 2\hat{e}_{di}\hat{e}_i^{\mathrm{T}} + 2\hat{e}_i\hat{e}_{di}^{\mathrm{T}} - 2\bar{e}_{di}\hat{e}_i^{\times} - 2\bar{e}_i\hat{e}_{di}^{\times}.$$

Remark 6. Let $\bar{V}_i = e_i^{\mathrm{T}}e_i$. It is shown in [2] that e_i is bounded, $\bar{V}_i = \eta_i^{\mathrm{T}}\eta_i$ and $\lim_{t\to\infty}\bar{V}_i(t) = 1$. Then, from the following inequality:

$$||\bar{V}_i - 1|| = ||\eta_i^{\mathrm{T}}\eta_i - q_0^{\mathrm{T}}q_0|| \leqslant k||\eta_i - q_0||, \tag{14}$$

for some $k > 0$, by Lemma 1, $\int_0^\infty ||\bar{V}_i(\tau) - 1||\mathrm{d}\tau$ exists and is bounded. Therefore, by Lemma 1 and (13),

$$\lim_{t\to\infty} e_{di}(t) = 0 \text{ and } \lim_{t\to\infty}\gamma_i(t) = 0, \tag{15}$$

and $\int_0^\infty ||e_{di}(\tau)||\mathrm{d}\tau$ and $\int_0^\infty ||\gamma_i(\tau)||\mathrm{d}\tau$ exist and are bounded.

Since, by Remark 6, we have $\lim_{t\to\infty} e_{di}(t) = 0$, Lemma 4.1 of [2] still applies to (12a). Thus we have the following lemma.

Lemma 2. Consider the subsystem (12a). For $i = 1, \ldots, N$, for any piecewise continuous time function $\bar{\omega}_i(t)$ defined for $t \geqslant 0$ satisfying $\lim_{t\to\infty}\bar{\omega}_i(t) = 0$, the solution of the subsystem (12a) is bounded for all $t \geqslant 0$ and $\lim_{t\to\infty}\hat{e}_i(t) = 0$.

In order to put equation (12b) in the standard form where the unknown parameters appear linearly, we adopt the parameter linearization operator $L(\cdot)$ introduced in [10]. Let J_i be denoted by

$$J_i = \begin{bmatrix} J_{i11} & J_{i12} & J_{i13} \\ J_{i12} & J_{i22} & J_{i23} \\ J_{i13} & J_{i23} & J_{i33} \end{bmatrix}.$$

Define $\Theta_i = \mathrm{col}(J_{i11}, J_{i22}, J_{i33}, J_{i23}, J_{i13}, J_{i12})$. Then it can be easily verified that

$$J_i x = L(x)\Theta_i. \tag{16}$$

Thus, equation (12b) can be rewritten as

$$J_i \dot{\bar{\omega}}_i = \chi_i \Theta_i + \gamma_i + u_i, \tag{17}$$

where

$$\chi_i = -\omega_i^{\times} L(\omega_i) + L((\bar{\omega}_i - k_{i1}\hat{e}_i)^{\times} C(e_i)\hat{\xi}_i - C(e_i)WS\xi_i + \frac{1}{2}k_{i1}(\hat{e}_i^{\times} + \bar{e}_i I_3)(\bar{\omega}_i - k_{i1}\hat{e}_i)). \tag{18}$$

For $i = 1, \ldots, N$, let

$$\dot{\hat{\Theta}}_i = \Lambda_i^{-1}\chi_i^{\mathrm{T}}\bar{\omega}_i, \tag{19a}$$

$$u_i = -\chi_i\hat{\Theta}_i - k_{i2}\bar{\omega}_i, \tag{19b}$$

where $k_{i2} > 0$, $\Lambda_i \in \mathbb{R}^{6\times 6}$ is the positive definite gain matrix.

Now we are ready to state the main result as follows.

Theorem 1. Given systems (1), (2) and the graph $\bar{\mathcal{G}}$, under Assumptions 1 and 2, Problem 1 is solvable by the control law composed of (8) and (19).

Proof. Let $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$. Then substituting (19b) into (17) gives

$$J_i \dot{\bar{\omega}}_i = \chi_i\tilde{\Theta}_i - k_{i2}\bar{\omega}_i + \gamma_i. \tag{20}$$

Let

$$V = \frac{1}{2}\sum_{i=1}^{N}\left(\bar{\omega}_i^{\mathrm{T}} J_i \bar{\omega}_i + \tilde{\Theta}_i^{\mathrm{T}}\Lambda_i\tilde{\Theta}_i\right). \tag{21}$$

Let $0 < \varepsilon_i < 2k_{i2}$ and $\tilde{k}_{i2} = k_{i2} - \varepsilon_i/2$. Then

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N}\left(\bar{\omega}_i^{\mathrm{T}}(\chi_i\tilde{\Theta}_i - k_{i2}\bar{\omega}_i + \gamma_i) - \tilde{\Theta}_i^{\mathrm{T}}\chi_i^{\mathrm{T}}\bar{\omega}_i\right) \\ &= \sum_{i=1}^{N}\left(-k_{i2}\bar{\omega}_i^{\mathrm{T}}\bar{\omega}_i + \bar{\omega}_i^{\mathrm{T}}\gamma_i\right) \\ &\leqslant \sum_{i=1}^{N}\left(-k_{i2}\bar{\omega}_i^{\mathrm{T}}\bar{\omega}_i + \frac{\varepsilon_i}{2}\bar{\omega}_i^{\mathrm{T}}\bar{\omega}_i + \frac{1}{2\varepsilon_i}||\gamma_i||^2\right) \\ &= -\varOmega(t) + \zeta(t), \end{aligned} \tag{22}$$

where

$$\varOmega(t) = \sum_{i=1}^{N}\tilde{k}_{i2}\bar{\omega}_i^{\mathrm{T}}\bar{\omega}_i,\ \zeta(t) = \sum_{i=1}^{N}\frac{1}{2\varepsilon_i}||\gamma_i||^2. \tag{23}$$

By Remark 6, $\int_0^\infty \zeta(\tau)\mathrm{d}\tau$ exists and is bounded. Since $\varOmega(t) \geqslant 0$ for all $t \geqslant 0$, we have

$$\dot{V} \leqslant \zeta(t). \tag{24}$$

Therefore

$$\int_0^\infty \dot{V}(\tau)\mathrm{d}\tau \leqslant \int_0^\infty \zeta(\tau)\mathrm{d}\tau. \tag{25}$$

Since $\int_0^\infty \zeta(\tau)\mathrm{d}\tau$ is bounded, V is bounded, which implies $\bar{\omega}_i$ and $\tilde{\Theta}_i$ are bounded. Since e_i, ξ_i and $\hat{\xi}_i$ are bounded, by (11b) and (18), ω_i and hence χ_i are bounded. Therefore, $\dot{\bar{\omega}}_i$ is bounded by (20). By (23), $\dot{\Omega}(t)$ is bounded, which implies $\Omega(t)$ is uniformly continuous.

We now claim

$$\lim_{t\to\infty} \Omega(t) = 0. \tag{26}$$

Otherwise, since $\Omega(t) \geqslant 0$ for all $t \geqslant 0$, there exists $\varepsilon > 0$ and a sequence $\{t_k\}$, $k = 1, 2, \dots$ satisfying $t_k \to \infty$ as $k \to \infty$ such that $\Omega(t_k) > \varepsilon$. Since $\Omega(t)$ is uniformly continuous, there exists $\delta > 0$, such that $|\Omega(t_k) - \Omega(t)| < \varepsilon/2$ whenever $|t_k - t| < \delta$. Without loss of generality, we can always choose $\{t_k\}$ such that $t_1 > \delta$ and $t_{k+1} - t_k > 2\delta$. Therefore, $\Omega(t) > \varepsilon/2$ whenever $|t_k - t| < \delta$. Therefore,

$$\int_0^\infty \Omega(\tau)\mathrm{d}\tau \geqslant \sum_{k=1}^{\infty} \int_{t_k-\delta}^{t_k+\delta} \Omega(\tau)\mathrm{d}\tau = +\infty. \tag{27}$$

Since $\int_0^\infty \zeta(\tau)\mathrm{d}\tau$ is bounded,

$$\int_0^\infty \dot{V}(\tau)\mathrm{d}\tau = V(\infty) - V(0) = -\infty, \tag{28}$$

which contradicts the fact that $V(t) \geqslant 0$ for all $t \geqslant 0$. Thus we have proved our claim, which implies

$$\lim_{t\to\infty} \bar{\omega}_i(t) = 0. \tag{29}$$

Then, by Lemma 2,

$$\lim_{t\to\infty} \hat{e}_i(t) = 0. \tag{30}$$

Since by Remark 6 $\lim_{t\to\infty} \bar{V}_i(t) = 1$, we have $\lim_{t\to\infty} |\bar{e}_i(t)| = 1$, and therefore

$$\lim_{t\to\infty} C(e_i(t)) = I_3. \tag{31}$$

By Lemma 1,

$$\lim_{t\to\infty} (\eta_i(t) - q_0(t)) = 0, \tag{32}$$

and therefore by (30), $\lim_{t\to\infty} \hat{\epsilon}_i(t) = 0$, and hence

$$\lim_{t\to\infty} C(\epsilon_i(t)) = I_3. \tag{33}$$

By (11b), we have

$$\lim_{t\to\infty} (\omega_i(t) - C(e_i(t))\hat{\xi}_i(t)) = 0. \tag{34}$$

By Lemma 1,

$$\lim_{t\to\infty} (\hat{\xi}_i(t) - \omega_0(t)) = 0. \tag{35}$$

Therefore, by (31), (33) and the following identity,

$$\hat{\omega}_i = \omega_i - C(\epsilon_i)\omega_0 = \omega_i - C(e_i)\hat{\xi}_i + C(e_i)\hat{\xi}_i - C(\epsilon_i)\omega_0, \tag{36}$$

we have

$$\lim_{t\to\infty} \hat{\omega}_i(t) = 0.$$

5 Parameter convergence

It is well known that an adaptive control law that solves the adaptive control problem does not guarantee the convergence of the estimated unknown parameters to the actual values of the unknown parameters [11, 12]. Specifically, the assumptions for Theorem 1 may not guarantee the convergence of the quantity $\tilde{\Theta}_i$ to the origin as t tends to infinity. In this section, we will further consider the conditions under which $\lim_{t\to\infty}\tilde{\Theta}_i(t) = 0$. For this purpose, we introduce the concept of persistent excitation [11].

Definition 1. A piecewise continuous time function $f(t) : [0, \infty) \to \mathbb{R}^n$ is a persistent excitation (PE) if there exist constants $t_0, T_0, \epsilon > 0$, such that

$$\frac{1}{T_0}\int_t^{t+T_0} |c^{\mathrm{T}} f(\tau)| \mathrm{d}\tau \geqslant \epsilon, \tag{37}$$

for all $t \geqslant t_0$ and for any unit vector $c \in \mathbb{R}^n$.

Our analysis will rely on Lemma 4.1 of [11]. For convenience, we rephrase Lemma 4.1 of [11] as follows.

Lemma 3. Let $g : [0, \infty) \to \mathbb{R}^n$ be a continuously differentiable function and $f : [0, \infty) \to \mathbb{R}^n$ be a bounded piecewise continuous function. If f is a PE and

$$\lim_{t\to\infty} \dot{g}(t) = 0, \tag{38a}$$

$$\lim_{t\to\infty} g(t)^{\mathrm{T}} f(t) = 0, \tag{38b}$$

then

$$\lim_{t\to\infty} g(t) = 0. \tag{39}$$

Theorem 2. Let $\rho = \omega_0^{\mathrm{T}} L(\dot{\omega}_0)$. If $\rho(t)$ is a PE. Then

$$\lim_{t\to\infty} \tilde{\Theta}_i(t) = 0, \quad i = 1, \ldots, N. \tag{40}$$

Proof. By (20), we have

$$J_i\ddot{\bar{\omega}}_i = \dot{\chi}_i\tilde{\Theta}_i + \chi_i\dot{\tilde{\Theta}}_i - k_{i2}\dot{\bar{\omega}}_i + \dot{\gamma}_i. \tag{41}$$

From Section 4, we know that $\dot{e}_i$ and $\dot{\bar{\omega}}_i$ are bounded. By (11b), $\dot{\omega}_i$ is bounded. Then by (18), $\dot{\chi}_i$ is bounded. By (19a), $\dot{\tilde{\Theta}}_i$ is bounded. By (13b), $\dot{\gamma}_i$ is bounded. Therefore, $\ddot{\bar{\omega}}_i$ is bounded. By (29) and Barbalat's Lemma,

$$\lim_{t\to\infty} \dot{\bar{\omega}}_i(t) = 0. \tag{42}$$

Then again by (20), we have

$$\lim_{t\to\infty} \chi_i(t)\tilde{\Theta}_i(t) = 0. \tag{43}$$

Let

$$\varrho(t) = \omega_0^{\times} L(\omega_0) + L(\dot{\omega}_0). \tag{44}$$

By (29), (30) and (31), for $i = 1, \ldots, N$, we have

$$\lim_{t\to\infty} (\chi_i(t) + \varrho(t)) = 0. \tag{45}$$

Thus, for $i = 1, \ldots, N$,

$$\lim_{t\to\infty} \varrho(t)\tilde{\Theta}_i(t) = 0. \tag{46}$$

By (46), since ω_0 is bounded, we have

$$\lim_{t\to\infty} \omega_0^{\mathrm{T}}(t)\varrho(t)\tilde{\Theta}_i(t) = 0. \tag{47}$$

Since

$$\omega_0^{\mathrm{T}}\varrho = \omega_0^{\mathrm{T}}(\omega_0^{\times} L(\omega_0) + L(\dot{\omega}_0)) = \omega_0^{\mathrm{T}} L(\dot{\omega}_0) = \rho, \tag{48}$$

we have,

$$\lim_{t\to\infty} \rho(t)\tilde{\Theta}_i(t) = 0. \tag{49}$$

By (19a), we know that

$$\lim_{t\to\infty} \dot{\tilde{\Theta}}_i(t) = 0. \tag{50}$$

Therefore, by Lemma 3, if $\rho(t)$ is a PE, then

$$\lim_{t\to\infty} \tilde{\Theta}_i(t) = 0, \tag{51}$$

for $i = 1, \ldots, N$.

Remark 7. For the attitude tracking problem of a single uncertain spacecraft system, Theorem 2 of [10] gave a result on determining the parameter convergence based on the invariant set principle for a periodic system (Theorem 4 of [13]). However, Theorem 2 of [10] is not applicable to our case since, due to the presence of the nonzero perturbation term $\gamma_i(t)$ in (22), $\dot{V} = 0$ does not imply $\bar{\omega}_i = 0$ any more. Therefore, we need to provide an independent analysis here. It is also worth noting that, unlike in [10], our approach does not require ω_0 to be periodic, and it suffices to require ρ to be a PE.

6 Simulation

Consider four follower systems whose motion equations are described by (1), with the following parameters

$$\Theta_1 = \mathrm{col}\begin{pmatrix} 1.2 & 3.5 & 4.7 & 0 & 0 & 0 \end{pmatrix}, \ \Theta_2 = \mathrm{col}\begin{pmatrix} 1.3 & 3.4 & 5.2 & 0 & 0 & 0 \end{pmatrix},$$
$$\Theta_3 = \mathrm{col}\begin{pmatrix} 1.9 & 2.1 & 3.5 & 0 & 0 & 0 \end{pmatrix}, \ \Theta_4 = \mathrm{col}\begin{pmatrix} 2.1 & 5.1 & 7.1 & 0 & 0 & 0 \end{pmatrix}.$$

Let the desired angular velocity ω_0 be given by (6) with

$$S = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & -8 & 0 \end{pmatrix}, \ W = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Then, by letting

$$v(0) = \mathrm{col}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix},$$

we have

$$\omega_0(t) = \mathrm{col}\begin{pmatrix} 1 + \sin 2t & 2 + \sin 4t & 3 + \sin 8t \end{pmatrix}.$$

Therefore,

$$\dot{\omega}_0(t) = \mathrm{col}\begin{pmatrix} 2\cos 2t & 4\cos 4t & 8\cos 8t \end{pmatrix}.$$

Let $\rho(t) = (\ \rho_1(t)\ \rho_2(t)\ \rho_3(t)\ \rho_4(t)\ \rho_5(t)\ \rho_6(t)\ )$. Then we have

$$\rho_1(t) = 2\cos 2t + \sin 4t, \ \rho_4(t) = 16\cos 8t + 6\sin 6t - 2\sin 2t + 12\cos 4t,$$
$$\rho_2(t) = 8\cos 4t + 2\sin 8t, \ \rho_5(t) = 8\cos 8t + 5\sin 5t - 3\sin 3t + 6\cos 2t,$$
$$\rho_3(t) = 24\cos 8t + 4\sin 16t, \ \rho_6(t) = 4\cos 4t + 3\sin 3t - \sin t + 4\cos 2t.$$

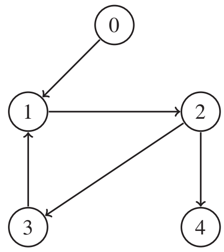

Figure 1 The network topology $\bar{\mathcal{G}}$.

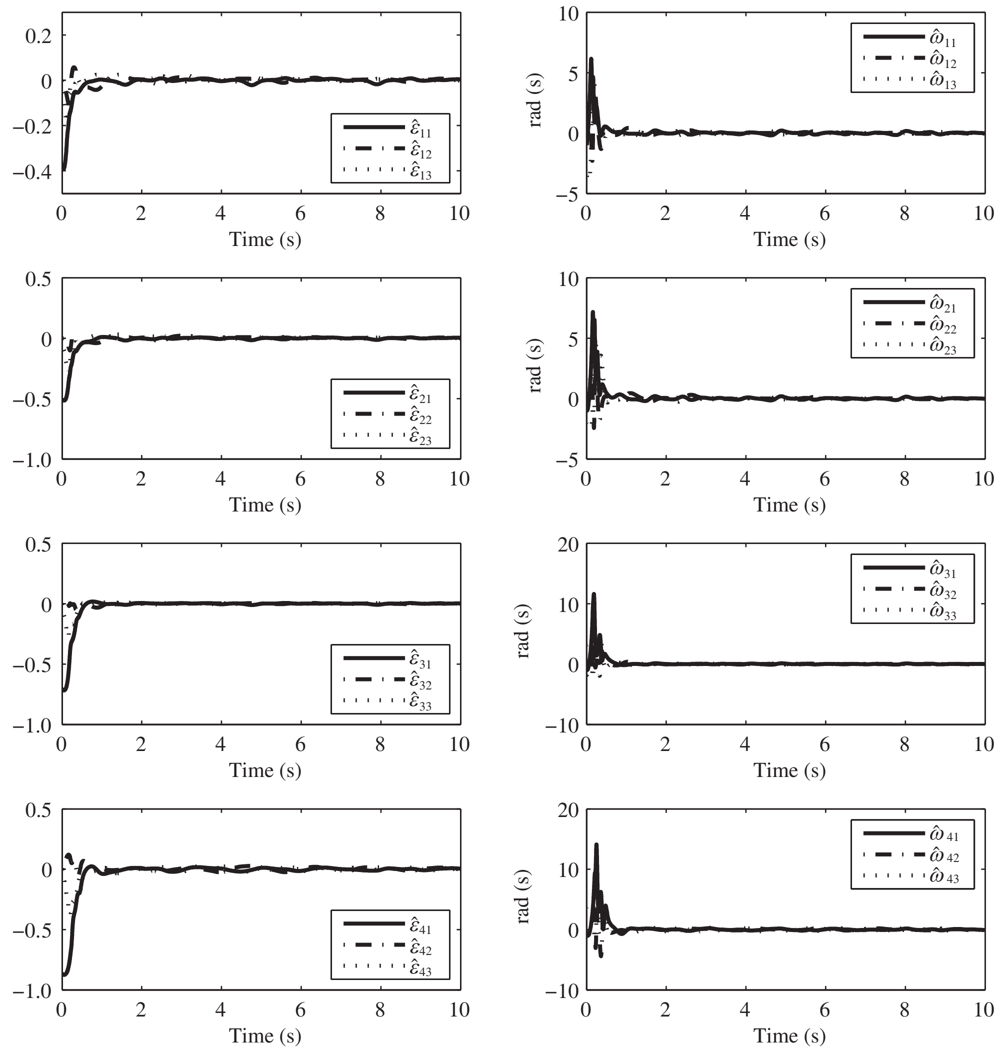

Figure 2 Tracking performance of attitude and angular velocity.

Note that $\rho(t)$ is a smooth periodic function with the period $T_0 = 2\pi$ and $\rho_i(t)$, $i = 1, \ldots, 6$, are linear independent in $\mathbb{R}^6$. Therefore, for any given unit vector $c \in \mathbb{R}^6$, $\rho(t)c$ is a nonzero smooth periodic function with the period $T_0 = 2\pi$. Hence, there exists an $\epsilon > 0$ such that

$$\frac{1}{T_0}\int_t^{t+T_0} |\rho(\tau)c|\mathrm{d}\tau = \epsilon, \tag{52}$$

for all $t \geqslant 0$. Since the value of c is limited to the compact set $\{x \in \mathbb{R}^6, ||x|| = 1\}$, there must be a minimum $\epsilon^* > 0$ such that for any unit vector $c \in \mathbb{R}^6$,

$$\frac{1}{T_0}\int_t^{t+T_0} |\rho(\tau)c|\mathrm{d}\tau \geqslant \epsilon^*, \tag{53}$$

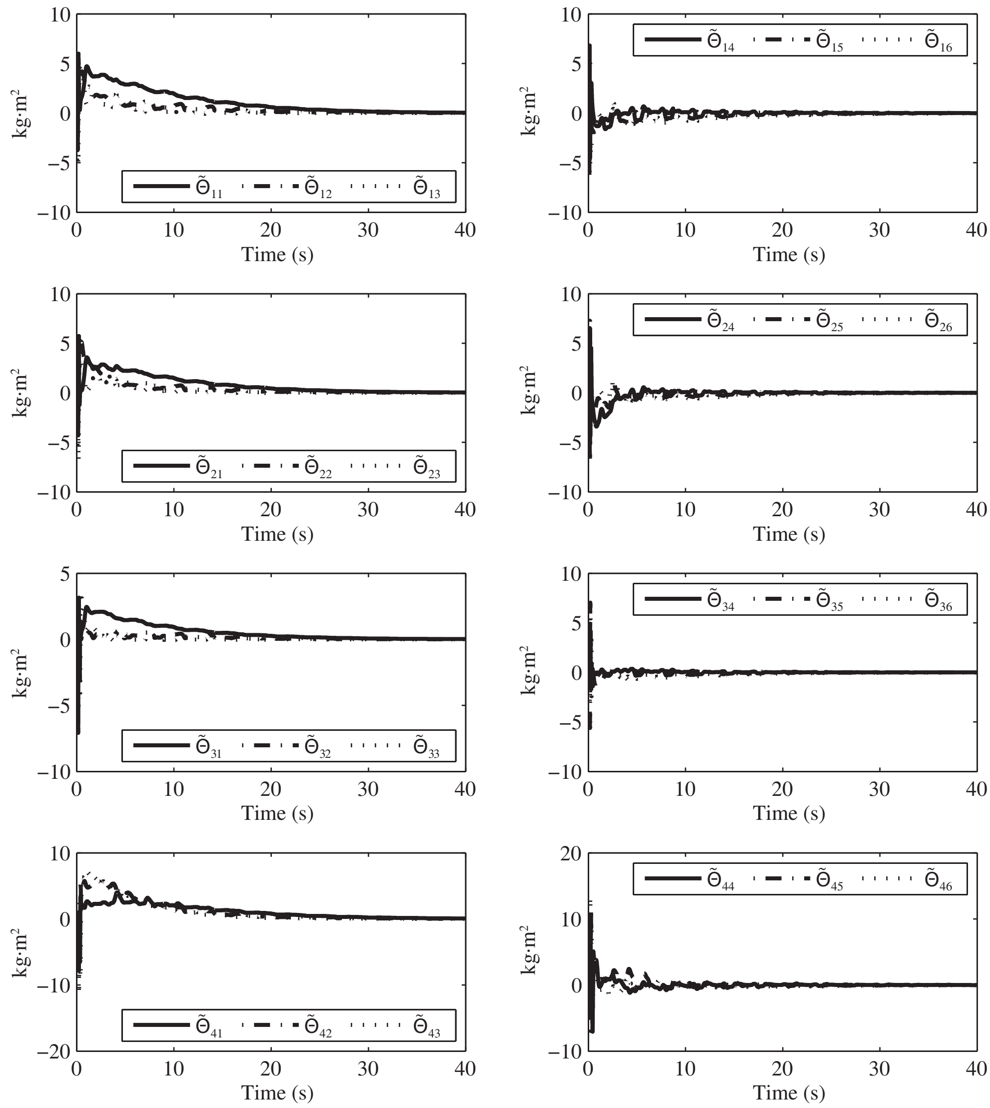

Figure 3 Parameter convergence.

for all $t \geqslant 0$. Therefore, by Definition 1, $\rho(t)$ is a PE. Then according to Theorem 2, the estimated parameters $\hat{\Theta}_i$ will converge to their actual values Θ_i.

$\bar{\mathcal{G}}$ is shown in Figure 1 which satisfies Assumption 1. Thus we can design a control law of the form of (8) and (19) with the following design parameters: $\mu_1 = 20$, $\mu_2 = 20$, $k_{i1} = 20$, $k_{i2} = 20$, $\Lambda_i = I_6$. Let $a_{ij} = 1$ whenever $(j, i) \in \bar{\mathcal{E}}$. The performance of the control law is simulated with the following initial conditions $q_0(0) = \mathrm{col}(\,0\ 0\ 0\ 1\,)$, $q_1(0) = \mathrm{col}(\,-\sin(\pi/8)\ 0\ 0\ \cos(\pi/8)\,)$, $q_2(0) = \mathrm{col}(\,-\sin(\pi/6)\ 0\ 0\ \cos(\pi/6)\,)$, $q_3(0) = \mathrm{col}(\,-\sin(\pi/4)\ 0\ 0\ \cos(\pi/4)\,)$, $q_4(0) = \mathrm{col}(\,-\sin(\pi/3)\ 0\ 0\ \cos(\pi/3)\,)$, and $\omega_i(0) = 0$, $\xi_i(0) = 0$, $\eta_i(0) = 0$, $\hat{\Theta}_i(0) = 0$.

Figures 2 shows the tracking performance of the attitude and angular velocity of each coordinate of q_i and ω_i, respectively, for $i = 1, 2, 3, 4$. It can be observed that the tracking performance is satisfactory. Figure 3 shows the result of the parameter convergence.

7 Conclusion

In this paper, we have presented a result on the leader-following consensus problem for a multiple uncertain rigid spacecraft system. Compared with our previous result in [2], by employing the adaptive control technique and making use of a new Lyapunov function, we allow the parameters of the spacecraft system to be unknown and allow the communication among the followers to be directed. Moreover, we have thoroughly addressed the condition for the estimated unknown parameters to converge to the true values of the unknown parameters. Our future work will consider the case where the spacecraft systems are also subject to external disturbances.

Acknowledgements This work has been supported in part by National Natural Science Foundation of China (Grant No. 61174049) and Research Grants Council of the Hong Kong Special Administration Region (Grant No. 412813).

Conflict of interest The authors declare that they have no conflict of interest.

References

1 Bai H, Arcak M, Wen J T. Rigid body attitude coordination without inertial frame information. Automatica, 2008, 44: 3170–3175
2 Cai H, Huang J. The leader-following attitude control of multiple rigid spacecraft systems. Automatica, 2014, 50: 1109–1115
3 Dimarogonas D V, Tsiotras P, Kyriakopoulos K J. Leader-follower cooperative attitude control of multiple rigid bodies. Syst Control Lett, 2009, 58: 429–435
4 Ren W. Formation keeping and attitude alignment for multiple spacecraft through local interactions. J Guid Control Dyn, 2007, 30: 633–638
5 Scharf D P, Hadaegh F Y, Ploen S R. A survey of spacecraft formation flying guidance and control (part II): control. In: Proceedings of the American Control Conference, Boston, 2004. 2976–2985
6 VanDyke M C, Hall C D. Decentralized coordinated attitude control within a formation of spacecraft. J Guid Control Dyn, 2006, 29: 1101–1109
7 Wang N, Zhang T W, Xu J Q. Formation control for networked spacecraft in deep space: with or without communication delays and with switching topology. Sci China Inf Sci, 2011, 54: 469–481
8 Chen Z, Huang J. Attitude tracking and disturbance rejection of rigid spacecraft by adaptive control. IEEE Trans Autom Control, 2009, 54: 600–605
9 Yuan J S C. Closed-loop manipulator control using quaternion feedback. IEEE J Robotic Autom, 1988, 4: 434–440
10 Ahmed J, Coppola V T, Bernstein D. Adaptive asymptotic tracking of spacecraft attitude motion with inertia matrix identification. J Guid Control Dyn, 1998, 21: 684–691
11 Liu L, Chen Z, Huang J. Parameter convergence and minimal internal model with an adaptive output regulation problem. Automatica, 2009, 45: 1306–1311
12 Boyd S, Sastry S. On parameter convergence in adaptive control. Syst Control Lett, 1983, 3: 311–319
13 Lasalle J P. Asymptotic stability criteria. In: Proceedings of Symposia in Applied Mathematics, Providence, 1962. 13: 299–307
14 Godsil C, Royal G. Algebraic Graph Theory. New York: Springer-Verlag, 2001. 163–164, 279–281
15 Horn R, Johnson C. Topics in Matrix Analysis. Cambridge: Cambridge University Press, 1991. 113–114
16 Su Y, Huang J. Cooperative output regulation of linear multi-agent systems. IEEE Trans Autom Control, 2012, 57: 1062–1066
17 Lewis F L, Jagannathan S, Yesildirek A. Neural Network Control of Robot Manipulators and Nonlinear Systems. Philadelphia: Taylor and Francis Inc, 1998. 112
18 Khalil H K. Nonlinear Systems. 3rd ed. Upper Saddle River: Prentice Hall, 2002. 102–103

Appendix A Graph

Some notation from graph theory is summarized here [14]. A digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a finite set of nodes $\mathcal{V} = \{1, \ldots, N\}$ and an edge set $\mathcal{E} = \{(i, j), i, j \in \mathcal{V}, i \neq j\}$. An edge from node i to node j is denoted by (i, j), and node i is called the neighbor of node j. If the digraph $\mathcal{G}$ contains a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})$, then the set $\{(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})\}$ is called a path of $\mathcal{G}$ from i_1 to i_{k+1}, and node i_{k+1} is said to be reachable from node i_1. A graph is said to contain a spanning tree if there exists a node i such that any other node is reachable from node i. The node i is called the root of the spanning tree. The edge (i, j) is called undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. The graph is called undirected if every edge in $\mathcal{E}$ is undirected. A graph $\mathcal{G}_s = (\mathcal{V}_s, \mathcal{E}_s)$ is called a subgraph of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ if $\mathcal{V}_s \subseteq \mathcal{V}$ and $\mathcal{E}_s \subseteq \mathcal{E} \cap (\mathcal{V}_s \times \mathcal{V}_s)$. The weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ is defined as $a_{ii} = 0$; for $i \neq j$, $a_{ij} > 0 \Leftrightarrow (j, i) \in \mathcal{E}$ and $a_{ij} = a_{ji}$ if (i, j) is an undirected edge of $\mathcal{E}$. The Laplacian of $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$, $l_{ij} = -a_{ij}$ for $i \neq j$.

Appendix B Proof of Lemma 1

To prove Lemma 1, we need the following Lemma.

Lemma 4. (Theorem 2.5.3 of [15]) A matrix $A \in \mathbb{R}^{N\times N}$ is called an M-matrix if $a_{ij} \leqslant 0$ for $i \neq j$, $i, j = 1, \ldots, N$ and all the eigenvalues of A have positive real parts. Then, A is an M-matrix if and only if there is a positive definite diagonal matrix $D = \mathrm{diag}\{d_1, \ldots, d_N\}$ such that $DA + A^{\mathrm{T}}D$ is positive definite.

Now, we are ready to prove Lemma 1 as follows.

Proof. Consider the system composed of (6) and (8b). Let $\mathcal{L}$ be the Laplacian of $\mathcal{G}$. Let $\xi = \mathrm{col}(\xi_1, \ldots, \xi_N)$ and $\omega = \xi - 1_N \otimes v$. Then we have

$$\dot{\omega} = ((I_N \otimes S) - \mu_2(H \otimes I_q))\omega, \tag{B1}$$

where $H = \mathcal{L} + \mathrm{diag}\{a_{10}, \ldots, a_{N0}\}$. By Lemma 1 of [16], under Assumption 1, all the eigenvalues of H have positive real parts and therefore H is an M-matrix. By Remark 4 of [16], for any $\mu_2 > 0$, for $i = 1, \ldots, N$,

$$\lim_{t\to\infty} (\xi_i(t) - v(t)) = 0,$$

exponentially. It follows from the following inequality

$$||\xi_i(\tau) - \xi_j(\tau)|| \leqslant ||\xi_i(\tau) - v(\tau)|| + ||\xi_j(\tau) - v(\tau)||,$$

that $\int_0^\infty ||\xi_i(\tau) - \xi_j(\tau)||\mathrm{d}\tau$ also exists and is bounded for $i, j = 0, 1, \ldots, N$. By (6b), we have

$$\lim_{t\to\infty} (\hat{\xi}_i(t) - \omega_0(t)) = \lim_{t\to\infty} W(\xi_i(t) - v(t)) = 0, \tag{B2}$$

exponentially. Hence, $\int_0^\infty ||\hat{\xi}_i(\tau) - \omega_0(\tau)||\mathrm{d}\tau$ exists and is bounded. Under Assumption 2, v is bounded, so are ω_0, ξ_i and $\hat{\xi}_i$.

Since H is an M-matrix, by Lemma 4, there is a positive definite diagonal matrix $D = \mathrm{diag}\{d_1, \ldots, d_N\}$ such that $\bar{H} = DH + H^{\mathrm{T}}D$ is positive definite. Let $\eta = \mathrm{col}(\eta_1, \ldots, \eta_N)$, $x = \eta - 1_N \otimes q_0$ and

$$\bar{V} = x^{\mathrm{T}}(D \otimes I_4)x. \tag{B3}$$

Then,

$$\begin{aligned}
\dot{\bar{V}} &= \sum_{i=1}^N d_i[\hat{\eta}_i^{\mathrm{T}} - \hat{q}_0^{\mathrm{T}}, \bar{\eta}_i - \bar{q}_0] \begin{bmatrix} \hat{\eta}_i^\times \hat{\xi}_i + \bar{\eta}_i\hat{\xi}_i + \sum_{j=0}^N 2a_{ij}\mu_1(\hat{\eta}_j - \hat{\eta}_i) - \hat{q}_0^\times\omega_0 - \bar{q}_0\omega_0 \\ -\hat{\eta}_i^{\mathrm{T}}\hat{\xi}_i + \sum_{j=0}^N 2a_{ij}\mu_1(\bar{\eta}_j - \bar{\eta}_i) + \hat{q}_0^{\mathrm{T}}\omega_0 \end{bmatrix} \\
&= \sum_{i=1}^N\sum_{j=0}^N 2d_i a_{ij}\mu_1[\eta_i^{\mathrm{T}} - q_0^{\mathrm{T}}][\eta_j - \eta_i] \\
&\quad + \sum_{i=1}^N d_i\left(-\hat{q}_0^{\mathrm{T}}\hat{\eta}_i^\times\hat{\xi}_i + \bar{\eta}_i\hat{\eta}_i^{\mathrm{T}}\hat{\xi}_i - \bar{\eta}_i\hat{q}_0^{\mathrm{T}}\hat{\xi}_i - \hat{\eta}_i^{\mathrm{T}}\hat{q}_0^\times\omega_0 - \bar{q}_0\hat{\eta}_i^{\mathrm{T}}\omega_0 + \bar{q}_0\hat{q}_0^{\mathrm{T}}\omega_0 - \bar{\eta}_i\hat{\eta}_i^{\mathrm{T}}\hat{\xi}_i + \bar{q}_0\hat{\eta}_i^{\mathrm{T}}\hat{\xi}_i + \bar{\eta}_i\hat{q}_0^{\mathrm{T}}\omega_0 - \bar{q}_0\hat{q}_0^{\mathrm{T}}\omega_0\right) \\
&= \sum_{i=1}^N\sum_{j=0}^N 2d_i a_{ij}\mu_1[\eta_i^{\mathrm{T}} - q_0^{\mathrm{T}}][\eta_j - \eta_i] + \sum_{i=1}^N d_i\left(\hat{q}_0^{\mathrm{T}}(\hat{\xi}_i^\times - \omega_0^\times)\hat{\eta}_i + (\bar{q}_0\hat{\eta}_i^{\mathrm{T}} - \bar{\eta}_i\hat{q}_0^{\mathrm{T}})(\hat{\xi}_i - \omega_0)\right) \\
&= -x^{\mathrm{T}}(2\mu_1 DH \otimes I_4)x + \phi(t) \\
&= -x^{\mathrm{T}}(\mu_1\bar{H} \otimes I_4)x + \phi(t),
\end{aligned} \tag{B4}$$

where

$$\phi(t) = \sum_{i=1}^N d_i\left(\hat{q}_0^{\mathrm{T}}(\hat{\xi}_i^\times - \omega_0^\times)\hat{\eta}_i + (\bar{q}_0\hat{\eta}_i^{\mathrm{T}} - \bar{\eta}_i\hat{q}_0^{\mathrm{T}})(\hat{\xi}_i - \omega_0)\right).$$

By (B2), the rest of the proof is similar to the proof of Lemma 3.1 of [2]. By invoking Theorem 2.5.7 of [17], we can conclude that η_i is bounded and therefore

$$\lim_{t\to\infty} \phi(t) = 0, \tag{B5}$$

exponentially. Next, by (B3) and (B4), there exists $a > 0$ such that

$$\dot{\bar{V}} \leqslant -a\bar{V} + \phi(t). \tag{B6}$$

Since $\lim_{t\to\infty}\phi(t) = 0$ exponentially, and $\bar{V}(t) \geqslant 0$, by the comparison lemma as can be found in, say, Lemma 3.4 of [18], we can conclude that

$$\lim_{t\to\infty} \bar{V}(t) = 0, \tag{B7}$$

exponentially, which in turn implies that

$$\lim_{t\to\infty} (\eta_i(t) - q_0(t)) = 0, \tag{B8}$$

exponentially and hence $\int_0^\infty ||\eta_i(\tau) - q_0(\tau)||\mathrm{d}\tau$ exists and is bounded. It follows from the following inequality

$$||\eta_i(\tau) - \eta_j(\tau)|| \leqslant ||\eta_i(\tau) - q_0(\tau)|| + ||\eta_j(\tau) - q_0(\tau)||,$$

that, for $i, j = 0, 1, \ldots, N$, $\int_0^\infty ||\eta_i(\tau) - \eta_j(\tau)||\mathrm{d}\tau$ also exists and is bounded.