



自动驾驶车辆道路跟驰与状态一致性控制

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摘要 针对含有未知输入和外部干扰的非线性自动驾驶车辆时变跟驰队列系统, 研究系统部分状态可测情况下的车辆道路跟驰和状态一致性控制问题. 基于车辆跟驰和二自由度动力学模型, 得到含有外部干扰和未知输入的离散化状态方程, 利用前导跟驰特性, 得到自动驾驶车辆跟驰队列系统; 利用比例积分状态观测器解决系统部分状态不可测问题, 提出一种基于观测器实现含有未知输入和外部干扰的自动驾驶车辆状态一致性控制协议; 将观测器估计效果和一致性控制问题转化为误差系统的稳定性问题, 由此构造 Lyapunov-Krasovkii 函数, 利用离散系统稳定性理论推导出一个充分条件; 通过求解线性矩阵不等式 (linear matrix inequality, LMI) 得到跟驰系统的增益矩阵和参数矩阵, 利用 H_∞ 性能指标分析系统鲁棒性. 仿真结果表明: 所设计观测器能够估计未知输入、外部干扰和系统状态, 并且基于观测器设计能够使自动驾驶车辆道路跟驰和状态达到一致.

关键词 智慧交通, 自动驾驶车辆, 道路跟驰, 多智能体系统, 一致性控制, 外部干扰

1 引言

近年来, 在智慧交通概念提出后, 相关政策频繁发布, 其作为实现智慧城市交通智能化管理的必要手段, 能够有效解决城市交通堵塞、环境污染等问题, 受到国家、研究人员和行业的高度重视^[1~4]. 多智能体系统因具备低成本、分布式、协调性、鲁棒性强等优势, 成为智慧交通研究中重要的技术手段, 具有巨大的应用前景. 多智能体系统的一致性作为协调控制的基础问题, 在智慧城市和智慧交通中也有着广泛应用, 如自动驾驶车辆^[5~7].

在智慧交通中, 针对自动驾驶车辆跟驰队列系统的一致性控制问题, 现阶段研究主要围绕自动驾驶车辆队列系统控制、路径规划和道路跟驰控制等方面展开. 文献 [8] 针对自动驾驶车辆在不确定性

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影响下的车辆跟踪控制问题, 提出基于 LMI 的滑模控制器, 实现系统对非匹配不确定性因素具有完全不变性. 文献 [9] 针对复杂交通环境下的智能车辆避撞问题, 提出了基于模型预测控制的一体式车辆避撞轨迹规划和跟踪控制方法, 实现了复杂环境下智能车辆避撞过程车辆姿态正确、车辆轨迹和控制动作平滑. 文献 [10] 考虑智能车辆高速情况下的动态跟踪问题, 提出基于实时更新、绝对动态安全距离的速度差控制数学模型, 为高速情况下的车辆跟踪控制提供更加安全、高效的方法. 文献 [11] 针对自动驾驶车辆车道保持问题, 将仿人智能控制 (human-simulated intelligent control, HSIC) 概念引入车辆横向控制, 提出了基于 HSIC 的自动驾驶车辆车道保持方法, 实现了曲线跟踪的前馈控制和间歇性纠错的“行为与等待”控制. 文献 [12] 针对智能网联车道路跟驰和通信时延问题, 提出分布式非线性道路跟驰控制器, 实现了对智能网联车的道路跟驰控制.

未知输入和外界干扰在系统中普遍存在, 文献 [13] 研究带有结构不确定干扰的网联车辆, 提出基于鲁棒对等变换的模型预测编队控制方法, 实现了网联车辆的编队控制问题并保障了车辆的编队安全性. 文献 [14] 研究了存在有界扰动情况下的路径跟踪问题, 提出基于事件触发的模型预测控制方法, 实现了在有界扰动情况下非线性自动驾驶车辆模型的路径跟踪. 此外, 大多系统都或多或少包含非线性, 文献 [15] 针对通信时延下的车辆队列系统问题, 提出基于观测器的车辆队列系统纵向控制器, 用分层控制策略克服非线性扰动的影响, 实现通信时延下的自动驾驶车辆队列系统协同控制. 文献 [16] 研究了自动驾驶车辆间非线性作用和通信时延问题, 提出了基于车辆三阶动力学模型的分布式异质车辆队列系统非线性纵向控制器, 完成了通信时延下的车辆状态一致性控制.

然而, 在上述研究中, 大多未能把未知输入、外部干扰、非线性项等复杂情况加入自动驾驶车辆模型中, 许多现有成果采用较为简单的模型, 利用已有定理直接处理 LMI 得出结果^[17~19]. 许多成果并没有具体考虑实际系统中可能存在的部分状态不可测、系统外部输入的未知性和干扰未知性等问题, 限制了实用性^[20, 21]. 如今, 更加安全的零信任网络架构已经逐渐融入智能网联车系统, 在其“永不信任、始终验证”的理念下, 智能网联车面临的通信问题更加复杂, 外部网络攻击也更具策略, 这都将导致系统的未知性问题增多. 为了得到更加实用的结果, 有必要给予考虑. 为此, 针对含有未知输入和外部干扰的自动驾驶车辆跟驰系统, 设计积分观测器用于估计车辆部分状态不可测、未知输入以及外部干扰, 并基于所设计观测器提出一致性控制协议来实现自动驾驶车辆道路跟驰和状态一致性控制.

2 系统模型和问题描述

2.1 图论

用 $\overline{G}_g = (V, \varepsilon_g, A_g)$ 表示一个拓扑图, 其中, $V = \{V_1, V_2, \dots, V_N\}$ 表示节点集合, $\varepsilon_g \in V \times V$ 表示连接节点集合, 节点 (i, j) 表示连接起点 i 和终点 j 的有向弧线. 如果 $(i, j) \in \varepsilon_g$, 则称节点 j 是节点 i 的邻节点, 否则非邻节点. $V_N = \{j \mid (i, j) \in \varepsilon_g\}$ 表示邻节点集合, $A_g = [a_{ij}] \in \mathbb{R}^{N \times N}$ 表示邻接权重矩阵. 如果 $(i, j) \in \varepsilon_g$, 则 $a_{ij} > 0$, 意味着节点 i 能接收到节点 j 的信息, 否则 $a_{ij} = 0$. 且若拓扑图是无向的, 则 $a_{ij} = a_{ji}$, 否则 $a_{ij} \neq a_{ji}$. Laplacian 矩阵能定义为 $L_a = D_a - A_g$, 其中, $D_a = \text{diag}\{d_1, d_2, \dots, d_N\}$, $d_i = \sum_{j=N_i} a_{ij}$, $i \in N_{1:N}$.

针对上述 (跟驰) 节点集合 N , 再引入一个 (前导) 节点 0, 同时此前导节点扮演着一个特别角色. 它能影响跟驰节点, 且不被跟驰节点影响. 就可以得到一个由前导节点及其连接跟驰节点的弧线和拓扑图 \overline{G}_g 构成的增广拓扑图 G_g . 在增广拓扑图 G_g 中, 如果存在至少一个 (跟驰) 节点能够获得节点 0 的信息, 并且拓扑图 \overline{G}_g 是连通的, 则称此增广拓扑图是连通的. 增广拓扑图下的矩阵可以定义为

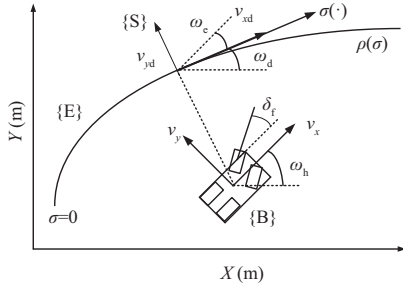


图 1 车辆路径跟驰示意图

Figure 1 Vehicle path following schematic diagram

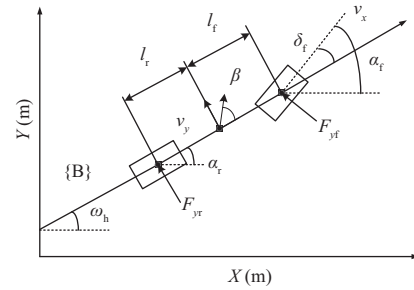


图 2 车辆模型

Figure 2 Vehicle model

$\mathcal{L} = \mathbf{L}_a + \mathbf{B}_g$, 其中, $\mathbf{B}_g = \text{diag} \{b_1, b_2, \dots, b_N\}$ 表示领随节点连通矩阵. 如果跟驰节点 i 能接收到前导节点的信息, 则 $b_i > 0$, 否则 $b_i = 0$. 通常设定, 节点 i 均不存在自环情况, 也就是说 $a_{ii} = 0$, 同时, 每个节点视为智能体, 其中, 前导节点表示前导者智能体, 跟驰节点表示跟驰者智能体.

2.2 系统模型

考虑一个前导车辆和 N 个跟驰车辆组成的自动驾驶车辆跟驰队列系统, 其车辆道路跟驰示意图如图 1 所示. 在车辆坐标系 $\{B\}$ 下的车辆模型如图 2 所示. 当 v_x 不变时, 该模型仅能通过控制 δ_f 来改变横向和横摆两个自由度的运动, 因此称为二自由度模型. 对该模型自动驾驶车辆列出动力学微分方程, 包含横向和横摆两个分量:

$$\begin{cases} F_{yf} \cos \delta_f + F_{yr} = m(\dot{v}_y + v_x r_e), \\ l_f F_{yr} \cos \delta_f - l_r F_{yr} = I_z \dot{r}_e, \end{cases} \quad (1)$$

其中, F_{yf} , F_{yr} 为前后轮胎的侧偏力; δ_f 为前轮转向角; m 为车辆的质量; v_x , v_y 为 $\{B\}$ 下的横向和纵向速度; 定义 ω_d 为目标道路航向角, ω_h 为车辆航向角, 即车身与 $\{B\}$ 下 X 轴的夹角; $\omega_e = \omega_h - \omega_d$ 为车辆航向角偏差, 如图 1 所示. 因此航向角偏差导数为 $r_e = \dot{\omega}_e = \dot{\omega}_h - \dot{\omega}_d$; l_f , l_r 分别为车辆质心到前后轮轴距离; I_z 为车辆的转动惯量.

定义 α_f , α_r 为前后轮胎的侧偏角, 车轮实际运动方向与车轮纵向中心线的夹角, 但后轮不能转向, 所以 α_r 为后轮运动方向与车身夹角. 并引入自动驾驶车辆模型假设:

- (1) 二自由度模型, 在 $\{B\}$ 下的横向速度 v_x 恒定;
- (2) 在前进过程中航向角 ω_e , α_f , α_r 都比较小.

从而有

- (1) $\cos(\omega_e) = 1, \sin(\omega_e) = \omega_e$,
- (2) $\alpha_f = \delta_f - \frac{l_f r}{v_x}, \alpha_r = \frac{l_r r}{v_x} - \frac{v_y}{v_x}$,
- (3) 车辆轮胎工作在线性区, 即 $F_{yf} = C_f \alpha_f, F_{yr} = C_r \alpha_r$, C_f, C_r 均为正,

其中, $r = \dot{\omega}_h$ 为车辆航向角导数; C_f, C_r 分别为前后轮胎的转向刚度值.

在 Serret-Frenet 坐标系 $\{S\}$ 下的自动驾驶车辆路径跟驰运动学方程^[22] 为

$$\begin{cases} \dot{e} = v_x \sin \omega_e + v_y \cos \omega_e, \\ \dot{\omega}_e = r - \rho(\sigma) \dot{\sigma}, \end{cases} \quad (2)$$

其中, e 为车辆质心 O 到目标点的距离; $\rho(\sigma)$ 为目标路径, 即弧长 σ 处的曲率.

在 $\{B\}$ 下的二自由度动力学模型^[23] 为

$$\begin{cases} \dot{\beta} = \frac{1}{mv_x} (F_{yf} + F_{yr}) - r_e - \rho(\sigma)v_x, \\ \dot{r}_e = \frac{1}{I_z} (l_f F_{yf} - l_r F_{yr}) - \dot{\rho}(\sigma)v_x, \end{cases} \quad (3)$$

其中, β 为质心侧偏角.

由图 2 可得 $\tan \beta = v_y/v_x$, 并且由车辆模型小角度假设有 $\beta = v_y/v_x$, 由二自由度模型假设 v_x 为给定常数, 则有 $\dot{\beta} = \dot{v}_y/v_x, \dot{\sigma} = v_x$ 并且将 $\rho(\sigma)$ 简写为 ρ , 可以得到二自由度动力学模型^[8]:

$$\begin{cases} \dot{e} = v_x \omega_e + v_x \beta, \\ \dot{\omega}_e = r_e, \\ \dot{\beta} = A_{33}\beta + A_{34}r_e + B_1 \delta_f + A_{34}\rho v_x, \\ \dot{r}_e = A_{43}\beta + A_{44}r_e + B_2 \delta_f + A_{44}\rho v_x - \dot{\rho}v_x. \end{cases} \quad (4)$$

由图 1 可知 \dot{e} 为车辆在大地坐标系 $\{E\}$ 中 Y 轴的位置, 且可以由此计算 X 轴位置, 得到车位置

$$\begin{cases} \dot{X} = v_x - v_x \omega_e, \\ \dot{Y} = v_x \omega_e + v_x \beta. \end{cases} \quad (5)$$

定义状态向量为 $\mathbf{x}_i(t) = [e, \omega_e, \beta, r_e]^T$, 控制输入为 $\mathbf{u}_i(t) = \delta_f$, 干扰项包含内部噪声和外部干扰, 为 $[E_1, E_2, E_3 + A_{34}\rho v_x, E_4 + A_{44}\rho v_x - \dot{\rho}v_x]^T$, 其中 E_1, E_2, E_3, E_4 为外界干扰. 考虑每个智能体系统均存在未知输入和非线性项, 由于控制系统大多是通过计算机完成控制的, 并且实际系统一定存在非线性项和未知输入, 则对原系统进行离散化得到状态方程:

$$\mathbf{x}_i(k+1) = \begin{bmatrix} 0 & v_x & v_x & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & A_{33} & A_{34} \\ 0 & 0 & A_{43} & A_{44} \end{bmatrix} \mathbf{x}_i(k) + \begin{bmatrix} 0 \\ 0 \\ B_1 \\ B_2 \end{bmatrix} \mathbf{u}_i(k) + \begin{bmatrix} E_1 \\ E_2 \\ E_3 + A_{34}\rho v_x \\ E_4 + A_{44}\rho v_x - \dot{\rho}v_x \end{bmatrix} \omega_i(k) + \mathbf{D}\boldsymbol{\varphi}_i(k) + \mathbf{G}\mathbf{d}_i(k),$$

其中, $A_{33} = -\frac{C_f + C_r}{mv_x}$, $A_{34} = -(1 + \frac{l_f C_f - l_r C_r}{mv_x^2})$, $A_{43} = \frac{l_f C_f + l_r C_r}{I_z}$, $A_{44} = -\frac{l_f^2 C_f + l_r^2 C_r}{I_z v_x}$, $B_1 = \frac{C_f}{mv_x^2}$, $B_2 = \frac{l_f C_f}{I_z}$, $\boldsymbol{\varphi}_i(k)$ 为考虑的系统非线性项, $\mathbf{d}_i(k)$ 为考虑的未知输入.

因此前导车辆的状态方程和第 i 个跟驰车辆的动力学模型分别为

$$\begin{cases} \mathbf{x}_0(k+1) = \mathbf{A}\mathbf{x}_0(k) + \mathbf{D}\boldsymbol{\varphi}_0(k) + \mathbf{E}\boldsymbol{\omega}_0(k) + \mathbf{G}\mathbf{d}_0(k), \\ \mathbf{y}_0(k) = \mathbf{C}\mathbf{x}_0(k) + \mathbf{F}\boldsymbol{\omega}_0(k), \end{cases} \quad (6)$$

$$\begin{cases} \mathbf{x}_i(k+1) = \mathbf{A}\mathbf{x}_i(k) + \mathbf{B}\mathbf{u}_i(k) + \mathbf{D}\boldsymbol{\varphi}_i(k) + \mathbf{E}\boldsymbol{\omega}_i(k) + \mathbf{G}\mathbf{d}_i(k), \\ \mathbf{y}_i(k) = \mathbf{C}\mathbf{x}_i(k) + \mathbf{F}\boldsymbol{\omega}_i(k), \end{cases} \quad (7)$$

其中, $\mathbf{x}_0, \mathbf{x}_i(k) \in \mathbb{R}^{4 \times 1}$ 表示前导车辆状态和第 i 个跟驰车辆的状态向量; $\mathbf{u}_i(k) \in \mathbb{R}^{1 \times 1}$ 表示控制系统输入向量; $\mathbf{y}_0, \mathbf{y}_i(k) \in \mathbb{R}^{2 \times 1}$ 表示系统输出向量; $\mathbf{d}_0, \mathbf{d}_i(k) \in \mathbb{R}^{1 \times 1}$ 表示有界的未知输入向量;

$\omega_0, \omega_i(k) \in \mathbb{R}^{1 \times 1}$ 表示系统外部扰动; $\varphi_0, \varphi_i(k) \in \mathbb{R}^{1 \times 1}$ 表示连续非线性函数; $i = 1, 2, \dots, N$, 并且 $\mathbf{A} \in \mathbb{R}^{4 \times 4}, \mathbf{B} \in \mathbb{R}^{4 \times 1}, \mathbf{E} \in \mathbb{R}^{4 \times 1}, \mathbf{D} \in \mathbb{R}^{4 \times 1}, \mathbf{G} \in \mathbb{R}^{4 \times 1}, \mathbf{C} \in \mathbb{R}^{2 \times 4}, \mathbf{F} \in \mathbb{R}^{2 \times 1}$ 为已知的参数矩阵.

可以验证

$$M_c = \text{rank} \begin{bmatrix} \mathbf{B} & \mathbf{A}\mathbf{B} & \mathbf{A}^2\mathbf{B} & \mathbf{A}^3\mathbf{B} \end{bmatrix} = 4$$

并且

$$N_o = \text{rank} \begin{bmatrix} (\overline{\mathbf{C}})^T & (\overline{\mathbf{C}}\mathbf{A})^T & (\overline{\mathbf{C}}\mathbf{A}^2)^T & (\overline{\mathbf{C}}\mathbf{A}^3)^T \end{bmatrix}^T = 4.$$

因此, $[\mathbf{A} \ \mathbf{C}]$ 可观, $[\mathbf{A} \ \mathbf{B}]$ 可控.

根据系统跟驰特性, 定义全局状态跟驰误差和输出误差为

$$\begin{cases} \delta_i(k) = \mathbf{x}_0(k) - \mathbf{x}_i(k), \\ \mathbf{Y}_i(k) = \mathbf{y}_0(k) - \mathbf{y}_i(k), \end{cases} \quad (8)$$

其中, $\delta_i(k)$ 表示全局状态跟驰误差, $\mathbf{x}_0(k)$ 表示前导车辆状态, $\mathbf{x}_i(k)$ 表示第 i 个跟驰车辆状态, $\mathbf{Y}_i(k)$ 表示全局输出误差, $\mathbf{y}_0(k)$ 表示前导车辆输出, $\mathbf{y}_i(k)$ 表示第 i 个跟驰车辆的输出.

则由式 (6)~(8) 可得

$$\begin{cases} \delta_i(k+1) = \mathbf{A}\delta_i(k) - \mathbf{B}u_i(k) + \mathbf{D}\overline{\varphi}_i(k) + \mathbf{E}\overline{\omega}_i(k) + \mathbf{G}\overline{\mathbf{d}}_i(k), \\ \mathbf{Y}_i(k) = \mathbf{C}\delta_i(k) + \mathbf{F}\overline{\omega}_i(k), \end{cases} \quad (9)$$

其中, $\overline{\varphi}_i(k) = \varphi_0(k) - \varphi_i(k)$, $\overline{\mathbf{d}}_i(k) = \mathbf{d}_0(k) - \mathbf{d}_i(k)$, $\overline{\omega}_i(k) = \omega_0(k) - \omega_i(k)$. 通过变换, 将系统中的跟驰车辆与前导车辆道路跟驰一致性问题转化为误差系统的稳定性问题.

3 状态估计与一致性设计

3.1 状态估计

利用全局跟驰误差系统状态与干扰项构建增广系统:

$$\begin{cases} \mathbf{M}\mathfrak{J}_i(k+1) = (\overline{\mathbf{A}} + \overline{\mathbf{E}})\mathfrak{J}_i(k) - \mathbf{B}u_i(k) + \mathbf{D}\overline{\varphi}_i(k) + \mathbf{G}\overline{\mathbf{d}}_i(k), \\ \mathbf{Y}_i(k) = \overline{\mathbf{C}}\mathfrak{J}_i(k), \end{cases} \quad (10)$$

其中, $\mathfrak{J}_i(k) = [\delta_i^T(k) \ \omega_i^T(k)]^T \in \mathbb{R}^{5 \times 1}$, $\mathbf{M} = [\mathbf{I} \ \mathbf{0}]$, $\overline{\mathbf{A}} = [\mathbf{A} \ \mathbf{0}]$, $\overline{\mathbf{E}} = [\mathbf{0} \ \mathbf{E}]$, $\overline{\mathbf{C}} = [\mathbf{C} \ \mathbf{F}]$, $\overline{\varphi}_i(k) = \overline{\varphi}_i(\mathbf{M}\mathfrak{J}_i, k)$. 由于

$$\text{rank} \begin{pmatrix} \begin{bmatrix} \mathbf{M} \\ \overline{\mathbf{C}} \end{bmatrix} \end{pmatrix} = \text{rank} \begin{pmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{C} & \mathbf{F} \end{bmatrix} \end{pmatrix} = 5$$

满秩, 所以一定存在满秩矩阵 \mathbf{T} 和矩阵 \mathbf{N} 使得 $\mathbf{T}\mathbf{M} + \mathbf{N}\overline{\mathbf{C}} = \mathbf{I}_5$ 成立. 则增广系统可以改写为

$$\begin{cases} \mathfrak{J}_i(k+1) = \mathbf{T}(\overline{\mathbf{A}} + \overline{\mathbf{E}})\mathfrak{J}_i(k) - \mathbf{T}\mathbf{B}u_i(k) + \mathbf{T}\mathbf{D}\overline{\varphi}_i(k) + \mathbf{T}\mathbf{G}\overline{\mathbf{d}}_i(k) + \mathbf{N}\mathbf{Y}_i(k+1), \\ \mathbf{Y}_i(k) = \overline{\mathbf{C}}\mathfrak{J}_i(k). \end{cases} \quad (11)$$

引理1 由于 $[\mathbf{A} \ \mathbf{C}]$ 是可观的, $[\mathbf{A} \ \mathbf{B}]$ 是可控的, 则增广系统 (11) 也是可观且可控的.

证明 由于 $A \in \mathbb{R}^{4 \times 4}$, $B \in \mathbb{R}^{4 \times 1}$, $E \in \mathbb{R}^{4 \times 1}$, $C \in \mathbb{R}^{2 \times 4}$, $F \in \mathbb{R}^{2 \times 1}$, 则令 $T(\bar{A} + \bar{E}) = A_5$, $TB = B_5$, 且 $A_5 \in \mathbb{R}^{5 \times 5}$, $\mathfrak{J}_i(k) \in \mathbb{R}^5$, $\bar{C} \in \mathbb{R}^{2 \times 5}$, $B_5 \in \mathbb{R}^{5 \times 1}$, 并且系统参数矩阵都是满秩的, 则由秩判据有

$$M_c = \text{rank} \begin{bmatrix} B_5 & A_5 B_5 & A_5^2 B_5 & A_5^3 B_5 & A_5^4 B_5 \end{bmatrix} = 5,$$

$$N_o = \text{rank} \begin{bmatrix} \bar{C}^T & (\bar{C}A_5)^T & (\bar{C}A_5^2)^T & (\bar{C}A_5^3)^T & (\bar{C}A_5^4)^T \end{bmatrix}^T = 5,$$

则系统 (11) 是可观且可控的, 证毕.

对于可控可观的增广系统 (11), 采用比例积分观测器^[24]设计方法, 设计观测器:

$$\begin{cases} \hat{\mathfrak{J}}_i(k+1) = T(\bar{A} + \bar{E})\hat{\mathfrak{J}}_i(k) - TBu_i(k) + TD\hat{\varphi}_i(k) + TG\hat{d}_i(k) \\ \quad + NY_i(k+1) + L_1(Y_i(k) - \bar{C}\hat{\mathfrak{J}}_i(k)), \\ \hat{d}_i(k+1) = \hat{d}_i(k) + L_2(Y_i(k) - \bar{C}\hat{\mathfrak{J}}_i(k)), \end{cases} \quad (12)$$

其中, $\hat{\mathfrak{J}}_i(k) \in \mathbb{R}^{5 \times 1}$ 是系统状态向量 $\mathfrak{J}_i(k)$ 的估计, $\hat{\varphi}_i(k) = \hat{\varphi}_i(M\mathfrak{J}_i, k)$ 是非线性函数 $\varphi_i(k)$ 的估计, L_1, L_2 为所设计观测器的增益矩阵.

定义状态估计误差和未知输入误差:

$$\begin{cases} \epsilon_i(k) = \mathfrak{J}_i(k) - \hat{\mathfrak{J}}_i(k), \\ \partial_i(k) = \bar{d}_i(k) - \hat{d}_i(k). \end{cases} \quad (13)$$

显然, 若存在合适的观测器增益 L_1, L_2 能够实现 $\lim_{k \rightarrow \infty} \epsilon_i(k) = 0$ 和 $\lim_{k \rightarrow \infty} \partial_i(k) = 0$. 说明所设计的观测器 (12) 符合要求, 可作为系统的观测器.

由式 (12) 和 (13) 可得

$$\begin{cases} \epsilon_i(k+1) = (T(\bar{A} + \bar{E}) - L_1\bar{C})\epsilon_i(k) + TD\tilde{\varphi}_i(k) + TG\partial_i(k), \\ \partial_i(k+1) = \partial_i(k) - L_2\bar{C}\epsilon_i(k) + \Delta d_i(k), \end{cases} \quad (14)$$

其中, $\tilde{\varphi}_i(k) = \varphi_i(k) - \hat{\varphi}_i(k)$, $\Delta d_i(k) = \bar{d}_i(k+1) - \bar{d}_i(k)$, 由式 (14) 设计广义系统观测器估计误差:

$$\begin{aligned} \begin{bmatrix} \epsilon_i(k+1) \\ \partial_i(k+1) \end{bmatrix} &= \begin{bmatrix} T(\bar{A} + \bar{E}) - L_1\bar{C} & TG \\ -L_2\bar{C} & I \end{bmatrix} \begin{bmatrix} \epsilon_i(k) \\ \partial_i(k) \end{bmatrix} + \begin{bmatrix} TD \\ 0 \end{bmatrix} \tilde{\varphi}_i(k) + \begin{bmatrix} 0 \\ I \end{bmatrix} \Delta d_i(k) \\ &= \left(\begin{bmatrix} T(\bar{A} + \bar{E}) & TG \\ 0 & I \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} \bar{C} & 0 \end{bmatrix} \right) \begin{bmatrix} \epsilon_i(k) \\ \partial_i(k) \end{bmatrix} + \begin{bmatrix} TD \\ 0 \end{bmatrix} \tilde{\varphi}_i(k) + \begin{bmatrix} 0 \\ I \end{bmatrix} \Delta d_i(k). \end{aligned}$$

取广义系统观测器估计误差:

$$\xi_i(k) = \begin{bmatrix} \epsilon_i(k) \\ \partial_i(k) \end{bmatrix}, \tilde{A} = \begin{bmatrix} T(\bar{A} + \bar{E}) & TG \\ 0 & I \end{bmatrix}, L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \tilde{C} = \begin{bmatrix} \bar{C} & 0 \end{bmatrix}, \bar{D} = \begin{bmatrix} TD \\ 0 \end{bmatrix}, H = \begin{bmatrix} 0 \\ I \end{bmatrix},$$

则有

$$\xi_i(k+1) = (\tilde{A} - L\tilde{C})\xi_i(k) + \bar{D}\tilde{\varphi}_i(k) + H\Delta d_i(k).$$

定义 $\boldsymbol{\xi}(k) = (\boldsymbol{\xi}_1^T(k), \boldsymbol{\xi}_2^T(k), \dots, \boldsymbol{\xi}_N^T(k))^T$, $\tilde{\boldsymbol{\varphi}}(k) = (\tilde{\boldsymbol{\varphi}}_1^T(k), \tilde{\boldsymbol{\varphi}}_2^T(k), \dots, \tilde{\boldsymbol{\varphi}}_N^T(k))^T$, $\Delta \mathbf{d}(k) = (\Delta \mathbf{d}_1^T(k), \Delta \mathbf{d}_2^T(k), \dots, \Delta \mathbf{d}_N^T(k))^T$. 则由 Kronecker 积的相关性质得广义系统观测器估计误差为

$$\boldsymbol{\xi}(k+1) = (\mathbf{I} \otimes (\tilde{\mathbf{A}} - \tilde{\mathbf{L}}\tilde{\mathbf{C}}))\boldsymbol{\xi}(k) + (\mathbf{I} \otimes \tilde{\mathbf{D}})\tilde{\boldsymbol{\varphi}}(k) + (\mathbf{I} \otimes \mathbf{H})\Delta \mathbf{d}(k), \quad (15)$$

关于 \mathbf{L} 的求解, 在定理 1 中给出.

3.2 一致性设计

根据广义系统观测器的估计结果, 分布式一致性控制协议:

$$\mathbf{u}_i(k) = \mathbf{K} \sum_{j \in N_i}^N a_{ij} \left(\hat{\mathfrak{J}}_j(k) - \hat{\mathfrak{J}}_i(k) \right), \quad (16)$$

其中, $\sum_{j \in N_i}^N a_{ij}$ 表示节点 i 的入度, \mathbf{K} 表示控制器一致性控制增益矩阵.

定义状态同步误差以及未知输入同步误差:

$$\begin{cases} \boldsymbol{\varepsilon}_i(k) = \mathfrak{J}_i(k) - \frac{1}{N} \sum_{j=1}^N \mathfrak{J}_j(k), \\ \boldsymbol{\vartheta}_i(k) = \bar{\mathbf{d}}_i(k) - \frac{1}{N} \sum_{j=1}^N \bar{\mathbf{d}}_j(k). \end{cases} \quad (17)$$

由式 (11) 和 (17) 可以得到

$$\begin{aligned} \boldsymbol{\varepsilon}_i(k+1) &= \mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}})\boldsymbol{\varepsilon}_i(k) + \mathbf{T}\mathbf{G}\boldsymbol{\vartheta}_i(k) + \mathbf{N}\bar{\mathbf{C}}(\Delta \boldsymbol{\varepsilon}_i(k) + \boldsymbol{\varepsilon}_i(k)) + \mathbf{T}\mathbf{D}\bar{\boldsymbol{\varphi}}_i(k) \\ &\quad - \mathbf{T}\mathbf{B}\mathbf{K} \left(\sum_{j \in N_i}^N a_{ij} (\hat{\mathfrak{J}}_j(k) - \hat{\mathfrak{J}}_i(k)) \right) + \frac{1}{N} \mathbf{T}\mathbf{B}\mathbf{K} \sum_{j=1}^N \left(\alpha \sum_{j \in N_i}^N a_{ij} (\hat{\mathfrak{J}}_j(k) - \hat{\mathfrak{J}}_i(k)) \right), \end{aligned} \quad (18)$$

其中, $\bar{\boldsymbol{\varphi}}_i(k) = \boldsymbol{\varphi}_i(k) - \frac{1}{N} \sum_{j=1}^N \boldsymbol{\varphi}_j(k)$, $\Delta \boldsymbol{\varepsilon}_i(k) = \boldsymbol{\varepsilon}_i(k+1) - \boldsymbol{\varepsilon}_i(k)$.

前导跟驰自动驾驶车辆的一致性通过线性变换后, 最终转化为误差系统的稳定性问题. 若存在合适的反馈增益矩阵 \mathbf{K} 能够使得 $\lim_{k \rightarrow \infty} \boldsymbol{\varepsilon}_i(k) = 0$, $\lim_{k \rightarrow \infty} \boldsymbol{\vartheta}_i(k) = 0$, 则表明前导跟驰车辆队列系统 (6) 和 (7) 能够在一致性协议 (16) 下实现一致性.

根据无向拓扑图结构的对称性, 即 $a_{ij} = a_{ji}$, 可知

$$\frac{1}{N} \mathbf{T}\mathbf{B}\mathbf{K} \sum_{j=1}^N \left(\sum_{j \in N_i}^N a_{ij} (\hat{\mathfrak{J}}_j(k) - \hat{\mathfrak{J}}_i(k)) \right) = 0, \quad (19)$$

并且有

$$\sum_{j \in N_i}^N a_{ij} (\hat{\mathfrak{J}}_j(k) - \hat{\mathfrak{J}}_i(k)) = \sum_{j \in N_i}^N a_{ij} (\mathfrak{J}_j(k) - \mathfrak{J}_i(k) + \boldsymbol{\varepsilon}_i(k) - \boldsymbol{\varepsilon}_j(k)). \quad (20)$$

由式 (18)~(20) 整理可以得到

$$\begin{aligned} \boldsymbol{\varepsilon}_i(k+1) &= (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})\boldsymbol{\varepsilon}_i(k) - \mathbf{T}\mathbf{B}\mathbf{K} \sum_{j \in N_i}^N a_{ij} (\boldsymbol{\varepsilon}_j(k) - \boldsymbol{\varepsilon}_i(k)) \\ &\quad - \mathbf{T}\mathbf{B}\mathbf{K} \sum_{j \in N_i}^N a_{ij} (\boldsymbol{\varepsilon}_i(k) - \boldsymbol{\varepsilon}_j(k)) + \mathbf{T}\mathbf{D}\bar{\boldsymbol{\varphi}}_i(k) + \mathbf{T}\mathbf{G}\boldsymbol{\vartheta}_i(k) + \mathbf{N}\bar{\mathbf{C}}\Delta \boldsymbol{\varepsilon}_i(k). \end{aligned} \quad (21)$$

定义 $\varepsilon(k) = (\varepsilon_1^T(k), \varepsilon_2^T(k), \dots, \varepsilon_N^T(k))^T$, $\bar{\varphi}(k) = (\bar{\varphi}_1^T(k), \bar{\varphi}_2^T(k), \dots, \bar{\varphi}_N^T(k))^T$ 并且令 $\epsilon(k) = (\mathbf{I} \otimes \mathbf{Q})\xi(k)$, 其中, $\mathbf{Q} = [\mathbf{I} \ 0]$, 并且基于一致性控制考虑, 假设 $\Delta\varepsilon_i(k) = \varepsilon_i(k+1) - \varepsilon_i(k) = 0$, 则由自动驾驶车辆拓扑结构图的 Laplacian 矩阵和 Kronecker 积有

$$\begin{aligned} \varepsilon(k+1) = & (\mathbf{I} \otimes (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}))\varepsilon(k) + (\mathbf{I} \otimes \mathbf{T}\mathbf{D})\bar{\varphi}(k) + (\mathbf{I} \otimes \mathbf{T}\mathbf{G})\vartheta(k) \\ & - (\mathbf{L}_a \otimes \mathbf{T}\mathbf{B}\mathbf{K})\varepsilon(k) - (\mathbf{L}_a \otimes \mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})\xi(k). \end{aligned} \quad (22)$$

增益矩阵 \mathbf{K} 的求解在定理 1 中给出.

定理1 对于 1 个前导车辆和 N 个跟驰车辆组成的自动驾驶车辆队列系统, 在经过前导跟驰车辆队列系统的处理后, 设计观测器估计系统未知状态和未知输入, 并基于所设计观测器设计一致性控制协议以实现对系统的道路跟驰和状态一致性控制, 给出 H_∞ 性能指标 ι , 增广 Laplacian 矩阵特征值 λ_i ($i = 1, 2, \dots, N$) 和非负标量常数 γ , 如果下列条件满足: 存在对称正定矩阵 $\mathbf{P}_1 \in \mathbb{R}^{6 \times 6}$, $\mathbf{P}_2 \in \mathbb{R}^{5 \times 5}$, 以及适维矩阵 $\mathbf{U}_1 \in \mathbb{R}^{6 \times 2}$, $\mathbf{U}_2 \in \mathbb{R}^{5 \times 5}$ 使得 LMI (23) 成立:

$$\begin{bmatrix} \Omega_{11} & * & * & * & * & * & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * & * & * & * & * & * \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & * & * & * & * & * & * \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} & * & * & * & * & * \\ \Omega_{51} & 0 & 0 & \Omega_{54} & \Omega_{55} & * & * & * & * \\ \Omega_{61} & 0 & 0 & 0 & \Omega_{65} & \Omega_{66} & * & * & * \\ \Omega_{71} & 0 & 0 & 0 & 0 & 0 & \Omega_{77} & * & * \\ \Omega_{81} & 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{88} & * \\ 0 & \Omega_{92} & 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{99} \end{bmatrix} < 0, \quad (23)$$

其中,

$$\begin{aligned} \Omega_{11} = & \mathbf{I}(\kappa\gamma^2 + 1) - \mathbf{P}_1, \Omega_{21} = -\lambda_i(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}), \\ \Omega_{22} = & (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^T \mathbf{P}_2(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) - \lambda_i(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}) \\ & - \lambda_i(\mathbf{T}\mathbf{B}\mathbf{K})^T \mathbf{P}_2(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) - \mathbf{P}_2 + \kappa\gamma^2 \mathbf{I} - \mathbf{P}_2, \\ \Omega_{31} = & -\lambda_i(\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}), \Omega_{32} = (\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) - \lambda_i(\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}), \\ \Omega_{33} = & (\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}\mathbf{G}), \Omega_{41} = -\lambda_i(\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}), \\ \Omega_{42} = & (\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) - \lambda_i(\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}), \Omega_{43} = (\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}\mathbf{G}), \\ \Omega_{44} = & (\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}\mathbf{D}) - \kappa \mathbf{I}, \Omega_{51} = \bar{\mathbf{D}}^T \mathbf{P}_1(\tilde{\bar{\mathbf{A}}} - \tilde{\mathbf{L}}\tilde{\bar{\mathbf{C}}}), \Omega_{55} = \bar{\mathbf{D}}^T \mathbf{P}_1 \bar{\mathbf{D}} - \kappa \mathbf{I}, \\ \Omega_{61} = & \mathbf{H}^T \mathbf{P}_1(\tilde{\bar{\mathbf{A}}} - \tilde{\mathbf{L}}\tilde{\bar{\mathbf{C}}}), \Omega_{65} = \mathbf{H}^T \mathbf{P}_1 \bar{\mathbf{D}}, \Omega_{66} = \mathbf{H}^T \mathbf{P}_1 \mathbf{H} - \iota^2 \mathbf{I}, \Omega_{71} = \mathbf{P}_1(\tilde{\bar{\mathbf{A}}} - \tilde{\mathbf{L}}\tilde{\bar{\mathbf{C}}}), \\ \Omega_{81} = & \lambda_i \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}), \Omega_{92} = \lambda_i \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}), \Omega_{77} = -\mathbf{P}_1, \Omega_{88} = -\mathbf{P}_2, \Omega_{99} = -\mathbf{P}_2, \end{aligned}$$

则所得到的误差系统满足 H_∞ 性能指标 $\|\xi(k)\| \leq \iota \|\Delta d(k)\|$, 并可以得到非负常数 κ , 观测器增益矩阵 $\mathbf{L} = \mathbf{P}_1^{-1} \mathbf{U}_1$, 一致性控制增益 $\mathbf{K} = ((\mathbf{T}\mathbf{B})^T \mathbf{T}\mathbf{B})^{-1} (\mathbf{T}\mathbf{B})^T \mathbf{P}_2^{-1} \mathbf{U}_2$.

证明 根据前导跟驰队列系统构建的广义系统以及广义系统观测器得到其误差系统再将前导跟驰队列系统的一致性问題转化为误差系统的稳定性问題, 构造 Lyapunov-Krasovskii 函数:

$$\mathbf{V}(k) = \boldsymbol{\xi}^T(k) (\mathbf{I} \otimes \mathbf{P}_1) \boldsymbol{\xi}(k) + \boldsymbol{\varepsilon}^T(k) (\mathbf{I} \otimes \mathbf{P}_2) \boldsymbol{\varepsilon}(k),$$

则有

$$\begin{aligned} \Delta \mathbf{V}(k) &= \mathbf{V}(k+1) - \mathbf{V}(k) = \boldsymbol{\xi}^T(k+1) (\mathbf{I} \otimes \mathbf{P}_1) \boldsymbol{\xi}(k+1) \\ &+ \boldsymbol{\varepsilon}^T(k+1) (\mathbf{I} \otimes \mathbf{P}_2) \boldsymbol{\varepsilon}(k+1) - \boldsymbol{\xi}^T(k) (\mathbf{I} \otimes \mathbf{P}_1) \boldsymbol{\xi}(k) - \boldsymbol{\varepsilon}^T(k) (\mathbf{I} \otimes \mathbf{P}_2) \boldsymbol{\varepsilon}(k). \end{aligned}$$

当非线性函数 $\varphi_i(k)$ 满足 Lipschitz 条件, 即 $\|\varphi_i(k) - \varphi_j(k)\| \leq \gamma \|\mathbf{x}_i(k) - \mathbf{x}_j(k)\|$, 则在估计误差系统中有 $\|\bar{\varphi}_i(k) - \hat{\varphi}_i(k)\| \leq \gamma \|\mathfrak{J}_i(k) - \hat{\mathfrak{J}}_i(k)\|$, 则有 $\|\bar{\varphi}(k)\| \leq \gamma \|\boldsymbol{\varepsilon}(k)\|$, 并且有 $\|\boldsymbol{\varepsilon}(k)\| \leq \|\boldsymbol{\xi}(k)\|$, 所以有 $\|\bar{\varphi}(k)\| \leq \gamma \|\boldsymbol{\xi}(k)\|$, 最后可得

$$0 \leq \gamma^2 \boldsymbol{\xi}^T(k) \boldsymbol{\xi}(k) - \bar{\varphi}^T(k) \bar{\varphi}(k). \quad (24)$$

在一致性控制误差系统中 $\|\bar{\varphi}_i(k) - \frac{1}{N} \sum_{j=1}^N \bar{\varphi}_j(k)\| \leq \gamma \|\mathfrak{J}_i(k) - \frac{1}{N} \sum_{j=1}^N \mathfrak{J}_j(k)\|$, 则有 $\|\bar{\varphi}(k)\| \leq \gamma \|\boldsymbol{\varepsilon}(k)\|$, 可以得到

$$0 \leq \gamma^2 \boldsymbol{\varepsilon}^T(k) \boldsymbol{\varepsilon}(k) - \bar{\varphi}^T(k) \bar{\varphi}(k). \quad (25)$$

由无向图性质可以知道存在正交矩阵 $\boldsymbol{\Pi}$, 使得 $\mathbf{L}_a = \boldsymbol{\Pi} \boldsymbol{\Lambda} \boldsymbol{\Pi}^T$, 其中, $\boldsymbol{\Lambda} = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_N\}$, $\{\lambda_1, \lambda_2, \dots, \lambda_N\}$ 为 Laplacian 矩阵 \mathbf{L}_a 的特征值, 并且 $\lambda_1 = 0$. 定义

$$\begin{aligned} \boldsymbol{\Xi}(k) &= (\boldsymbol{\Xi}_1^T(k), \dots, \boldsymbol{\Xi}_N^T(k))^T = (\boldsymbol{\Pi}^T \otimes \mathbf{I}) \boldsymbol{\xi}(k), \\ \boldsymbol{\Gamma}(k) &= (\boldsymbol{\Gamma}_1^T(k), \dots, \boldsymbol{\Gamma}_N^T(k))^T = (\boldsymbol{\Pi}^T \otimes \mathbf{I}) \boldsymbol{\varepsilon}(k), \\ \boldsymbol{\Theta}(k) &= (\boldsymbol{\Theta}_1^T(k), \dots, \boldsymbol{\Theta}_N^T(k))^T = (\boldsymbol{\Pi}^T \otimes \mathbf{I}) \boldsymbol{\vartheta}(k), \\ \boldsymbol{\varsigma}(k) &= (\boldsymbol{\varsigma}_1^T(k), \dots, \boldsymbol{\varsigma}_N^T(k))^T = (\boldsymbol{\Pi}^T \otimes \mathbf{I}) \bar{\varphi}(k), \\ \boldsymbol{\zeta}(k) &= (\boldsymbol{\zeta}_1^T(k), \dots, \boldsymbol{\zeta}_N^T(k))^T = (\boldsymbol{\Pi}^T \otimes \mathbf{I}) \tilde{\varphi}(k), \\ \boldsymbol{\Psi}(k) &= (\boldsymbol{\Psi}_1^T(k), \dots, \boldsymbol{\Psi}_N^T(k))^T = (\boldsymbol{\Pi}^T \otimes \mathbf{I}) \Delta \mathbf{d}(k). \end{aligned}$$

根据式 (16) 和 (22) 将 $\Delta \mathbf{V}(k)$ 中的变量 $\boldsymbol{\xi}(k), \boldsymbol{\varepsilon}(k), \boldsymbol{\vartheta}(k), \bar{\varphi}(k), \tilde{\varphi}(k), \Delta \mathbf{d}(k)$ 利用 Laplacian 引理分别转化为 $\boldsymbol{\Xi}(k), \boldsymbol{\Gamma}(k), \boldsymbol{\Theta}(k), \boldsymbol{\varsigma}(k), \boldsymbol{\zeta}(k), \boldsymbol{\Psi}(k)$, 则有变换后的 $\Delta \bar{\mathbf{V}}(k)$:

$$\begin{aligned} \Delta \bar{\mathbf{V}}(k) &= \sum_{i=1}^N \left(\boldsymbol{\Xi}^T(k) \left((\tilde{\mathbf{A}} - \mathbf{L}\tilde{\mathbf{C}})^T \mathbf{P}_1 (\tilde{\mathbf{A}} - \mathbf{L}\tilde{\mathbf{C}}) - \mathbf{P}_1 \right) \boldsymbol{\Xi}(k) + \boldsymbol{\Xi}^T(k) \left((\tilde{\mathbf{A}} - \mathbf{L}\tilde{\mathbf{C}})^T \mathbf{P}_1 \bar{\mathbf{D}} \right) \boldsymbol{\zeta}(k) \right. \\ &+ \boldsymbol{\Xi}^T(k) \left((\tilde{\mathbf{A}} - \mathbf{L}\tilde{\mathbf{C}})^T \mathbf{P}_1 \mathbf{H} \right) \boldsymbol{\Psi}(k) + \boldsymbol{\zeta}^T(k) \left(\bar{\mathbf{D}}^T \mathbf{P}_1 (\tilde{\mathbf{A}} - \mathbf{L}\tilde{\mathbf{C}}) \right) \boldsymbol{\Xi}(k) + \boldsymbol{\zeta}^T(k) \left(\bar{\mathbf{D}}^T \mathbf{P}_1 \bar{\mathbf{D}} \right) \boldsymbol{\zeta}(k) \\ &+ \boldsymbol{\zeta}^T(k) \left(\bar{\mathbf{D}}^T \mathbf{P}_1 \mathbf{H} \right) \boldsymbol{\Psi}(k) + \boldsymbol{\Psi}^T(k) \left(\mathbf{H}^T \mathbf{P}_1 (\tilde{\mathbf{A}} - \mathbf{L}\tilde{\mathbf{C}}) \right) \boldsymbol{\Xi}(k) + \boldsymbol{\Psi}^T(k) \left(\mathbf{H}^T \mathbf{P}_1 \bar{\mathbf{D}} \right) \boldsymbol{\zeta}(k) \\ &+ \boldsymbol{\Psi}^T(k) \left(\mathbf{H}^T \mathbf{P}_1 \mathbf{H} \right) \boldsymbol{\Psi}(k) + \boldsymbol{\Gamma}^T(k) \left((\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^T \mathbf{P}_2 (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) - \mathbf{P}_2 \right) \boldsymbol{\Gamma}(k) \\ &+ \boldsymbol{\Gamma}^T(k) \left((\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^T \mathbf{P}_2 (\mathbf{T}\bar{\mathbf{D}}) \right) \boldsymbol{\varsigma}(k) + \boldsymbol{\Gamma}^T(k) \left((\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^T \mathbf{P}_2 (\mathbf{T}\mathbf{G}) \right) \boldsymbol{\Theta}(k) \end{aligned}$$

$$\begin{aligned}
 & -\lambda_i \mathbf{\Gamma}^T(k) \left((\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}) \right) \mathbf{\Gamma}(k) \\
 & -\lambda_i \mathbf{\Gamma}^T(k) \left((\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}) \right) \mathbf{\Xi}(k) \\
 & +\mathbf{\varsigma}^T(k) \left((\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) \right) \mathbf{\Gamma}(k) + \mathbf{\varsigma}^T(k) \left((\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}\mathbf{D}) \right) \mathbf{\varsigma}(k) \\
 & +\mathbf{\varsigma}^T(k) \left((\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}\mathbf{G}) \right) \mathbf{\Theta}(k) - \lambda_i \mathbf{\varsigma}^T(k) (\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}) \mathbf{\Gamma}(k) \\
 & -\lambda_i \mathbf{\varsigma}^T(k) \left((\mathbf{T}\mathbf{D})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}) \right) \mathbf{\Xi}(k) + \mathbf{\Theta}^T(k) \left((\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) \right) \mathbf{\Gamma}(k) \\
 & +\mathbf{\Theta}^T(k) \left((\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}\mathbf{D}) \right) \mathbf{\varsigma}(k) + \mathbf{\Theta}^T(k) \left((\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}\mathbf{G}) \right) \mathbf{\Theta}(k) \\
 & -\lambda_i \mathbf{\Theta}^T(k) \left((\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}) \right) \mathbf{\Gamma}(k) - \lambda_i \mathbf{\Theta}^T(k) \left((\mathbf{T}\mathbf{G})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}) \right) \mathbf{\Xi}(k) \\
 & -\lambda_i \mathbf{\Gamma}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K})^T \mathbf{P}_2(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) \right) \mathbf{\Gamma}(k) - \lambda_i \mathbf{\Gamma}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K})^T \mathbf{P}_2(\mathbf{T}\mathbf{D}) \right) \mathbf{\varsigma}(k) \\
 & -\lambda_i \mathbf{\Gamma}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K})^T \mathbf{P}_2(\mathbf{T}\mathbf{G}) \right) \mathbf{\Theta}(k) + \lambda_i^2 \mathbf{\Gamma}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}) \right) \mathbf{\Gamma}(k) \\
 & +\lambda_i^2 \mathbf{\Gamma}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}) \right) \mathbf{\Xi}(k) - \lambda_i \mathbf{\Xi}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^T \mathbf{P}_2(\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) \right) \mathbf{\Gamma}(k) \\
 & -\lambda_i \mathbf{\Xi}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^T \mathbf{P}_2(\mathbf{T}\mathbf{D}) \right) \mathbf{\varsigma}(k) - \lambda_i \mathbf{\Xi}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^T \mathbf{P}_2(\mathbf{T}\mathbf{G}) \right) \mathbf{\Theta}(k) \\
 & +\lambda_i^2 \mathbf{\Xi}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}) \right) \mathbf{\Gamma}(k) + \lambda_i^2 \mathbf{\Xi}^T(k) \left((\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^T \mathbf{P}_2(\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}) \right) \mathbf{\Xi}(k).
 \end{aligned}$$

由 $\Delta\bar{\mathbf{V}}(k)$ 和式 (24) 和 (25) 易得

$$\Delta\bar{\mathbf{V}}(k) \leq \Delta\bar{\bar{\mathbf{V}}}(k) = \Delta\bar{\mathbf{V}}(k) + \kappa (\gamma^2 \mathbf{\Gamma}^T(k)\mathbf{\Gamma}(k) - \mathbf{\varsigma}^T(k)\mathbf{\varsigma}(k)) + \kappa (\gamma^2 \mathbf{\Xi}^T(k)\mathbf{\Xi}(k) - \mathbf{\zeta}^T(k)\mathbf{\zeta}(k)). \quad (26)$$

定义如下性能指标 $J = \sum_{k=1}^{\infty} (\mathbf{\xi}^T(k)\mathbf{\xi}(k) - \iota^2 \Delta\mathbf{d}^T(k)\Delta\mathbf{d}(k))$, 其中, 将变量 $\mathbf{\xi}(k), \Delta\mathbf{d}(k)$ 替换为 $\mathbf{\xi}(k), \mathbf{\Psi}(k)$ 可以得到

$$J \leq \sum_{k=1}^{\infty} (\Delta\mathbf{V} + \mathbf{\Xi}^T(k)\mathbf{\Xi}(k) - \iota^2 \mathbf{\Psi}^T(k)\mathbf{\Psi}(k)). \quad (27)$$

由式 (26) 和 (27) 易得

$$J \leq \sum_{k=1}^{\infty} (\Delta\mathbf{V} + \mathbf{\Xi}^T(k)\mathbf{\Xi}(k) - \iota^2 \mathbf{\Psi}^T(k)\mathbf{\Psi}(k)) \leq \sum_{k=1}^{\infty} (\Delta\bar{\bar{\mathbf{V}}} + \mathbf{\Xi}^T(k)\mathbf{\Xi}(k) - \iota^2 \mathbf{\Psi}^T(k)\mathbf{\Psi}(k)). \quad (28)$$

定义 $\boldsymbol{\rho} = [\mathbf{\Xi}^T(k), \mathbf{\Gamma}^T(k), \mathbf{\Theta}^T(k), \mathbf{\varsigma}^T(k), \mathbf{\zeta}^T(k), \mathbf{\Psi}^T(k)]^T$, 必存在

$$(\Delta\bar{\bar{\mathbf{V}}} + \mathbf{\Xi}^T(k)\mathbf{\Xi}(k) - \iota^2 \mathbf{\Psi}^T(k)\mathbf{\Psi}(k)) = \boldsymbol{\rho}^T \boldsymbol{\Phi} \boldsymbol{\rho},$$

从而有

$$\Delta\mathbf{V} + \mathbf{\Xi}^T(k)\mathbf{\Xi}(k) - \iota^2 \mathbf{\Psi}^T(k)\mathbf{\Psi}(k) \leq \boldsymbol{\rho}^T \boldsymbol{\Phi} \boldsymbol{\rho},$$

其中

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_{11} & * & * & * & * & * \\ \boldsymbol{\Phi}_{21} & \boldsymbol{\Phi}_{22} & * & * & * & * \\ \boldsymbol{\Phi}_{31} & \boldsymbol{\Phi}_{32} & \boldsymbol{\Phi}_{33} & * & * & * \\ \boldsymbol{\Phi}_{41} & \boldsymbol{\Phi}_{42} & \boldsymbol{\Phi}_{43} & \boldsymbol{\Phi}_{44} & * & * \\ \boldsymbol{\Phi}_{51} & 0 & 0 & \boldsymbol{\Phi}_{54} & \boldsymbol{\Phi}_{55} & * \\ \boldsymbol{\Phi}_{61} & 0 & 0 & 0 & \boldsymbol{\Phi}_{65} & \boldsymbol{\Phi}_{66} \end{bmatrix}, \quad (29)$$

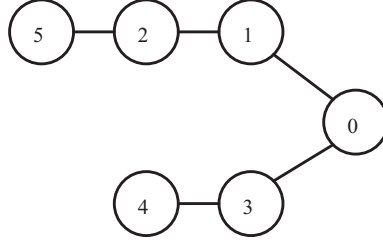


图 3 系统通信拓扑结构

Figure 3 System communication topology

$$\begin{aligned}
\Phi_{11} &= (\tilde{\mathbf{A}} - \tilde{\mathbf{L}}\tilde{\mathbf{C}})^\top \mathbf{P}_1 (\tilde{\mathbf{A}} - \tilde{\mathbf{L}}\tilde{\mathbf{C}}) + \lambda_i^2 (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}) + \mathbf{I}(\kappa\gamma^2 + 1) - \mathbf{P}_1, \\
\Phi_{12} &= -\lambda_i (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^\top \mathbf{P}_2 (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) + \lambda_i^2 (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}), \\
\Phi_{13} &= -\lambda_i (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{G}), \Phi_{14} = -\lambda_i (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{D}), \Phi_{15} = (\tilde{\mathbf{A}} - \tilde{\mathbf{L}}\tilde{\mathbf{C}})^\top \mathbf{P}_1 \bar{\mathbf{D}}, \\
\Phi_{16} &= (\tilde{\mathbf{A}} - \tilde{\mathbf{L}}\tilde{\mathbf{C}})^\top \mathbf{P}_1 \mathbf{H}, \Phi_{21} = -\lambda_i (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}) + \lambda_i^2 (\mathbf{T}\mathbf{B}\mathbf{K})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}), \\
\Phi_{22} &= (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^\top \mathbf{P}_2 (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) - \lambda_i (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}) \\
&\quad - \lambda_i (\mathbf{T}\mathbf{B}\mathbf{K})^\top \mathbf{P}_2 (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) + \lambda_i^2 (\mathbf{T}\mathbf{B}\mathbf{K})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}) + \kappa\gamma^2 \mathbf{I} - \mathbf{P}_2, \\
\Phi_{23} &= (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{G}) - \lambda_i (\mathbf{T}\mathbf{B}\mathbf{K})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{G}), \\
\Phi_{24} &= (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{D}) - \lambda_i (\mathbf{T}\mathbf{B}\mathbf{K})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{D}), \Phi_{31} = -\lambda_i (\mathbf{T}\mathbf{G})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}), \\
\Phi_{32} &= (\mathbf{T}\mathbf{G})^\top \mathbf{P}_2 (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) - \lambda_i (\mathbf{T}\mathbf{G})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}), \Phi_{33} = (\mathbf{T}\mathbf{G})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{G}), \\
\Phi_{34} &= (\mathbf{T}\mathbf{G})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{D}), \Phi_{41} = -\lambda_i (\mathbf{T}\mathbf{D})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}\mathbf{Q}), \\
\Phi_{42} &= (\mathbf{T}\mathbf{D})^\top \mathbf{P}_2 (\mathbf{T}(\bar{\mathbf{A}} + \bar{\mathbf{E}}) - \mathbf{N}\bar{\mathbf{C}}) - \lambda_i (\mathbf{T}\mathbf{D})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{B}\mathbf{K}), \Phi_{43} = (\mathbf{T}\mathbf{D})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{G}), \\
\Phi_{44} &= (\mathbf{T}\mathbf{D})^\top \mathbf{P}_2 (\mathbf{T}\mathbf{D}) - \kappa \mathbf{I}, \Phi_{51} = \bar{\mathbf{D}}^\top \mathbf{P}_1 (\tilde{\mathbf{A}} - \tilde{\mathbf{L}}\tilde{\mathbf{C}}), \Phi_{55} = \bar{\mathbf{D}}^\top \mathbf{P}_1 \bar{\mathbf{D}} - \kappa \mathbf{I}, \\
\Phi_{56} &= \bar{\mathbf{D}}^\top \mathbf{P}_1 \mathbf{H}, \Phi_{61} = \mathbf{H}^\top \mathbf{P}_1 (\tilde{\mathbf{A}} - \tilde{\mathbf{L}}\tilde{\mathbf{C}}), \Phi_{65} = \mathbf{H}^\top \mathbf{P}_1 \bar{\mathbf{D}}, \Phi_{66} = \mathbf{H}^\top \mathbf{P}_1 \mathbf{H} - \iota^2 \mathbf{I}.
\end{aligned}$$

证毕.

4 仿真验证

在前导跟驰车辆队列系统中, 由于前导车辆不受跟驰车辆影响, 且能够对跟驰产生影响, 故可以在仿真验证中, 通过控制前导车辆的运动来完成系统状态控制, 这样可以极大简化控制过程、降低控制成本. 为了验证设计方法的有效性, 考虑系统模型为 1 个前导车辆和 5 个跟驰车辆组成的前导跟驰车辆队列系统, 该车辆队列系统的通信拓扑结构如图 3 所示, 其中智能体 0 表示前导车辆, 智能体 1~5 表示跟驰车辆. 采用文献 [8] 给出的前导车辆质量及系统参数, 如表 1 所列.

假设车辆队列系统参数仅车体质量不同, 其中跟驰车辆质量分别为 $m_1 = 2500$ kg, $m_2 = 3000$ kg, $m_3 = 3500$ kg, $m_4 = 4000$ kg, $m_5 = 2000$ kg. 取 $\iota = 10$, $\gamma = 0.05$, $v_x = 15$ m/s, 假设未知输入为 $\mathbf{d}_i(k) = \mathcal{U}_1 \times \sin(\mathbf{x}_i(\mathcal{U}_2, k))$, 连续非线性函数为 $\varphi_i(k) = \mathcal{U}_2 \times i \times e^{-k} - \mathcal{U}_1 \times \sin(\mathbf{x}_i(\mathcal{U}_2, k))$, 系统外部扰动为 $\omega_i(k) = -\mathcal{U}_1 \times \sin(\mathcal{U}_2 \times k)$, 曲率为 $\rho = 50 \times \sin(k)$, 且 $i = 1, 2, \dots, 5$, $\mathcal{U}_1 \in [0, 1]$, $\mathcal{U}_2 \in [1, 3]$. 前导

表 1 系统参数
Table 1 System parameters

Parameter	Parameter value	Unit
m_0	1500	kg
I_z	2500	kg · m ²
l_s	0.8	m
l_f	1.3	m
l_r	1.4	m
C_f, C_r	45000	N/rad

车辆 (6)、跟驰车辆 (7) 系数矩阵:

$$\begin{aligned}
 \mathbf{A}_0 &= \begin{bmatrix} 0 & 15 & 15 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -4 & 0.98667 \\ 0 & 0 & 48.6 & -4.38 \end{bmatrix}, \mathbf{A}_1 = \begin{bmatrix} 0 & 15 & 15 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0.99333 \\ 0 & 0 & 48.6 & -4.38 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 15 & 15 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2.4 & 0.992 \\ 0 & 0 & 48.6 & -4.38 \end{bmatrix}, \\
 \mathbf{A}_3 &= \begin{bmatrix} 0 & 15 & 15 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1.7143 & 0.99429 \\ 0 & 0 & 48.6 & -4.38 \end{bmatrix}, \mathbf{A}_4 = \begin{bmatrix} 0 & 15 & 15 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1.5 & 0.995 \\ 0 & 0 & 48.6 & -4.38 \end{bmatrix}, \mathbf{A}_5 = \begin{bmatrix} 0 & 15 & 15 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & 0.99 \\ 0 & 0 & 48.6 & -4.38 \end{bmatrix}, \\
 \mathbf{B}_1 &= \begin{bmatrix} 0 \\ 0 \\ 0.08 \\ 25.2 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0.06667 \\ 25.2 \end{bmatrix}, \mathbf{B}_3 = \begin{bmatrix} 0 \\ 0 \\ 0.05714 \\ 25.2 \end{bmatrix}, \mathbf{B}_4 = \begin{bmatrix} 0 \\ 0 \\ 0.05 \\ 25.2 \end{bmatrix}, \mathbf{B}_5 = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ 25.2 \end{bmatrix}.
 \end{aligned}$$

前导跟驰车辆队列系统其余参数假设:

$$\mathbf{E} = \begin{bmatrix} 0.015 \\ 0.015 \\ 0.075 - 74.46\rho \\ 0.065 - 58.3995\rho - 15\dot{\rho} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.3 \\ 0.1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

由 $\mathbf{TM} + \mathbf{NC} = \mathbf{I}$ 可以求出其中一组 \mathbf{T}, \mathbf{N} 分别为

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -1 & -1 \end{bmatrix}, \mathbf{N} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.5 & 0.5 \end{bmatrix}.$$

将数据代入线性矩阵不等式 (23) 由 Matlab 中的工具箱可以求出 $\kappa = 8.2319$. 以及如下增益:

$$\mathbf{L}_{11} = \begin{bmatrix} 0.0236 & 0.0253 \\ 0.0026 & 0.0041 \\ 0.0540 & 0.0525 \\ -0.0481 & -0.0475 \\ -0.0295 & -0.0303 \end{bmatrix}, \mathbf{L}_{12} = \begin{bmatrix} 0.024 & 0.0249 \\ 0.0029 & 0.0038 \\ 0.0542 & 0.0534 \\ -0.0478 & -0.0475 \\ -0.0303 & -0.0307 \end{bmatrix}, \mathbf{L}_{13} = \begin{bmatrix} 0.0249 & 0.0239 \\ 0.0038 & 0.0030 \\ 0.0537 & 0.0545 \\ -0.0472 & -0.0479 \\ -0.0314 & -0.0305 \end{bmatrix},$$

$$\mathbf{L}_{14} = \begin{bmatrix} 0.0249 & 0.0239 \\ 0.0038 & 0.0029 \\ 0.0540 & 0.0548 \\ -0.0475 & -0.0476 \\ -0.0314 & -0.0312 \end{bmatrix}, \mathbf{L}_{15} = \begin{bmatrix} 0.0245 & 0.0247 \\ 0.0033 & 0.0034 \\ 0.0526 & 0.0526 \\ -0.0488 & -0.0488 \\ -0.0284 & -0.0285 \end{bmatrix},$$

$$\mathbf{L}_{21}^T = \begin{bmatrix} 0.0013 \\ 0.0020 \end{bmatrix}^T, \mathbf{L}_{22}^T = \begin{bmatrix} 0.0015 \\ 0.0018 \end{bmatrix}^T, \mathbf{L}_{23}^T = \begin{bmatrix} 0.0021 \\ 0.0012 \end{bmatrix}^T, \mathbf{L}_{24}^T = \begin{bmatrix} 0.0018 \\ 0.0016 \end{bmatrix}^T, \mathbf{L}_{25}^T = \begin{bmatrix} 0.0018 \\ 0.0019 \end{bmatrix}^T,$$

$$\mathbf{K}_1^T = \begin{bmatrix} -0.0115 \\ 0.0314 \\ 0.0196 \\ 0.0016 \\ -0.0013 \end{bmatrix}^T, \mathbf{K}_2^T = \begin{bmatrix} -0.0110 \\ 0.0307 \\ 0.0193 \\ 0.0014 \\ -0.0013 \end{bmatrix}^T, \mathbf{K}_3^T = \begin{bmatrix} -0.0107 \\ 0.0301 \\ 0.0190 \\ 0.0013 \\ -0.0013 \end{bmatrix}^T,$$

$$\mathbf{K}_4^T = \begin{bmatrix} -0.0104 \\ 0.0298 \\ 0.0189 \\ 0.0012 \\ -0.0013 \end{bmatrix}^T, \mathbf{K}_5^T = \begin{bmatrix} -0.0130 \\ 0.0337 \\ 0.0205 \\ 0.0002 \\ -0.0013 \end{bmatrix}^T.$$

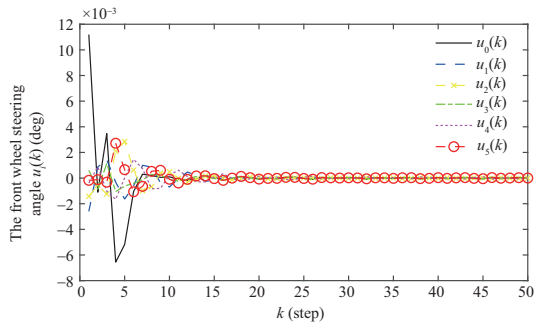


图 4 (网络版彩图) 前轮转角

Figure 4 (Color online) Front wheel steering angle

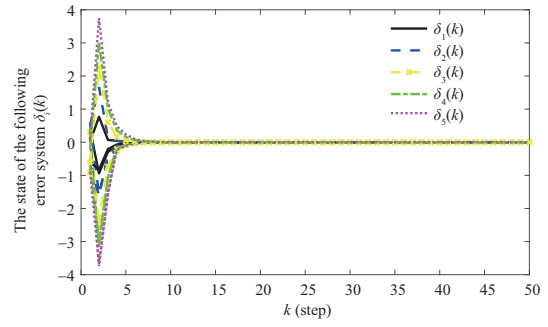


图 5 (网络版彩图) 全局跟驰误差

Figure 5 (Color online) Global following errors

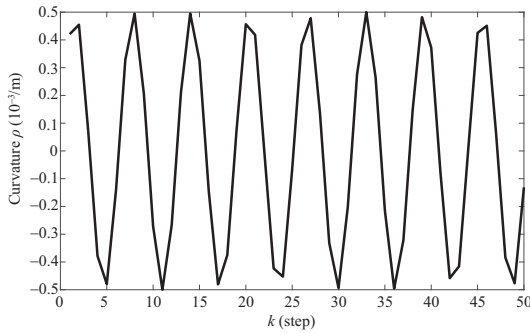


图 6 (网络版彩图) 曲率

Figure 6 (Color online) Curvature

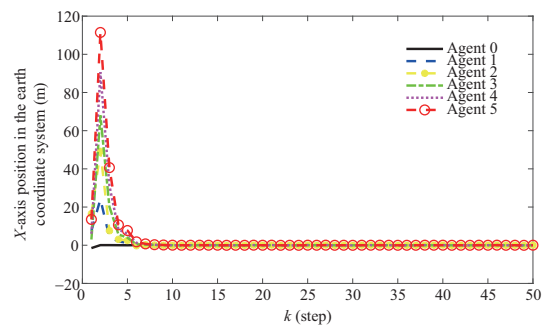


图 7 (网络版彩图) 大地坐标 X 轴位置

Figure 7 (Color online) X-axis position of geodetic coordinates

假设系统的初始状态为

$$\left\{ \begin{array}{l} \mathbf{x}_0(0) = \begin{bmatrix} -0.1 \\ -0.4 \\ -0.3 \\ -0.1 \end{bmatrix}, \mathbf{x}_1(0) = \begin{bmatrix} -0.5 \\ 0.2 \\ -0.3 \\ 0.5 \end{bmatrix}, \mathbf{x}_2(0) = \begin{bmatrix} -0.7 \\ 0.3 \\ -0.8 \\ 0.7 \end{bmatrix}, \\ \mathbf{x}_3(0) = \begin{bmatrix} -0.4 \\ 0.5 \\ 0.3 \\ -0.4 \end{bmatrix}, \mathbf{x}_4(0) = \begin{bmatrix} -0.5 \\ 0.3 \\ -0.1 \\ 0.5 \end{bmatrix}, \mathbf{x}_5(0) = \begin{bmatrix} -0.2 \\ 0.5 \\ -0.4 \\ 0.2 \end{bmatrix} \end{array} \right.$$

图 4 所示为一致性控制协议 (16) 作用下的前轮转角; 图 5 给出了系统全局跟驰误差变化, 表明系统在短时间内能实现跟驰车辆对前导车辆的跟驰. 验证了上述前导跟驰系统所设计的观测器 (10) 和控制协议 (16).

图 6 给出了运动道路的曲率; 图 7 为通过计算得到的大地坐标系 {E} 下的 X 轴位置; 图 8 为系

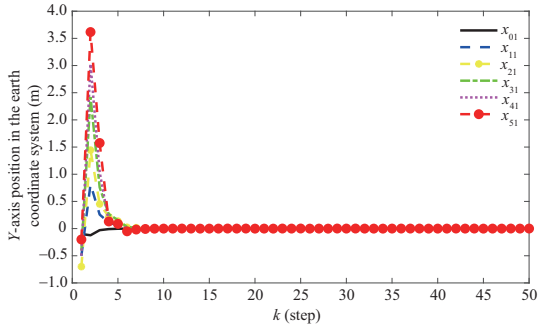


图 8 (网络版彩图) 大地坐标 Y 轴位置

Figure 8 (Color online) Y-axis position of geodetic coordinates

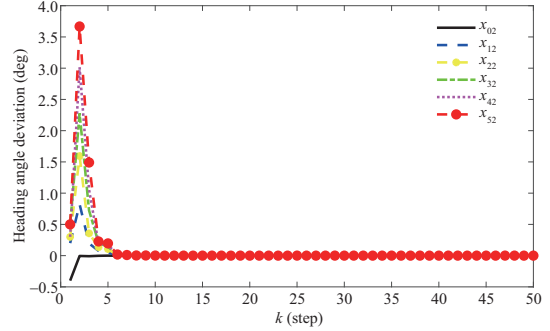


图 9 (网络版彩图) 航向角偏差

Figure 9 (Color online) Heading angle deviation

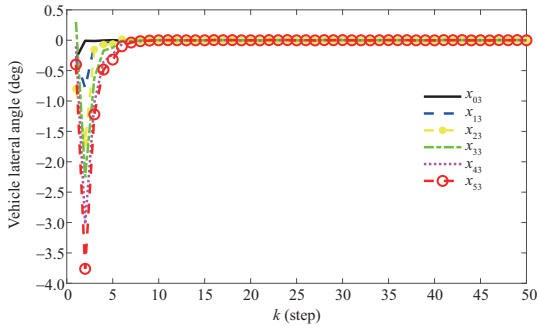


图 10 (网络版彩图) 车辆侧偏角

Figure 10 (Color online) Vehicle lateral angle

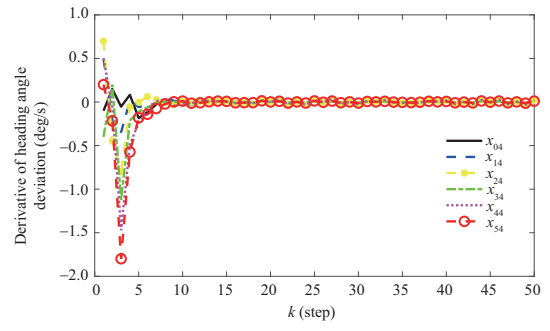


图 11 (网络版彩图) 航向角偏差导数

Figure 11 (Color online) Derivative of heading angle deviation

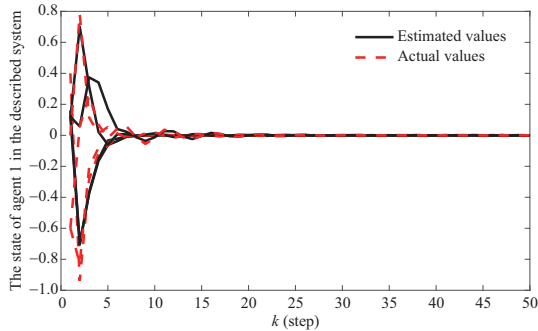


图 12 (网络版彩图) 跟驰队列系统状态估计值 $\hat{\mathfrak{J}}_1(k)$ 与真实值 $\mathfrak{J}_1(k)$

Figure 12 (Color online) Following system status estimates $\hat{\mathfrak{J}}_1(k)$ and true values $\mathfrak{J}_1(k)$

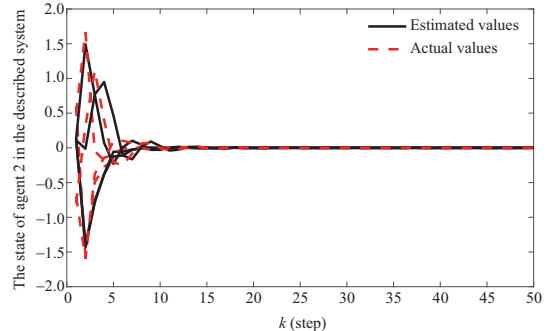


图 13 (网络版彩图) 跟驰队列系统状态估计值 $\hat{\mathfrak{J}}_2(k)$ 与真实值 $\mathfrak{J}_2(k)$

Figure 13 (Color online) Following system status estimates $\hat{\mathfrak{J}}_2(k)$ and true values $\mathfrak{J}_2(k)$

统状态 $x_{i1}(k)$, 其代表的是大地坐标系 $\{E\}$ 下的 Y 轴位置; 图 9 为系统状态 $x_{i2}(k)$, 其代表的是车辆导向角偏差随时间变化情况; 图 10 为系统状态 $x_{i3}(k)$, 其代表的是车辆侧偏角随时间变化情况; 图 11

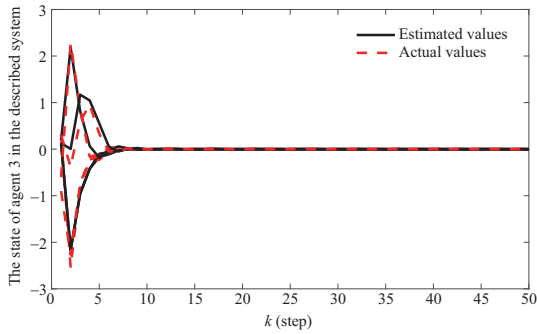


图 14 (网络版彩图) 跟驰队列系统状态估计值 $\hat{\mathfrak{J}}_3(k)$ 与真实值 $\mathfrak{J}_3(k)$

Figure 14 (Color online) Following system status estimates $\hat{\mathfrak{J}}_3(k)$ and true values $\mathfrak{J}_3(k)$

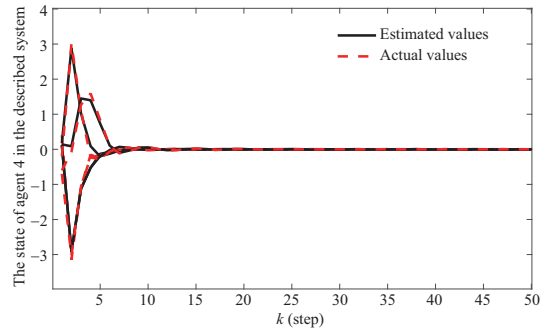


图 15 (网络版彩图) 跟驰队列系统状态估计值 $\hat{\mathfrak{J}}_4(k)$ 与真实值 $\mathfrak{J}_4(k)$

Figure 15 (Color online) Following system status estimates $\hat{\mathfrak{J}}_4(k)$ and true values $\mathfrak{J}_4(k)$

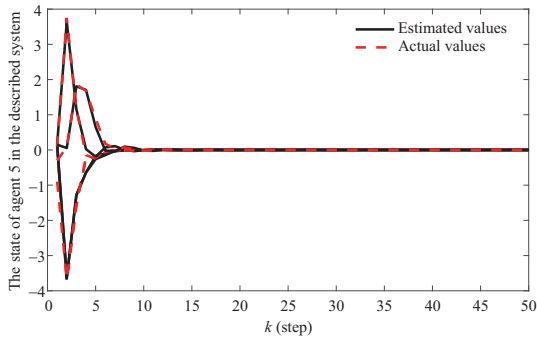


图 16 (网络版彩图) 跟驰系统状态估计值 $\hat{\mathfrak{J}}_5(k)$ 与真实值 $\mathfrak{J}_5(k)$

Figure 16 (Color online) Following system status estimates $\hat{\mathfrak{J}}_5(k)$ and true values $\mathfrak{J}_5(k)$

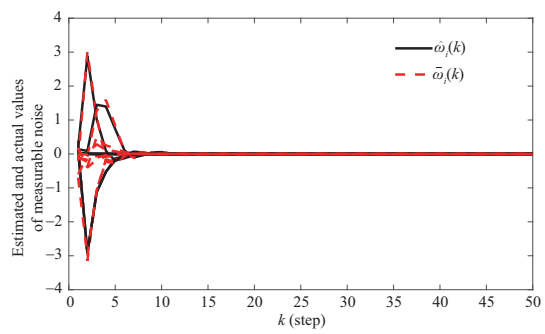


图 17 (网络版彩图) 可测噪声 $\bar{\omega}_i(k)$ 及其估计 $\hat{\omega}_i(k)$

Figure 17 (Color online) Measurable noise $\bar{\omega}_i(k)$ and its estimation $\hat{\omega}_i(k)$

为系统状态 $x_{i4}(k)$, 其代表的是车辆航向角偏差导数随时间变化情况; 图 12~16 给出了跟驰车辆队列系统的状态估计 $\hat{\mathfrak{J}}_i(k)$ 与真实值 $\mathfrak{J}_i(k)$ 情况, 表明了此自动驾驶车辆跟驰系统中跟驰车辆最终能按照前导车辆轨迹实现跟驰效果, 并达到一致; 图 17 显示的是利用观测器得到的可测噪声估计与其真实值的变化轨迹; 图 18 显示的是利用观测器得到的未知输入估计与未知输入真实值的变化轨迹; 图 19 明显看出自动驾驶车辆在 $\{E\}$ 下位置随时间变化, 并且在拓扑关系的影响下, 先后次序实现一致性控制.

为了验证所提方法的优越性, 与文献 [25] 的方法进行了对比, 仿真结果如图 20 和 21 所示. 从图 20, 4 与 21 中可以看出, 所提方法在控制精度、控制效率等方面更有优势.

5 总结

针对带有未知输入和外部干扰的自动驾驶车辆跟驰系统, 利用前导跟驰多智能体系统模型, 解决车辆间的跟驰问题, 将跟驰问题转化为误差问题, 设计观测器用来估计系统状态不可测部分、未知输入以及外部干扰, 以完成自动驾驶系统的状态检测.

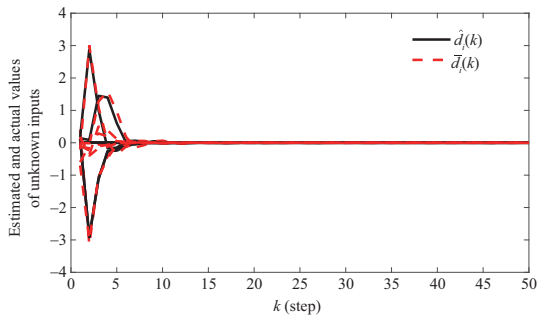


图 18 (网络版彩图) 未知输入 $\bar{d}_i(k)$ 及其估计误差 $\hat{d}_i(k)$

Figure 18 (Color online) Unknown inputs $\bar{d}_i(k)$ and the estimates $\hat{d}_i(k)$

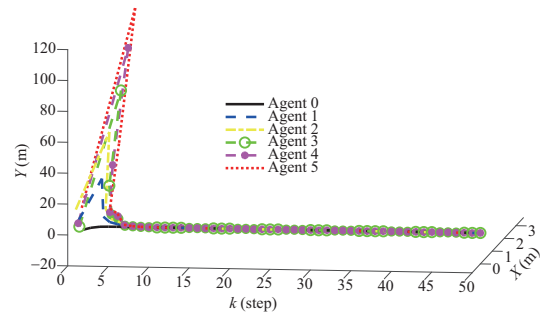


图 19 (网络版彩图) 车辆位置

Figure 19 (Color online) Vehicle location

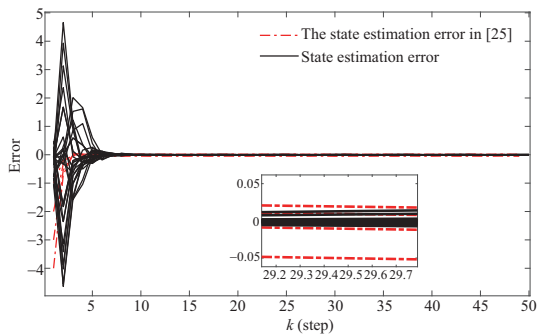


图 20 (网络版彩图) 状态估计误差对比

Figure 20 (Color online) Comparison of state estimation errors

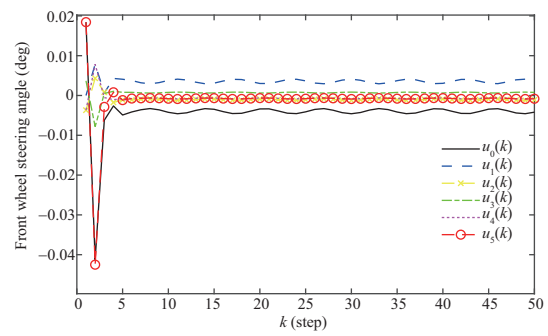


图 21 (网络版彩图) 对比方法 [25] 中的前轮转角

Figure 21 (Color online) Front wheel steering angle in [25]

基于所设计观测器得到的系统状态估计, 设计一致性控制协议, 将系统的一致性控制问题转化为误差系统的稳定性问题, 利用图论、Lyapunov-Krasovkii 函数, 对符合 Lipschitz 条件的非线性项, 通过求解线性矩阵不等式, 得到使误差系统稳定的增益矩阵和参数, 并利用 H_∞ 技术, 保证了自动驾驶车辆道路跟驰和状态一致性控制。

无向拓扑结构是有向拓扑的特殊情况, 有向拓扑突破了固定拓扑和无向拓扑的局限性, 前导跟驰车辆队列系统能一定程度上简化控制过程, 减少通信负担, 事件触发机制能更有效地减少智能体间的通信资源消耗问题。因此, 下一步的工作将研究在事件触发控制和有向拓扑切换下的自动驾驶车辆跟驰和状态一致性问题。

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Path following and status consistency control of autonomous vehicles

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Abstract The problem of path following and state consistency control in a nonlinear time-varying platooning system of autonomous vehicles with unknown inputs and external disturbances is investigated in this study. Specifically, the focus is on addressing the control challenges when only partial states of the system can be measured. First, using the vehicle-following and two-degree-of-freedom dynamics models, a discrete state equation is derived for the platooning system of autonomous vehicles, which considers external disturbances and unknown inputs. The leading-following characteristics are utilized to establish an autonomous vehicle platooning system. Then, the partial state unobservability problem of the system is addressed by employing a proportional-integral state observer. An observer-based approach is proposed to realize state consistency control for autonomous vehicle platooning systems. In addition, the estimation effectiveness of the observer and the problem of state consistency control are transformed into stability issues of the error system. This is realized by constructing a Lyapunov-Krasovskii function and deriving sufficient conditions using the discrete system stability theory. The robustness of the system is examined using the H_∞ performance index. Finally, the gain and parametric matrices of the platooning system are obtained by solving the linear matrix inequality (LMI). Simulation results verify the effectiveness of the observer-based design in allowing accurate path following and ensuring state consistency for autonomous vehicles with unknown inputs.

Keywords intelligent transportation, autonomous vehicle system, path following, multiagent system, consistency control, external disturbance