



# 输入死区下的多输入多输出系统自适应神经网络容错控制

周琪<sup>1,2</sup>, 林国怀<sup>1,2</sup>, 马慧<sup>1,2</sup>, 鲁仁全<sup>1,2\*</sup>

1. 广东工业大学自动化学院, 广州 510006

2. 广东省智能决策与协同控制重点实验室 (广东工业大学), 广州 510006

\* 通信作者. E-mail: lurenquan2012@163.com

收稿日期: 2019-11-19; 修回日期: 2020-01-31; 接受日期: 2020-03-07; 网络出版日期: 2021-02-23

国家自然科学基金 (批准号: 61973091)、广东省自然科学基金研究团队项目 (批准号: 2018B030312006) 和广州市科技计划项目 (批准号: 201904020006) 资助

**摘要** 针对一类具有传感器故障和不对称输入死区的非线性多输入多输出非严格反馈系统, 本文提出一种自适应神经网络容错控制方案. 控制器的设计以反步法为框架, 采用自适应神经网络控制方法处理传感器故障, 利用死区斜率的有界性补偿输入死区对系统性能造成的影响, 同时引入动态面控制技术克服“计算爆炸”的问题. 该控制方法不仅能够保证闭环系统中所有信号半全局一致最终有界, 而且能使跟踪误差收敛至原点附近的紧集内. 最后通过两个仿真实验验证该控制方法的有效性.

**关键词** 非线性系统, 传感器故障, 容错控制, 动态面控制, 输入死区

## 1 引言

近年来, 反步法已成为非线性系统控制设计的有效工具, 对非线性系统控制理论的发展起着重要的作用<sup>[1~3]</sup>. 自适应反步法属于系统化的递归式设计方法, 在反步法的基础上引入自适应律, 解决了在线估计系统中的未知参数问题<sup>[4]</sup>. 在自适应反步法的框架之下, 研究者可以利用神经网络或模糊逻辑系统逼近系统模型中的未知非线性函数, 进而设计出使系统稳定的控制器<sup>[5]</sup>. 然而, 自适应反步法仍然存在某些不足之处. 例如, 在设计控制器的过程中, 需要对虚拟控制器进行反复求导, 从而增加了计算负担, 出现“计算爆炸”的问题. Tong等<sup>[6]</sup>在反步法设计过程中结合动态面控制技术克服了此类问题.

多输入多输出非线性系统具有并行传送数据的特点, 且广泛存在于实际工程中. 借助于反步法和神经网络, 研究者们极大地推进了多输入多输出非线性系统控制理论的发展<sup>[7~9]</sup>. Chen等<sup>[7]</sup>研究了

**引用格式:** 周琪, 林国怀, 马慧, 等. 输入死区下的多输入多输出系统自适应神经网络容错控制. 中国科学: 信息科学, 2021, 51: 618–632, doi: 10.1360/SSI-2019-0198  
Zhou Q, Lin G H, Ma H, et al. Adaptive neural network fault-tolerant control for MIMO systems with dead zone inputs (in Chinese). Sci Sin Inform, 2021, 51: 618–632, doi: 10.1360/SSI-2019-0198

一类鲁棒自适应控制问题,为分析具有输入饱和与输入死区的多输入多输出严格反馈系统提供了新的思路.针对含有控制方向未知的多输入多输出系统,Zhou 等<sup>[8]</sup>结合 Nussbaum 函数,提出了一种新颖的自适应神经网络控制方法.值得注意的是,虽然上述研究为非线性系统控制器设计提供了有效的途径,但并不适用于非严格反馈形式下的非线性系统.

非严格反馈形式是一种常见的非线性系统形式.对于非线性非严格反馈系统,由于系统中的非线性函数包含全状态变量,在设计控制器时会产生代数环问题.因此,为了解决非严格反馈所带来的上述问题,Chen 等<sup>[10]</sup>和 Wang 等<sup>[11]</sup>应用变量分离的方法,统一在反步法的最后一步处理状态分离后的非线性函数,而 Li 等<sup>[12]</sup>和 Ma 等<sup>[13]</sup>利用模糊基函数的性质对非线性函数进行变换,最终设计出可以保证系统稳定的自适应模糊控制器.与上述文献不同的是,Wang 等<sup>[14]</sup>和 Sun 等<sup>[15]</sup>利用径向基神经网络的性质改变基函数向量的状态,使控制器的设计过程更为简洁直观.虽然学者们在非线性系统自适应控制的研究方面已经取得了许多成果,但对于实际应用中传感器故障和输入死区同时出现的情况,往往没有进行充分的考虑与分析.

事实上,随着现代科学技术的飞速发展,工业过程控制系统的规模和复杂度都在不断增加,系统一旦发生故障,轻则降低系统性能,重则破坏系统稳定性,造成不可预估的损失<sup>[16~20]</sup>.因此,提高控制系统的可靠性和安全性显得尤为关键,而容错控制的出现和发展有效地解决了此类问题.对于发生传感器故障的系统,许多学者提出了有效的自适应主动容错控制方法,通过适当调整控制器的参数来补偿故障,进而减小传感器故障对系统的影响<sup>[21~23]</sup>.Yan 等<sup>[24]</sup>基于滑模观测器提出了一种故障重构方法,利用重构信号逼近故障信号,从而使故障检测与诊断方法的实用性更强.针对一类具有传感器故障的二阶多输入多输出非线性系统,Khebbache 等<sup>[25]</sup>设计了一种基于自适应动态面控制方法的容错控制器,Bounemour 等<sup>[26]</sup>基于模糊逻辑系统提出了一种自适应模糊容错控制方法.Zhang 等<sup>[27]</sup>设计了一种包含自适应故障补偿机制的控制器,解决了传感器部分失效故障的问题.另一方面,输入死区作为一种非光滑的非线性特性,若在容错控制设计中忽视其存在,可能会严重地影响系统的控制性能.为了使具有死区的非线性系统保持稳定,许多学者对此提出了多种自适应控制方法<sup>[28~30]</sup>.Li 等<sup>[31]</sup>基于模糊逼近理论,利用死区斜率的有界性解决了输入死区问题.Luo 等<sup>[32]</sup>基于自适应反步法提出了一种补偿执行器死区的容错控制方法.针对一类具有未知死区和执行器故障的不确定非线性互联大系统,Chen 等<sup>[33]</sup>设计了一种自适应神经网络容错控制器.鉴于对以上文献的分析,在容错控制设计的过程中考虑输入死区对系统的影响是非常有必要的.因此,本文针对一类具有传感器故障和不对称输入死区的非线性多输入多输出非严格反馈系统,提出了一种新的自适应神经网络容错控制方法.

本文的主要贡献如下:(1)相比文献[25,26],本文考虑了  $m$  阶多输入多输出系统,更具有一般性.与此同时,本文考虑的  $m$  阶非线性系统还受输入死区影响,在不需要建立死区逆模型的情况下,我们利用死区斜率的有界性补偿不对称死区非线性的问题.(2)与文献[10,11]不同的是,本文不需要文献[10,11]中对非线性函数的假设,从而降低了设计的保守性,并使控制器的设计过程更简单直接.(3)该控制方法将未知神经网络权重向量作为估计参数,减少了自适应参数的数量,有效地降低了计算负担.

本文的后续部分安排如下:第 2 节介绍了文章的系统,并阐述了传感器故障和输入死区的问题;第 3 节介绍了自适应神经网络控制器的设计和相应的稳定性分析;第 4 节采用两个仿真实验对所提出的控制方法进行验证;第 5 节对全文进行总结.

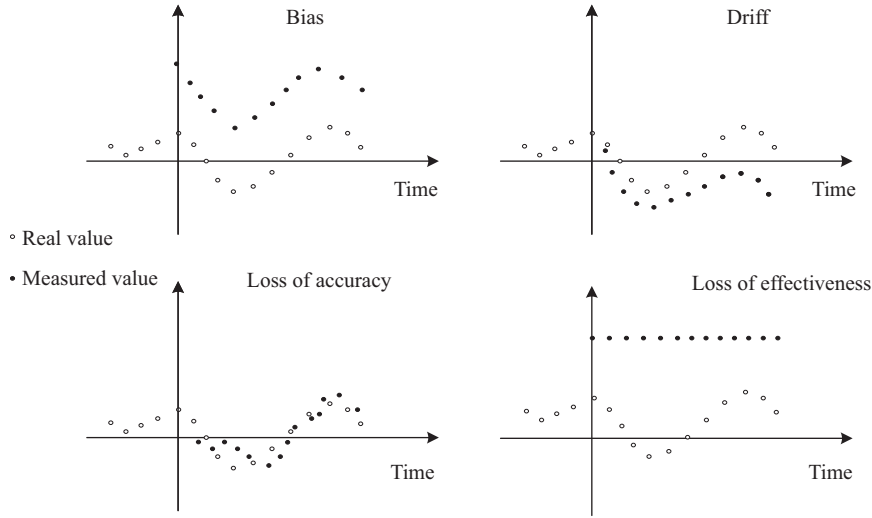


图 1 传感器故障类型示意图  
Figure 1 Types of sensor faults

## 2 问题阐述

考虑一类具有非严格反馈形式的多输入多输出非线性系统:

$$\begin{aligned}\dot{x}_{j,i} &= f_{j,i}(x) + x_{j,i+1}, \\ \dot{x}_{j,m} &= f_{j,m}(x) + u_j, \\ y_j &= h(x_{j,1}),\end{aligned}\quad (1)$$

其中,  $j = 1, 2, \dots, n$ ,  $i = 1, 2, \dots, m-1$ ,  $x = [x_1^T, x_2^T, \dots, x_n^T]^T$ ,  $x_j = [x_{j,1}, x_{j,2}, \dots, x_{j,m}]^T \in \mathbb{R}^m$ ,  $y_j$  为第  $j$  个子系统的输出,  $u_j \in \mathbb{R}$  为第  $j$  个子系统的输入,  $f_{j,i}(\cdot)$  为未知光滑非线性函数, 且满足  $f_{j,i}(0) = 0$ . 系统的传感器故障模型为:  $h(x_{j,1}) = K_j x_{j,1} + q_j(t)$ , 其中,  $K_j$  和  $q_j(t)$  为传感器故障的参数, 且满足以下条件:  $0 < \bar{K}_{j,\min} \leq K_j \leq 1$ ,  $-\bar{q}_j \leq q_j(t) \leq \bar{q}_j$ .  $\bar{K}_{j,\min}$  表示传感器最小影响值,  $\bar{q}_j$  与  $-\bar{q}_j$  分别表示  $q_j(t)$  的上界与下界. 传感器故障示意图如图 1 所示, 且故障类型总结如下:

(1) 当  $K_j = 1$ ,  $q_j(t)$  为常数, 传感器发生偏差故障; (2) 当  $K_j = 1$ ,  $|q_j(t)| = \varphi t$ ,  $0 < \varphi \ll 1$ , 传感器发生漂移故障; (3) 当  $K_j = 1$ ,  $|q_j(t)| < \bar{q}_j$ ,  $q_j(t) \rightarrow 0$ , 传感器发生精度下降故障; (4) 当  $0 < \bar{K}_{j,\min} \leq K_j < 1$ ,  $q_j(t) = 0$ , 传感器发生失效故障.

令  $f_{sj} = (K_j - 1)x_{j,1} + q_j(t)$ , 其中  $f_{sj} \in \mathbb{R}$  表示传感器故障向量, 且  $y_j = x_{j,1} + f_{sj}$ .  $y_j$  的导数为  $\dot{y}_j = \dot{x}_{j,1} + \dot{f}_{sj}$ . 其中,  $\dot{f}_{sj} = f_{psj}$ .  $u_j$  是一个不对称输入死区非线性函数, 定义如下:

$$u_j = D_j(\varpi_j) \triangleq \begin{cases} o_{jr}(\varpi_j - p_{jr}), & \varpi_j \geq p_{jr}, \\ 0, & -p_{jl} < \varpi_j < p_{jr}, \\ o_{jl}(\varpi_j + p_{jl}), & \varpi_j \leq -p_{jl}. \end{cases}\quad (2)$$

不对称死区非线性函数如图 2 所示,  $\varpi_j \in \mathbb{R}$  表示第  $j$  个死区的输入,  $o_{jl}$  和  $o_{jr}$  分别表示死区特性的左斜率和右斜率,  $p_{jl}$  和  $p_{jr}$  分别表示左断点和右断点.

假设 1 ([29]) 存在一个大于 0 的常数  $M_m$  满足:  $|\varpi_j| \leq M_m$ , 其中  $j = 1, 2, \dots, n$ .

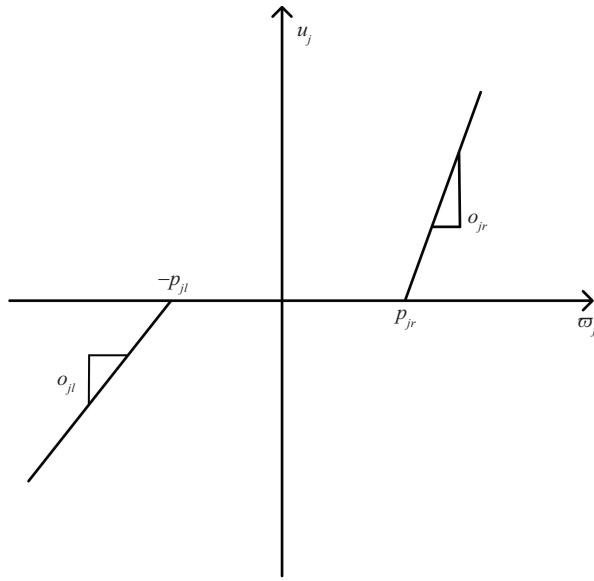


图 2 不对称死区非线性函数

Figure 2 Nonsymmetric dead-zone nonlinearity

假设2 ([29]) 参数  $o_{jr}, o_{jl}, p_{jr}, p_{jl}$  均为大于 0 的常数, 且  $o_{jr} \neq o_{jl}$ . 将式 (2) 改写为

$$u_j = o_j(t)\varpi_j(t) + \lambda_j(t), \tag{3}$$

其中,

$$o_j(t) = \begin{cases} o_{jl}, & \varpi_j \leq 0, \\ o_{jr}, & \varpi_j > 0, \end{cases} \tag{4}$$

$$\lambda_j(t) = \begin{cases} -o_{jr}p_{jr}, & \varpi_j \geq p_{jr}, \\ -o_j(t)\varpi_j(t), & -p_{jl} < \varpi_j < p_{jr}, \\ o_{jl}p_{jl}, & \varpi_j \leq -p_{jl}. \end{cases} \tag{5}$$

由式 (5) 可得

$$\lambda_j(t) \leq \bar{\lambda}_j, \quad \bar{\lambda}_j = \max\{o_{jl}p_{jl}, o_{jr}p_{jr}\}. \tag{6}$$

定义  $\bar{\alpha}_j = \max\{o_{jl}, o_{jr}\}$ ,  $\underline{\alpha}_j = \min\{o_{jl}, o_{jr}\}$ ,

$$\frac{o_j(t)}{\underline{\alpha}_j} = 1 + \rho_j(t), \tag{7}$$

其中,  $\rho_j(t) \geq 0$ . 由式 (4) 和 (7) 得

$$\rho_j(t) \leq \frac{\bar{\alpha}_j}{\underline{\alpha}_j} - 1. \tag{8}$$

将式 (7) 代入式 (3) 可得,

$$u_j = \alpha_j(1 + \rho_j(t))\varpi_j(t) + \lambda_j(t). \quad (9)$$

**假设3** ([8]) 对于  $j = 1, 2, \dots, n$ , 参考信号  $y_{dj}$  与其  $m$  阶导数均是连续有界的.

假设给定一个大于 0 的常数  $\varepsilon$ , 对于任意的连续函数  $f(\zeta)$ , 存在一个神经网络  $W^T S(\zeta)$  使得

$$f(\zeta) = W^T S(\zeta) + \delta(\zeta), \quad |\delta(\zeta)| \leq \varepsilon,$$

其中, 理想权重向量  $W = [w_1, w_2, \dots, w_\iota]^T \in \mathbb{R}^\iota$ , 且  $\iota > 1$ ,  $\iota$  表示神经网络的节点数,  $\zeta$  表示输入向量, 且  $\forall \zeta \in \Omega_\zeta \subset \mathbb{R}^n$ ,  $S(\zeta) = [S_1(\zeta), S_2(\zeta), \dots, S_\iota(\zeta)]^T$  代表高斯 (Gauss) 函数向量组, 且

$$S_j(\zeta) = \exp \left[ \frac{-(\zeta - \mu_j)^T (\zeta - \mu_j)}{\Phi_j^2} \right], \quad j = 1, 2, \dots, \iota,$$

其中,  $\mu_j = [\mu_{j1}, \mu_{j2}, \dots, \mu_{jn}]^T$  表示可取范围的中心,  $\Phi_j$  为高斯函数的宽度.

**引理1** ([14]) 定义  $\bar{x}_{j,q} = [x_{j,1}, x_{j,2}, \dots, x_{j,q}]^T$ , 其中  $S_j(\bar{x}_{j,q}) = [S_{j,1}(\bar{x}_{j,q}), S_{j,2}(\bar{x}_{j,q}), \dots, S_{j,m}(\bar{x}_{j,q})]^T$  为神经网络的基函数向量. 对于任意正数  $p \leq q$ , 以下不等式成立:

$$\|S_j(\bar{x}_{j,q})\|^2 \leq \|S_j(\bar{x}_{j,p})\|^2.$$

### 3 主要结果

基于反步法和动态面控制技术, 本文设计一种自适应神经网络容错控制器和自适应律, 使得闭环系统的所有信号都是半全局一致最终有界, 并使跟踪误差收敛到零的小邻域内.

在  $m$  步的反步设计过程中, 每一步都基于以下的坐标变换:

$$\begin{aligned} z_{j,1} &= y_j - y_{dj}, \\ z_{j,i} &= x_{j,i} - \alpha_{j,i,f}, \\ \chi_{j,i} &= \alpha_{j,i,f} - \alpha_{j,i-1}, \end{aligned}$$

其中,  $j = 1, 2, \dots, n$ ,  $i = 2, 3, \dots, m$ ,  $z_{j,1}$  为跟踪误差,  $z_{j,i}$  为误差面,  $x_{j,i}$  为状态变量,  $\chi_{j,i}$  为一阶滤波器的输出误差,  $\alpha_{j,i-1}$  和  $\alpha_{j,i,f}$  分别表示虚拟控制器信号和滤波器输出信号.

定义未知常数  $\theta_j$  和  $\Theta_j$  为:  $\theta_j = \max\{\|W_{j,i}\|^2\}$  和  $\Theta_j = \|W_{psj}\|^2$ , 其中,  $i = 1, 2, \dots, m$ .

第  $j, 1$  步的虚拟控制器  $\alpha_{j,1}$  和自适应律  $\hat{\Theta}_j$  为

$$\alpha_{j,1} = -k_{j,1}z_{j,1} - z_{j,1} - \frac{1}{2c_{j,1}^2}z_{j,1}\hat{\theta}_j S^T(Y_{j,1})S(Y_{j,1}) - \frac{1}{2c_{psj}^2}z_{j,1}\hat{\Theta}_j S_{psj}^T(\zeta_{psj})S_{psj}(\zeta_{psj}), \quad (10)$$

$$\dot{\hat{\Theta}}_j = \frac{\beta_j}{2c_{psj}^2}z_{j,1}^2 S_{psj}^T(\zeta_{psj})S_{psj}(\zeta_{psj}) - d_j \hat{\Theta}_j, \quad (11)$$

其中,  $j = 1, 2, \dots, n$ ,  $Y_{j,1} = [x_{j,1}, \dot{y}_{dj}]^T$ ,  $\zeta_{psj} = x_{psj}^T$ ,  $k_{j,1}$ ,  $c_{j,1}$ ,  $c_{psj}$ ,  $\beta_j$ ,  $d_j$  均为大于 0 的常数.  $\hat{\theta}_j$  是  $\theta_j$  的估计,  $\hat{\Theta}_j$  是  $\Theta_j$  的估计.

第  $j, i$  步的虚拟控制器  $\alpha_{j,i}$  为

$$\alpha_{j,i} = -k_{j,i}z_{j,i} - z_{j,i-1} - \frac{1}{2c_{j,i}^2}z_{j,i}\hat{\theta}_j S^T(Y_{j,i})S(Y_{j,i}), \quad (12)$$

其中,  $i = 2, \dots, m-1$ ,  $c_{j,i} > 0$ ,  $k_{j,i} > 0$ ,  $Y_{j,i} = \bar{x}_{j,i}$ . 此外,  $\bar{x}_{j,i} = [x_{j,1}, x_{j,2}, \dots, x_{j,i}]^T \in \mathbb{R}^i$  为系统中的第  $j$  个子系统的状态向量.

第  $j, m$  步的控制器  $\varpi_j(t)$  和自适应律  $\dot{\hat{\theta}}_j$  为

$$\varpi_j(t) = -\frac{1}{\underline{\alpha}_j} \left[ k_{j,m} z_{j,m} + \frac{1}{2} z_{j,m} + z_{j,m-1} + \frac{1}{2c_{j,m}^2} z_{j,m} \hat{\theta}_j S^T(Y_{j,m}) S(Y_{j,m}) \right] - \frac{1}{\underline{\alpha}_j} (\bar{\alpha}_j - \underline{\alpha}_j) M_m, \quad (13)$$

$$\dot{\hat{\theta}}_j = \sum_{i=1}^m \frac{r_j}{2c_{j,i}^2} z_{j,i}^2 S^T(Y_{j,i}) S(Y_{j,i}) - b_j \hat{\theta}_j, \quad (14)$$

其中,  $Y_{j,m} = x$ ,  $k_{j,m} > 0$ ,  $c_{j,m} > 0$ ,  $r_j > 0$ ,  $b_j > 0$ .

步骤  $j, 1$  选取 Lyapunov 函数为

$$V_{j,1} = \frac{1}{2} z_{j,1}^2 + \frac{1}{2r_j} \tilde{\theta}_j^2 + \frac{1}{2\beta_j} \tilde{\Theta}_j^2,$$

其中,  $\tilde{\theta}_j = \theta_j - \hat{\theta}_j$ ,  $\tilde{\Theta}_j = \Theta_j - \hat{\Theta}_j$ . 对  $V_{j,1}$  求导, 其导数为

$$\dot{V}_{j,1} = z_{j,1}(\bar{f}_{j,1}(\zeta_{j,1}) + \alpha_{j,1}) + z_{j,1}(\alpha_{j,2,f} - \alpha_{j,1}) + z_{j,1}z_{j,2} + z_{j,1}f_{psj} - \frac{1}{r_j} \tilde{\theta}_j \dot{\hat{\theta}}_j - \frac{1}{\beta_j} \tilde{\Theta}_j \dot{\hat{\Theta}}_j, \quad (15)$$

其中,  $\zeta_{j,1} = [x^T, \dot{y}_{dj}]^T$ ,  $\bar{f}_{j,1}(\zeta_{j,1}) = f_{j,1} - \dot{y}_{dj}$ . 通过使用神经网络, 对系统中的未知非线性函数  $\bar{f}_{j,1}(\zeta_{j,1})$  和  $f_{psj}(\zeta_{psj})$  进行逼近, 如下所示

$$\bar{f}_{j,1}(\zeta_{j,1}) = W_{j,1}^T S(\zeta_{j,1}) + \delta_{j,1}(\zeta_{j,1}),$$

其中,  $|\delta_{j,1}| \leq \varepsilon_{j,1}$ , 且  $\varepsilon_{j,1} > 0$ . 使用引理 1 和 Young's 不等式, 可得

$$\begin{aligned} z_{j,1} \bar{f}_{j,1}(\zeta_{j,1}) &= z_{j,1} (W_{j,1}^T S(\zeta_{j,1}) + \delta_{j,1}(\zeta_{j,1})) \\ &\leq \frac{1}{2} z_{j,1}^2 + \frac{1}{2} c_{j,1}^2 + \frac{1}{2} \varepsilon_{j,1}^2 + \frac{1}{2c_{j,1}^2} z_{j,1}^2 \theta_j S^T(Y_{j,1}) S(Y_{j,1}), \end{aligned} \quad (16)$$

其中,  $Y_{j,1} = [x_{j,1}, \dot{y}_{dj}]^T$ . 类似地,  $f_{psj}(\zeta_{psj}) = W_{psj}^T S_{psj}(\zeta_{psj}) + \delta_{psj}(\zeta_{psj})$ , 其中,  $|\delta_{psj}| \leq \varepsilon_{psj}$ , 且  $\varepsilon_{psj} > 0$ . 使用 Young's 不等式, 可得

$$\begin{aligned} z_{j,1} f_{psj} &= z_{j,1} (W_{psj}^T S_{psj}(\zeta_{psj}) + \delta_{psj}(\zeta_{psj})) \\ &\leq \frac{1}{2} z_{j,1}^2 + \frac{1}{2} c_{psj}^2 + \frac{1}{2} \varepsilon_{psj}^2 + \frac{1}{2c_{psj}^2} z_{j,1}^2 \Theta_j S_{psj}^T(\zeta_{psj}) S_{psj}(\zeta_{psj}), \end{aligned} \quad (17)$$

其中,  $\zeta_{psj} = x_{psj}$ . 将式 (16) 和 (17) 代入式 (15), 根据式 (10) 和 (11), 可得

$$\begin{aligned} \dot{V}_{j,1} &\leq -k_{j,1} z_{j,1}^2 + \frac{1}{2c_{j,1}^2} z_{j,1}^2 \tilde{\theta}_j S^T(Y_{j,1}) S(Y_{j,1}) + \frac{1}{2c_{psj}^2} z_{j,1}^2 \tilde{\Theta}_j S_{psj}^T(\zeta_{psj}) S_{psj}(\zeta_{psj}) + \frac{1}{2} c_{j,1}^2 \\ &\quad + \frac{1}{2} \varepsilon_{j,1}^2 + \frac{1}{2} c_{psj}^2 + \frac{1}{2} \varepsilon_{psj}^2 - \frac{1}{r_j} \tilde{\theta}_j \dot{\hat{\theta}}_j + z_{j,1} z_{j,2} - \frac{1}{\beta_j} \tilde{\Theta}_j \dot{\hat{\Theta}}_j + z_{j,1}(\alpha_{j,2,f} - \alpha_{j,1}) \\ &\leq -k_{j,1} z_{j,1}^2 + z_{j,1} z_{j,2} + \frac{1}{2} c_{j,1}^2 + \frac{1}{2} c_{psj}^2 + \frac{1}{2} \varepsilon_{psj}^2 + \frac{1}{r_j} \tilde{\theta}_j \left[ \frac{r_j}{2c_{j,1}^2} z_{j,1}^2 S^T(Y_{j,1}) S(Y_{j,1}) - \dot{\hat{\theta}}_j \right] \\ &\quad + \frac{1}{2} \varepsilon_{j,1}^2 + z_{j,1}(\alpha_{j,2,f} - \alpha_{j,1}) + \frac{d_j}{\beta_j} \tilde{\Theta}_j \dot{\hat{\Theta}}_j. \end{aligned} \quad (18)$$

对  $\frac{d_j}{\beta_j} \tilde{\Theta}_j \hat{\Theta}_j$  使用 Young's 不等式放缩, 可得

$$\frac{d_j}{\beta_j} \tilde{\Theta}_j \hat{\Theta}_j \leq -\frac{d_j}{2\beta_j} \tilde{\Theta}_j^2 + \frac{d_j}{2\beta_j} \Theta_j^2. \quad (19)$$

将式 (19) 代入式 (18), 可得

$$\dot{V}_{j,1} \leq -k_{j,1} z_{j,1}^2 + z_{j,1} z_{j,2} + \frac{1}{r_j} \tilde{\theta}_j \left[ \frac{r_j}{2c_{j,1}^2} z_{j,1}^2 S^T(Y_{j,1}) S(Y_{j,1}) - \dot{\hat{\theta}}_j \right] + z_{j,1} (\alpha_{j,2,f} - \alpha_{j,1}) + \Xi_{j,1} - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2,$$

其中,  $\Xi_{j,1} = \frac{1}{2} c_{j,1}^2 + \frac{1}{2} \varepsilon_{j,1}^2 + \frac{1}{2} c_{psj}^2 + \frac{1}{2} \varepsilon_{psj}^2 + \frac{d_j}{2\beta_j} \Theta_j^2$ .

根据文献 [6, 34, 35], 利用动态面控制技术, 将虚拟控制器信号  $\alpha_{j,1}$  进行滤波, 一阶滤波器的形式如下:

$$\omega_{j,2} \dot{\alpha}_{j,2,f} + \alpha_{j,2,f} = \alpha_{j,1}, \quad \alpha_{j,2,f}(0) = \alpha_{j,1}(0), \quad (20)$$

其中,  $\alpha_{j,2,f}$  表示滤波器输出信号,  $\omega_{j,2}$  表示时间常数, 定义  $\chi_{j,2} = \alpha_{j,2,f} - \alpha_{j,1}$ , 则  $\dot{\alpha}_{j,2,f} = -\frac{\chi_{j,2}}{\omega_{j,2}}$ , 且  $\dot{\chi}_{j,2} = \dot{\alpha}_{j,2,f} - \dot{\alpha}_{j,1} = -\frac{\chi_{j,2}}{\omega_{j,2}} + D_{j,2}$ , 其中  $D_{j,2} = -\frac{\partial \alpha_{j,1}}{\partial x_{j,1}} (f_{j,1} + x_{j,2}) - \sum_{l=0}^1 \frac{\partial \alpha_{j,1}}{\partial y_{dj}^{(l)}} y_{dj}^{(l+1)} - \frac{\partial \alpha_{j,1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j - \frac{\partial \alpha_{j,1}}{\partial \tilde{\Theta}_j} \dot{\tilde{\Theta}}_j$ . 那么

$$\dot{V}_{j,1} \leq -k_{j,1} z_{j,1}^2 + \frac{1}{r_j} \tilde{\theta}_j \left[ \frac{r_j}{2c_{j,1}^2} z_{j,1}^2 S^T(Y_{j,1}) S(Y_{j,1}) - \dot{\hat{\theta}}_j \right] - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2 + \Xi_{j,1} + z_{j,1} \chi_{j,2} + z_{j,1} z_{j,2}.$$

**注释1** 我们利用动态面控制技术将复杂的求偏导运算变成简单的代数运算, 从而减少计算负担, 使控制器的设计过程变得简单, 即在反步法控制设计的第  $i$  步 ( $i = 1, 2, \dots, m-1$ ) 中引入一阶滤波器, 利用滤波器的输出信号  $\alpha_{j,i+1,f}$  替代虚拟控制器  $\alpha_{j,i}$ , 在设计第  $i+1$  步的虚拟控制器  $\alpha_{j,i+1}$  时避免对第  $i$  步的虚拟控制器  $\alpha_{j,i}$  求偏导.

**步骤  $j, i$  ( $i = 2, 3, \dots, m-1$ )** 选取 Lyapunov 函数为

$$V_{j,i} = V_{j,i-1} + \frac{1}{2} z_{j,i}^2 + \frac{1}{2} \chi_{j,i}^2,$$

对  $V_{j,i}$  求导, 其导数为

$$\begin{aligned} \dot{V}_{j,i} &\leq -\sum_{l=1}^{i-1} k_{j,l} z_{j,l}^2 + z_{j,i-1} z_{j,i} + z_{j,i} z_{j,i+1} + \sum_{l=1}^{i-1} z_{j,l} \chi_{j,l+1} + z_{j,i} (\alpha_{j,i+1,f} - \alpha_{j,i}) + \Xi_{j,i-1} \\ &\quad + \sum_{l=2}^i \left( \frac{z_{j,l} \chi_{j,l}}{\omega_{j,l}} - \frac{\chi_{j,l}^2}{\omega_{j,l}} + \chi_{j,l} D_{j,l} \right) + \frac{1}{r_j} \tilde{\theta}_j \left[ \sum_{l=1}^{i-1} \frac{r_j}{2c_{j,l}^2} z_{j,l}^2 S^T(Y_{j,l}) S(Y_{j,l}) - \dot{\hat{\theta}}_j \right] \\ &\quad - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2 + z_{j,i} (\bar{f}_{j,i}(\zeta_{j,i}) + \alpha_{j,i}), \end{aligned} \quad (21)$$

其中,  $\Xi_{j,i-1} = \frac{1}{2} \sum_{l=1}^{i-1} (c_{j,l}^2 + \varepsilon_{j,l}^2) + \frac{1}{2} c_{psj}^2 + \frac{1}{2} \varepsilon_{psj}^2 + \frac{d_j}{2\beta_j} \Theta_j^2$ ,  $\zeta_{j,i} = x$ ,  $\bar{f}_{j,i}(\zeta_{j,i}) = f_{j,i}$ . 未知非线性函数  $\bar{f}_{j,i}(\zeta_{j,i})$  通过神经网络进行逼近, 类似地,  $\bar{f}_{j,i}(\zeta_{j,i}) = W_{j,i}^T S(\zeta_{j,i}) + \delta_{j,i}(\zeta_{j,i})$ , 其中,  $|\delta_{j,i}| \leq \varepsilon_{j,i}$ , 且  $\varepsilon_{j,i} > 0$ . 利用引理 1, 与式 (16) 类似, 可以得到

$$z_{j,i} \bar{f}_{j,i} \leq \frac{1}{2} z_{j,i}^2 + \frac{1}{2} c_{j,i}^2 + \frac{1}{2} \varepsilon_{j,i}^2 + \frac{1}{2c_{j,i}^2} z_{j,i}^2 \theta_j S^T(Y_{j,i}) S(Y_{j,i}), \quad (22)$$

其中,  $Y_{j,i} = \bar{x}_{j,i} = [x_{j,1}, x_{j,2}, \dots, x_{j,i}]^T$ . 将式 (22) 代入式 (21), 并根据式 (12), 可以得到

$$\begin{aligned} \dot{V}_{j,i} \leq & -\sum_{l=1}^i k_{j,l} z_{j,l}^2 + z_{j,i} z_{j,i+1} + \sum_{l=1}^{i-1} z_{j,l} \chi_{j,l+1} + \frac{1}{r_j} \tilde{\theta}_j \left[ \sum_{l=1}^i \frac{r_j}{2c_{j,l}^2} z_{j,l}^2 S^T(Y_{j,l}) S(Y_{j,l}) - \dot{\hat{\theta}}_j \right] \\ & + \sum_{l=2}^i \left( \frac{z_{j,l} \chi_{j,l}}{\omega_{j,l}} - \frac{\chi_{j,l}^2}{\omega_{j,l}} + \chi_{j,l} D_{j,l} \right) + \Xi_{j,i} + z_{j,i} (\alpha_{j,i+1,f} - \alpha_{j,i}) - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2, \end{aligned}$$

其中,  $\Xi_{j,i} = \frac{1}{2} \sum_{l=1}^i (c_{j,l}^2 + \varepsilon_{j,l}^2) + \frac{1}{2} c_{psj}^2 + \frac{1}{2} \varepsilon_{psj}^2 + \frac{d_j}{2\beta_j} \Theta_j^2$ .

类似地, 将虚拟控制器信号  $\alpha_{j,i}$  进行滤波, 一阶滤波器的形式如下:

$$\omega_{j,i+1} \dot{\alpha}_{j,i+1,f} + \alpha_{j,i+1,f} = \alpha_{j,i}, \quad \alpha_{j,i+1,f}(0) = \alpha_{j,i}(0), \quad (23)$$

其中,  $\alpha_{j,i+1,f}$  表示滤波器输出信号,  $\omega_{j,i+1}$  表示时间常数. 设  $\chi_{j,i+1} = \alpha_{j,i+1,f} - \alpha_{j,i}$ , 则  $\dot{\alpha}_{j,i+1,f} = -\frac{\chi_{j,i+1}}{\omega_{j,i+1}}$ , 且  $\dot{\chi}_{j,i+1} = -\frac{\chi_{j,i+1}}{\omega_{j,i+1}} + D_{j,i+1}$ , 其中  $D_{j,i+1} = -\sum_{l=0}^1 \frac{\partial \alpha_{j,i+1}}{\partial y_{d_j}^{(l+1)}} g_{d_j}^{(l+1)} - \sum_{l=1}^i \frac{\partial \alpha_{j,i}}{\partial x_{j,l}} (f_{j,l} + x_{j,l+1}) - \frac{\partial \alpha_{j,i}}{\partial \theta_j} \dot{\hat{\theta}}_j$ . 那么

$$\begin{aligned} \dot{V}_{j,i} \leq & -\sum_{l=1}^i k_{j,l} z_{j,l}^2 + z_{j,i} z_{j,i+1} + \sum_{l=1}^{i-1} z_{j,l} \chi_{j,l+1} + \frac{1}{r_j} \tilde{\theta}_j \left[ \sum_{l=1}^i \frac{r_j}{2c_{j,l}^2} z_{j,l}^2 S^T(Y_{j,l}) S(Y_{j,l}) - \dot{\hat{\theta}}_j \right] \\ & + \sum_{l=2}^i \left( \frac{z_{j,l} \chi_{j,l}}{\omega_{j,l}} - \frac{\chi_{j,l}^2}{\omega_{j,l}} + \chi_{j,l} D_{j,l} \right) + \Xi_{j,i} - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2. \end{aligned}$$

步骤  $j, m$  选取 Lyapunov 函数为

$$V_{j,m} = V_{j,m-1} + \frac{1}{2} z_{j,m}^2 + \frac{1}{2} \chi_{j,m}^2,$$

对  $V_{j,m}$  求导, 其导数为

$$\begin{aligned} \dot{V}_{j,m} \leq & -\sum_{l=1}^{m-1} k_{j,l} z_{j,l}^2 + z_{j,m-1} z_{j,m} + \sum_{l=1}^{m-1} z_{j,l} \chi_{j,l+1} + \frac{1}{r_j} \tilde{\theta}_j \left[ \sum_{l=1}^{m-1} \frac{r_j}{2c_{j,l}^2} z_{j,l}^2 S^T(Y_{j,l}) S(Y_{j,l}) - \dot{\hat{\theta}}_j \right] \\ & + z_{j,m} (\bar{f}_{j,m}(\zeta_{j,m}) + u_j) - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2 + \sum_{l=2}^m \left( \frac{z_{j,l} \chi_{j,l}}{\omega_{j,l}} - \frac{\chi_{j,l}^2}{\omega_{j,l}} + \chi_{j,l} D_{j,l} \right) + \Xi_{j,m-1}, \quad (24) \end{aligned}$$

其中,  $\Xi_{j,m-1} = \frac{1}{2} \sum_{l=1}^{m-1} (c_{j,l}^2 + \varepsilon_{j,l}^2) + \frac{1}{2} c_{psj}^2 + \frac{1}{2} \varepsilon_{psj}^2 + \frac{d_j}{2\beta_j} \Theta_j^2$ ,  $\zeta_{j,m} = x$ ,  $\bar{f}_{j,m}(\zeta_{j,m}) = f_{j,m}$ . 未知非线性函数  $\bar{f}_{j,m}(\zeta_{j,m})$  通过神经网络进行逼近, 类似地,  $\bar{f}_{j,m}(\zeta_{j,m}) = W_{j,m}^T S(\zeta_{j,m}) + \delta_{j,m}(\zeta_{j,m})$ , 其中,  $|\delta_{j,m}| \leq \varepsilon_{j,m}$ , 且  $\varepsilon_{j,m} > 0$ . 利用引理 1, 与式 (16) 和 (22) 类似, 可以得到

$$z_{j,m} \bar{f}_{j,m} \leq \frac{1}{2} z_{j,m}^2 + \frac{1}{2} c_{j,m}^2 + \frac{1}{2} \varepsilon_{j,m}^2 + \frac{1}{2c_{j,m}^2} z_{j,m}^2 \theta_j S^T(Y_{j,m}) S(Y_{j,m}), \quad (25)$$

其中,  $Y_{j,m} = x$ . 根据式 (6), (8), (9) 和假设 1, 可得

$$\underline{\alpha}_j \rho_j(t) \varpi_j(t) + \lambda_j(t) \leq (\bar{\alpha}_j - \underline{\alpha}_j) M_m + \bar{\lambda}_j. \quad (26)$$

将式 (25) 和 (26) 代入式 (24), 可以得到

$$\dot{V}_{j,m} \leq -\sum_{l=1}^{m-1} k_{j,l} z_{j,l}^2 + z_{j,m-1} z_{j,m} + \sum_{l=1}^{m-1} z_{j,l} \chi_{j,l+1} + \frac{1}{2} z_{j,m}^2 + \sum_{l=2}^m \left( \frac{z_{j,l} \chi_{j,l}}{\omega_{j,l}} - \frac{\chi_{j,l}^2}{\omega_{j,l}} + \chi_{j,l} D_{j,l} \right)$$



$$\begin{aligned}
& + \frac{1}{r_j} \tilde{\theta}_j \left[ \sum_{l=1}^m \frac{r_j}{2c_{j,l}^2} z_{j,l}^2 S^T(Y_{j,l}) S(Y_{j,l}) - \dot{\theta}_j \right] + \frac{1}{2c_{j,m}^2} z_{j,m}^2 \hat{\theta}_j S^T(Y_{j,m}) S(Y_{j,m}) - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2 \\
& + \Xi_{j,m} + z_{j,m} [\underline{\alpha}_j \varpi_j(t) + (\bar{\alpha}_j - \underline{\alpha}_j) M_m + \bar{\lambda}_j],
\end{aligned}$$

其中,  $\Xi_{j,m} = \frac{1}{2} \sum_{l=1}^m (c_{j,l}^2 + \varepsilon_{j,l}^2) + \frac{1}{2} c_{psj}^2 + \frac{1}{2} \varepsilon_{psj}^2 + \frac{d_j}{2\beta_j} \Theta_j^2$ . 对  $z_{j,m} \bar{\lambda}_j$  使用 Young's 不等式放缩, 可得

$$z_{j,m} \bar{\lambda}_j \leq \frac{z_{j,m}^2}{2} + \frac{\bar{\lambda}_j^2}{2}. \quad (27)$$

根据式 (13), (14) 和 (27), 可得

$$\begin{aligned}
\dot{V}_{j,m} & \leq - \sum_{l=1}^m k_{j,l} z_{j,l}^2 + \sum_{l=1}^{m-1} z_{j,l} \chi_{j,l+1} - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2 + \sum_{l=2}^m \left( \frac{z_{j,l} \chi_{j,l}}{\omega_{j,l}} - \frac{\chi_{j,l}^2}{\omega_{j,l}} + \chi_{j,l} D_{j,l} \right) + \Xi_{j,m} \\
& + \frac{b_j}{r_j} \tilde{\theta}_j \hat{\theta}_j + \frac{z_{j,m}^2}{2} + \frac{\bar{\lambda}_j^2}{2}.
\end{aligned} \quad (28)$$

使用 Young's 不等式, 可得

$$\sum_{l=1}^{m-1} z_{j,l} \chi_{j,l+1} \leq \sum_{l=1}^{m-1} \frac{1}{2} z_{j,l}^2 + \sum_{l=2}^m \frac{1}{2} \chi_{j,l}^2, \quad (29)$$

$$\frac{b_j}{r_j} \tilde{\theta}_j \hat{\theta}_j \leq -\frac{b_j}{2r_j} \tilde{\theta}_j^2 + \frac{b_j}{2r_j} \theta_j^2, \quad (30)$$

$$\sum_{l=2}^m \left( \frac{z_{j,l} \chi_{j,l}}{\omega_{j,l}} - \frac{\chi_{j,l}^2}{\omega_{j,l}} + \chi_{j,l} D_{j,l} \right) \leq \sum_{l=2}^m \left( \frac{z_{j,l}^2}{2\omega_{j,l}} - \frac{\chi_{j,l}^2}{2\omega_{j,l}} + \frac{\chi_{j,l}^2 D_{j,l}^2}{2\pi_j} + \frac{\pi_j}{2} \right), \quad (31)$$

其中,  $\pi_j > 0$  为设计参数. 将式 (29)~(31) 代入式 (28), 可得

$$\dot{V}_{j,m} \leq - \left( k_{j,1} - \frac{1}{2} \right) z_{j,1}^2 - \sum_{l=2}^m \left( k_{j,l} - \frac{1}{2} - \frac{1}{2\omega_{j,l}} \right) z_{j,l}^2 + \bar{\Xi}_{j,m} - \sum_{l=2}^m \left( -\frac{1}{2} + \frac{1}{2\omega_{j,l}} - \frac{D_{j,l}^2}{2\pi_j} \right) \chi_{j,l}^2 - \frac{b_j}{2r_j} \tilde{\theta}_j^2 - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2,$$

其中,  $\bar{\Xi}_{j,m} = \Xi_{j,m} + \frac{\pi_j(m-1)}{2} + \frac{b_j}{2r_j} \theta_j^2 + \frac{\bar{\lambda}_j^2}{2}$ .

选取系统的 Lyapunov 函数为

$$V = \sum_{j=1}^n V_{j,m}.$$

对  $V$  进行求导, 可得

$$\begin{aligned}
\dot{V} & \leq \sum_{j=1}^n \left[ - \left( k_{j,1} - \frac{1}{2} \right) z_{j,1}^2 - \sum_{l=2}^m \left( k_{j,l} - \frac{1}{2} - \frac{1}{2\omega_{j,l}} \right) z_{j,l}^2 - \sum_{l=2}^m \left( -\frac{1}{2} + \frac{1}{2\omega_{j,l}} - \frac{D_{j,l}^2}{2\pi_j} \right) \chi_{j,l}^2 \right. \\
& \left. - \frac{b_j}{2r_j} \tilde{\theta}_j^2 - \frac{d_j}{2\beta_j} \tilde{\Theta}_j^2 \right] + \Delta,
\end{aligned} \quad (32)$$

其中,  $\Delta = \sum_{j=1}^n \bar{\Xi}_{j,m}$ .

令  $C_j = \min\{(2k_{j,1} - 1), (2k_{j,2} - 1 - \frac{1}{\omega_{j,2}}), \dots, (2k_{j,m} - 1 - \frac{1}{\omega_{j,m}}), (-1 + \frac{1}{\omega_{j,2}} - \frac{D_{j,2}^2}{\pi_j}), \dots, (-1 + \frac{1}{\omega_{j,m}} - \frac{D_{j,m}^2}{\pi_j}), b_j, d_j \mid j = 1, 2, \dots, n\}$ , 其中,  $(2k_{j,1} - 1) > 0$ ,  $(2k_{j,l} - 1 - \frac{1}{\omega_{j,l}}) > 0$ ,  $(-1 + \frac{1}{\omega_{j,l}} - \frac{D_{j,l}^2}{\pi_j}) > 0$ ,  $b_j > 0$ ,  $d_j > 0$ ,  $l = 2, 3, \dots, m$ , 以及  $C = \min\{C_1, C_2, \dots, C_n\}$ .

那么, 式 (32) 可以写成

$$\dot{V} \leq -CV + \Delta. \quad (33)$$

对不等式 (33) 两边同时积分可得

$$0 \leq V(t) \leq V(0)e^{-Ct} + \frac{\Delta}{C}, \quad (34)$$

由式 (34) 可得闭环系统中的所有信号都是半全局一致最终有界的. 从而可得

$$\lim_{t \rightarrow \infty} |z_{j,1}| \leq \lim_{t \rightarrow \infty} \sqrt{2V(0)e^{-Ct} + \frac{2\Delta}{C}} \leq \sqrt{\frac{2\Delta}{C}}, \quad (35)$$

因此, 由式 (35) 可知, 通过选择适当的设计参数, 使  $C$  足够大或者  $\Delta$  足够小, 则能使系统的跟踪误差收敛至原点附近的紧集内.

**定理 1** 针对一类具有传感器故障和输入死区的非严格反馈非线性系统 (1), 设计虚拟控制器 (10), (12), 实际控制器 (13), 一阶滤波器 (20), (23) 及自适应律 (11), (14), 通过选择合适的参数使得闭环系统内所有信号都是半全局一致最终有界的, 且能使跟踪误差收敛至原点附近的紧集内.

根据以上的分析, 提出自适应神经网络容错控制的设计算法<sup>[36]</sup>, 如算法 1 所示.

---

#### Algorithm 1

---

**Step 1.** Determine the number of neural network nodes  $\iota$ , and design the Gaussian function  $S_j(\zeta)$ .

**Step 2.** Based on Step 1, define the neural network  $W^T S(\zeta)$  to identify the nonlinear functions of the MIMO systems (1).

**Step 3.** Select appropriate design parameters  $k_{j,1} > 0$ ,  $c_{j,1} > 0$ ,  $c_{psj} > 0$ ,  $\beta_j > 0$ ,  $d_j > 0$ , and design virtual control signal  $\alpha_{j,1}$  (10), adaptive law  $\hat{\theta}_j$  (11) and first-order filter (20), where  $j = 1, 2, \dots, n$ .

**Step 4.** Provide approximate design parameters  $k_{j,i} > 0$ ,  $c_{j,i} > 0$ , and design virtual control signals  $\alpha_{j,i}$  (12) and first-order filter (23), where  $i = 2, 3, \dots, m-1$ .

**Step 5.** Choose appropriate design parameters  $k_{j,m} > 0$ ,  $c_{j,m} > 0$ ,  $r_j > 0$ ,  $b_j > 0$ , and determine the actual control signal  $\varpi_j(t)$  (13) and adaptive law  $\hat{\theta}_j$  (14) for the MIMO nonlinear systems (1).

---

## 4 仿真算例

仿真 1. 通过一个数值仿真实验的结果验证本文所提出的控制方法的有效性, 考虑如下三阶系统:

$$\dot{x}_{j,i} = f_{j,i} + x_{j,i+1},$$

$$\dot{x}_{j,m} = f_{j,m} + u_j,$$

$$y_j = K_j x_{j,1} + q_j,$$

其中,  $j = 1, 2$ ,  $i = 1, 2$ ,  $m = 3$ , 非线性函数  $f_{1,1} = \frac{x_{1,1}x_{1,2}}{2} + \frac{1}{10(1+x_{1,3}^2)} + \frac{1}{2+x_{2,1}^2}$ ,  $f_{1,2} = x_{1,1}x_{1,2}x_{1,3} + \frac{1}{1+x_{2,3}^2}$ ,  $f_{1,3} = -x_{1,2}(\sin(x_{1,3}))^2 + x_{2,3} + \frac{1}{1+x_{1,1}^2}$ ,  $f_{2,1} = \frac{x_{2,1}x_{2,2}}{2} + \frac{1}{10(1+x_{2,3}^2)} + \frac{1}{2+x_{1,2}^2}$ ,  $f_{2,2} = x_{2,1}x_{2,2}x_{2,3} + \frac{1}{1+x_{1,3}^2}$ ,  $f_{2,3} = -x_{2,2}(\sin(x_{2,3}))^2 + x_{1,3} + \frac{1}{1+x_{2,1}^2}$ . 假设系统的参考信号为:  $y_{d1} = y_{d2} = \sin(t)$ .

根据上述系统, 选取适当的设计参数为:  $c_{1,1} = c_{2,1} = 15$ ,  $c_{1,2} = c_{2,2} = 18$ ,  $c_{1,3} = c_{2,3} = 0.5$ ,  $k_{1,1} = 24$ ,  $k_{1,2} = 85$ ,  $k_{1,3} = 100$ ,  $k_{2,1} = 18$ ,  $k_{2,2} = 75$ ,  $k_{2,3} = 100$ ,  $c_{ps1} = c_{ps2} = 1.5$ ,  $r_1 = r_2 = 0.1$ ,  $\beta_1 = 9$ ,  $\beta_2 = 12$ ,  $b_1 = b_2 = 3.2$ ,  $d_1 = d_2 = 2$ ,  $\omega_{1,2} = 0.08$ ,  $\omega_{1,3} = 0.01$ ,  $\omega_{2,2} = 0.32$ ,  $\omega_{2,3} = 0.012$ ,

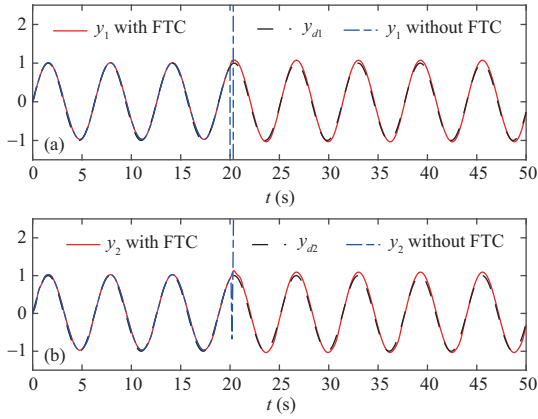


图 3 (网络版彩图) 仿真 1 输出信号  $y_j$  和参考信号  $y_{dj}$   
**Figure 3** (Color online) (a) Output  $y_1$  and reference signal  $y_{d1}$ ; (b) output  $y_2$  and reference signal  $y_{d2}$  in simulation 1.

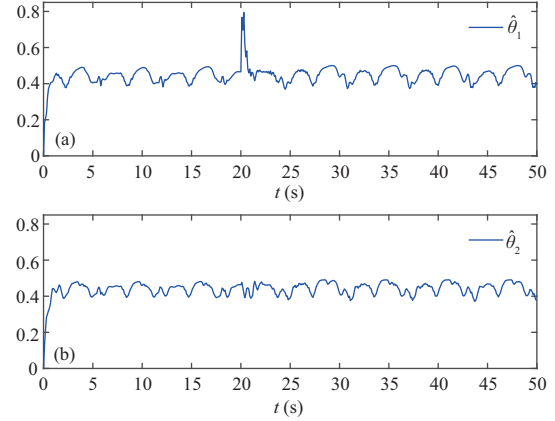


图 4 (网络版彩图) 仿真 1 自适应参数 (a)  $\hat{\theta}_1$  和 (b)  $\hat{\theta}_2$   
**Figure 4** (Color online) Adaptive parameters (a)  $\hat{\theta}_1$  and (b)  $\hat{\theta}_2$  in simulation 1.

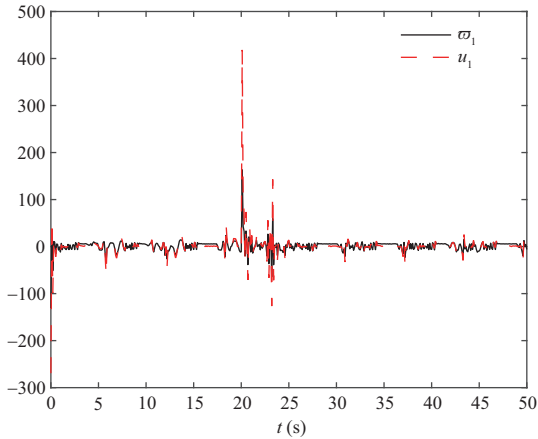


图 5 (网络版彩图) 仿真 1 死区输入  $w_1$  与死区输出  $u_1$   
**Figure 5** (Color online) Dead zone input  $w_1$  and dead zone output  $u_1$  in simulation 1.

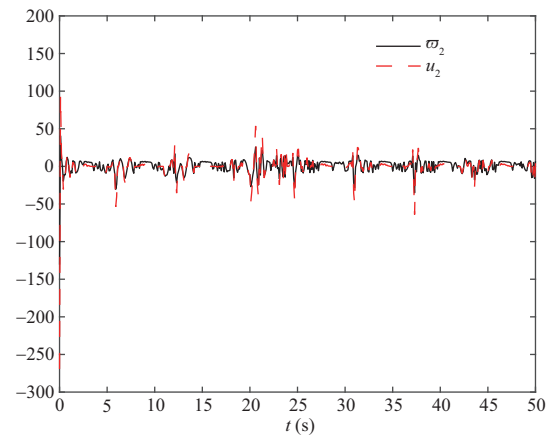


图 6 (网络版彩图) 仿真 1 死区输入  $w_2$  与死区输出  $u_2$   
**Figure 6** (Color online) Dead zone input  $w_2$  and dead zone output  $u_2$  in simulation 1.

$M_{m1} = M_{m2} = 580$ . 选取适当的传感器故障参数为:  $K_1 = K_2 = 0.9$ ,  $q_1 = q_2 = 0$ . 选取不对称输入死区的参数为:  $o_{1r} = o_{2r} = 2.65$ ,  $o_{1l} = o_{2l} = 2.15$ ,  $p_{1r} = p_{2r} = p_{1l} = p_{2l} = 5.4$ . 系统的初始状态取值如下:  $[x_{1,1}(0), x_{1,2}(0), x_{1,3}(0), x_{2,1}(0), x_{2,2}(0), x_{2,3}(0)]^T = [0.01, 0.01, 0.01, 0.01, 0.01, 0.01]^T$ . 假设自适应参数的初始状态为:  $\hat{\theta}_1(0) = \hat{\theta}_2(0) = 0$ ,  $\hat{\Theta}_{ps1}(0) = \hat{\Theta}_{ps2}(0) = 0$ .

仿真结果如图 3~6 所示. 图 3(a) 表示输出信号  $y_1$  与参考信号  $y_{d1}$  的跟踪效果图, 图 3(b) 表示输出信号  $y_2$  与参考信号  $y_{d2}$  的跟踪效果图, 由图 3 可知, 从  $t = 20$  s 开始, 系统发生传感器故障, 有容错的控制方案使系统的输出信号 (红色实线) 仍能较好地跟踪上参考信号 (黑色虚线), 而无容错的控制方案无法使系统的输出信号 (蓝色点划线) 跟踪上参考信号 (黑色虚线). 图 4(a) 和 (b) 分别表示自适应参数  $\hat{\theta}_1$  和  $\hat{\theta}_2$  的响应曲线, 图 5 表示系统的死区输入信号  $w_1$  和死区输出信号  $u_1$  的轨迹图, 图 6 表示系统的死区输入信号  $w_2$  和死区输出信号  $u_2$  的轨迹图. 由图 4~6 可知, 当  $t = 20$  s 时, 自适应参

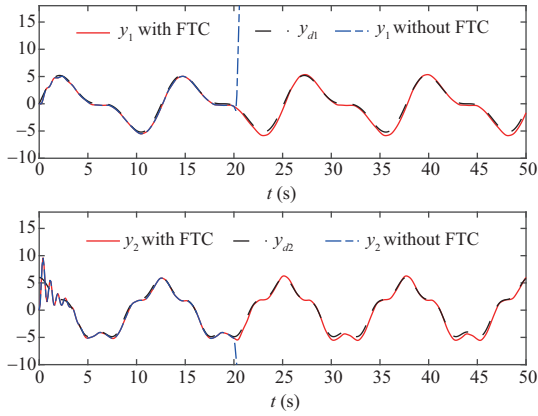


图 7 (网络版彩图) 仿真 2 输出信号  $y_j$  和参考信号  $y_{dj}$   
**Figure 7** (Color online) (a) Output  $y_1$  and reference signal  $y_{d1}$ ; (b) output  $y_2$  and reference signal  $y_{d2}$  in simulation 2.

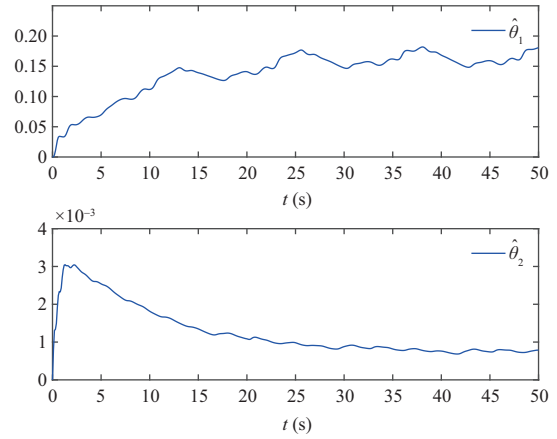


图 8 (网络版彩图) 仿真 2 自适应参数 (a)  $\hat{\theta}_1$  和 (b)  $\hat{\theta}_2$   
**Figure 8** (Color online) Adaptive parameters (a)  $\hat{\theta}_1$  and (b)  $\hat{\theta}_2$  in simulation 2.

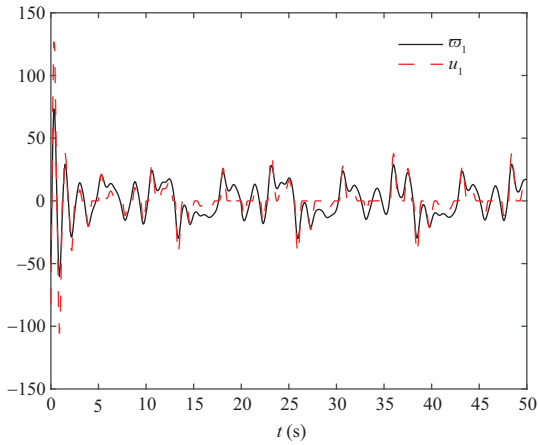


图 9 (网络版彩图) 仿真 2 死区输入  $\varpi_1$  与死区输出  $u_1$   
**Figure 9** (Color online) Dead zone input  $\varpi_1$  and dead zone output  $u_1$  in simulation 2.

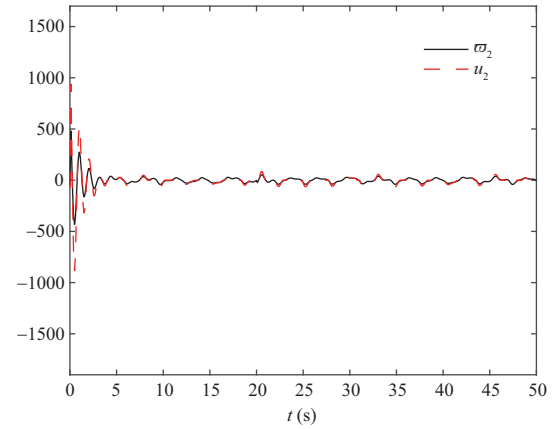


图 10 (网络版彩图) 仿真 2 死区输入  $\varpi_2$  与死区输出  $u_2$   
**Figure 10** (Color online) Dead zone input  $\varpi_2$  and dead zone output  $u_2$  in simulation 2.

数  $\hat{\theta}_1, \hat{\theta}_2$ , 死区输入信号  $\varpi_1, \varpi_2$  和死区输出信号  $u_1, u_2$  出现短暂的跳变, 但在  $t = 20$  s 后仍能保持有界.

仿真 2. 根据文献 [31], 具有传感器故障和输入死区的双倒立摆系统的动力学方程如下所示:

$$\begin{aligned} \dot{x}_{j,1} &= x_{j,2} + f_{j,1}(x), \\ \dot{x}_{j,2} &= \frac{u_j}{J_j} + A_j + f_{j,2}(x), \\ y_j &= K_j x_{j,1} + q_j, \end{aligned}$$

其中,  $j = 1, 2, A_1 = -\frac{nh^2}{4J_1} \sin(x_{2,1}), A_2 = -\frac{nh^2}{2J_2} \sin(x_{1,2})$ , 系统的参考信号为:  $y_{d1} = 4 \sin(\frac{t}{2}) + 2 \sin(t)$ ,

$y_{d2} = 5 \cos(\frac{t}{2}) + \cos(2t)$ . 假设  $f_{1,1}(x) = f_{2,1}(x) = 0$ ,  $f_{1,2}(x) = \frac{2nh^2}{4J_1} \sin(x_{2,1}) + (\frac{m_1gh}{J_1} - \frac{nh^2}{4J_1}) \sin(x_{1,1}) + \frac{nh(L-e)}{2J_1}$ ,  $f_{2,2}(x) = \frac{3nh^2}{4J_2} \sin(x_{1,2}) + (\frac{m_2gh}{J_2} - \frac{nh^2}{4J_2}) \sin(x_{2,1}) + \frac{nh(L-e)}{2J_2}$ ,  $m_1 = m_2 = 2 \text{ kg}$  表示连杆的末端质量,  $n = 10 \text{ N/m}$  表示弹簧的劲度系数,  $J_1 = J_2 = 6 \text{ kg} \cdot \text{m}^2$  表示连杆的转动惯量,  $h = 0.1 \text{ m}$  表示连杆的高度,  $g = 9.81 \text{ m/s}^2$  表示重力加速度,  $L = 0.5 \text{ m}$  表示弹簧的原长, 两个连杆之间的距离为  $e = 0.4 \text{ m}$ , 且  $e < L$ .

选取适当的设计参数为:  $c_{1,1} = c_{1,2} = 10$ ,  $c_{2,1} = c_{2,2} = 50$ ,  $c_{ps1} = c_{ps2} = 25$ ,  $r_1 = 0.1$ ,  $r_2 = 0.01$ ,  $\beta_1 = \beta_2 = 8$ ,  $b_1 = b_2 = 0.1$ ,  $d_1 = d_2 = 1$ ,  $k_{1,1} = k_{1,2} = k_{2,1} = k_{2,2} = 15$ ,  $\omega_{1,2} = 0.01$ ,  $\omega_{2,2} = 0.02$ ,  $M_{m1} = M_{m2} = 1000$ . 选取适当的传感器故障参数为:  $K_1 = K_2 = 0.9$ ,  $q_1 = q_2 = 0$ . 选取适当的死区参数为:  $p_{1r} = p_{1l} = 10$ ,  $p_{2r} = p_{2l} = 10$ ,  $o_{1r} = o_{2r} = 2$ ,  $o_{1l} = o_{2l} = 2.1$ . 假设自适应参数的初始状态如下:  $\hat{\theta}_1(0) = \hat{\theta}_2(0) = 0$ ,  $\hat{\Theta}_{ps1}(0) = \hat{\Theta}_{ps2}(0) = 0$ , 上述非线性系统的初始状态取值如下:  $[x_{1,1}(0), x_{1,2}(0), x_{2,1}(0), x_{2,2}(0)]^T = [0.01, 0.01, 0.01, 0.01]^T$ . 由图 7~10 可知, 双倒立摆系统在受传感器故障和输入死区影响的情况下, 本文所提出的自适应神经网络容错控制方法仍能保证系统的稳定性, 并能获得较好的跟踪效果.

## 5 结论

本文针对一类具有非严格反馈形式的多输入多输出非线性系统, 在同时发生传感器故障和不对称输入死区的情况下, 结合反步法和动态面控制技术设计了一种自适应神经网络容错控制方案. 在不需构造死区逆模型的基础上, 利用死区斜率的有界性有效地减小了输入死区对非线性系统造成的影响. 基于 Lyapunov 稳定性理论, 保证了闭环系统中的所有信号都是有界的, 并通过两个仿真例子验证了该方法的有效性.

## 参考文献

- 1 Ge S S, Wang C. Adaptive NN control of uncertain nonlinear pure-feedback systems. *Automatica*, 2002, 38: 671–682
- 2 Chen C L P, Wen G X, Liu Y J, et al. Observer-based adaptive backstepping consensus tracking control for high-order nonlinear semi-strict-feedback multiagent systems. *IEEE Trans Cybern*, 2016, 46: 1591–1601
- 3 Wang F K, Chen W S, Dai H, et al. Backstepping control of a quadrotor unmanned aerial vehicle based on multi-rate sampling. *Sci China Inf Sci*, 2019, 62: 019203
- 4 Liu Y J, Tong S C. Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems. *Automatica*, 2017, 76: 143–152
- 5 Chen B, Liu X P, Liu K F, et al. Direct adaptive fuzzy control of nonlinear strict-feedback systems. *Automatica*, 2009, 45: 1530–1535
- 6 Tong S C, Li Y M, Feng G, et al. Observer-based adaptive fuzzy backstepping dynamic surface control for a class of MIMO nonlinear systems. *IEEE Trans Syst Man Cybern B*, 2011, 41: 1124–1135
- 7 Chen M, Ge S S, How B. Robust adaptive neural network control for a class of uncertain MIMO nonlinear systems with input nonlinearities. *IEEE Trans Neural Netw*, 2010, 21: 796–812
- 8 Zhou Q, Shi P, Tian Y, et al. Approximation-based adaptive tracking control for MIMO nonlinear systems with input saturation. *IEEE Trans Cybern*, 2015, 45: 2119–2128
- 9 Zheng H Y, Wang P, Zou T, et al. A framework for multi-variable, semi-adaptive predictive control system. *Sci Sin Inform*, 2019, 49: 57–73 [郑洪宇, 王鹏, 邹涛, 等. 多变量半自适应预测控制系统架构. *中国科学: 信息科学*, 2019, 49: 57–73]
- 10 Chen B, Liu X P, Ge S S, et al. Adaptive fuzzy control of a class of nonlinear systems by fuzzy approximation approach. *IEEE Trans Fuzzy Syst*, 2012, 20: 1012–1021

- 11 Wang H Q, Chen B, Lin C. Approximation-based adaptive fuzzy control for a class of non-strict-feedback stochastic nonlinear systems. *Sci China Inf Sci*, 2014, 57: 032203
- 12 Li Y M, Tong S C. Command-filtered-based fuzzy adaptive control design for MIMO-switched nonstrict-feedback nonlinear systems. *IEEE Trans Fuzzy Syst*, 2017, 25: 668–681
- 13 Ma H, Zhou Q, Bai L, et al. Observer-based adaptive fuzzy fault-tolerant control for stochastic nonstrict-feedback nonlinear systems with input quantization. *IEEE Trans Syst Man Cybern Syst*, 2019, 49: 287–298
- 14 Wang F, Chen B, Lin C, et al. Distributed adaptive neural control for stochastic nonlinear multiagent systems. *IEEE Trans Cybern*, 2017, 47: 1795–1803
- 15 Sun Y M, Chen B, Lin C, et al. Finite-time adaptive control for a class of nonlinear systems with nonstrict feedback structure. *IEEE Trans Cybern*, 2018, 48: 2774–2782
- 16 Chai T Y. Industrial process control systems: research status and development direction. *Sci Sin Inform*, 2016, 46: 1003–1015 [柴天佑. 工业过程控制系统研究现状与发展方向. *中国科学: 信息科学*, 2016, 46: 1003–1015]
- 17 Lu R Q, Yu W W, Lu J H, et al. Synchronization on complex networks of networks. *IEEE Trans Neural Netw Learn Syst*, 2014, 25: 2110–2118
- 18 Chai Y, Mao W B, Ren H, et al. Research on operational safety assessment for spacecraft launch system: progress and challenges. *Act Autom Sin*, 2019, 45: 1829–1845 [柴毅, 毛万标, 任浩, 等. 航天发射系统运行安全性评估研究进展与挑战. *自动化学报*, 2019, 45: 1829–1845]
- 19 Zhang K, Zhou D H, Chai Y. Review of multiple fault diagnosis methods. *Control Theory Appl*, 2015, 32: 1143–1157 [张可, 周东华, 柴毅. 复合故障诊断技术综述. *控制理论与应用*, 2015, 32: 1143–1157]
- 20 Shaker M S, Patton R J. Active sensor fault tolerant output feedback tracking control for wind turbine systems via T-S model. *Eng Appl Artif Intell*, 2014, 34: 1–12
- 21 Li H Y, Gao Y B, Shi P, et al. Observer-based fault detection for nonlinear systems with sensor fault and limited communication capacity. *IEEE Trans Autom Control*, 2016, 61: 2745–2751
- 22 Zhai D, An L W, Li X J, et al. Adaptive fault-tolerant control for nonlinear systems with multiple sensor faults and unknown control directions. *IEEE Trans Neural Netw Learn Syst*, 2018, 29: 4436–4446
- 23 Lu R Q, Xu Y, Xue A K, et al. Networked control with state reset and quantized measurements: observer-based case. *IEEE Trans Ind Electron*, 2013, 60: 5206–5213
- 24 Yan X G, Edwards C. Nonlinear robust fault reconstruction and estimation using a sliding mode observer. *Automatica*, 2007, 43: 1605–1614
- 25 Khebbache H, Tadjine M, Labiod S. Adaptive sensor-fault tolerant control for a class of MIMO uncertain nonlinear systems: adaptive nonlinear filter-based dynamic surface control. *J Franklin Inst*, 2016, 353: 1313–1338
- 26 Bounemour A, Chemachema M, Essounbouli N. Indirect adaptive fuzzy fault-tolerant tracking control for MIMO nonlinear systems with actuator and sensor failures. *ISA Trans*, 2018, 79: 45–61
- 27 Zhang L L, Yang G H. Observer-based fuzzy adaptive sensor fault compensation for uncertain nonlinear strict-feedback systems. *IEEE Trans Fuzzy Syst*, 2018, 26: 2301–2310
- 28 Wang W, Xie B, Zuo Z Y, et al. Adaptive backstepping control of uncertain gear transmission servosystems with asymmetric dead-zone nonlinearity. *IEEE Trans Ind Electron*, 2019, 66: 3752–3762
- 29 Yu J P, Shi P, Dong W J, et al. Adaptive fuzzy control of nonlinear systems with unknown dead zones based on command filtering. *IEEE Trans Fuzzy Syst*, 2018, 26: 46–55
- 30 Li H Y, Zhao S Y, He W, et al. Adaptive finite-time tracking control of full state constrained nonlinear systems with dead-zone. *Automatica*, 2019, 100: 99–107
- 31 Li Y M, Tong S C, Li T S. Observer-based adaptive fuzzy tracking control of MIMO stochastic nonlinear systems with unknown control directions and unknown dead zones. *IEEE Trans Fuzzy Syst*, 2015, 23: 1228–1241
- 32 Luo X Y, Wu X J, Guan X P. Adaptive backstepping fault-tolerant control for unmatched non-linear systems against actuator dead-zone. *IET Control Theory Appl*, 2010, 4: 879–888
- 33 Chen M, Tao G. Adaptive fault-tolerant control of uncertain nonlinear large-scale systems with unknown dead zone. *IEEE Trans Cybern*, 2016, 46: 1851–1862
- 34 Zhang T P, Ge S S. Adaptive dynamic surface control of nonlinear systems with unknown dead zone in pure feedback form. *Automatica*, 2008, 44: 1895–1903
- 35 Shen Q K, Jiang B, Cocquempot V. Adaptive fuzzy observer-based active fault-tolerant dynamic surface control for a

- class of nonlinear systems with actuator faults. *IEEE Trans Fuzzy Syst*, 2014, 22: 338–349
- 36 Li H Y, Wang L J, Du H P, et al. Adaptive fuzzy backstepping tracking control for strict-feedback systems with input delay. *IEEE Trans Fuzzy Syst*, 2017, 25: 642–652

## Adaptive neural network fault-tolerant control for MIMO systems with dead zone inputs

Qi ZHOU<sup>1,2</sup>, Guohuai LIN<sup>1,2</sup>, Hui MA<sup>1,2</sup> & Renquan LU<sup>1,2\*</sup>

1. *School of Automation, Guangdong University of Technology, Guangzhou 510006, China;*

2. *Guangdong Province Key Laboratory of Intelligent Decision and Cooperative Control, Guangdong University of Technology, Guangzhou 510006, China*

\* Corresponding author. E-mail: lurenquan2012@163.com

**Abstract** In this paper, an adaptive neural network fault-tolerant control (FTC) strategy is proposed for a class of multi-input and multi-output (MIMO) nonlinear systems in nonstrict-feedback form, in which sensor faults and nonsymmetric dead zone inputs are considered. Based on the backstepping method, the desired controller is designed. The adaptive neural network control approach is proposed to handle the problem of sensor faults. The effect of dead zone inputs on the system performance can be compensated by virtue of the boundedness of dead zone slopes. Furthermore, the dynamic surface control technique is introduced to prevent an “explosion of complexity”. The proposed control method ensures that all the signals of the closed-loop system are semi-globally uniformly ultimately bounded, and that the tracking errors converge to a compact set around the origin. Finally, simulation results are provided to demonstrate the effectiveness of the proposed approach.

**Keywords** nonlinear systems, sensor faults, fault-tolerant control, dynamic surface control, dead zone inputs



**Qi ZHOU** was born in 1983. She received her B.S. and M.S. degrees in mathematics from Bohai University, Jinzhou, China, in 2006 and 2009, and her Ph.D. degree in control science and engineering from Nanjing University of Science and Technology, Nanjing, China, in 2013, respectively. She is a professor at the Guangdong University of Technology, Guangdong, China. Her research interests include adaptive control, fuzzy control, neural control, and

robust control for nonlinear systems.



**Guohuai LIN** was born in 1995. He received his B.S. degree from Guangdong University of Petrochemical Technology, Maoming, China, in 2018. He is currently pursuing his M.S. degree in control science and engineering at Guangdong University of Technology, Guangzhou, China. His current research interests include adaptive control and fault-tolerant control for nonlinear systems.



**Hui MA** was born in 1994. She received her B.S. degree in mathematics from Harbin Normal University, Harbin, China, in 2016, and her M.S. degree in applied mathematics from Bohai University, Jinzhou, China, in 2019. She is studying for a Ph.D. degree in control science and engineering at Guangdong University of Technology, Guangzhou, China. Her current research interests include fuzzy control and adaptive control for nonlinear systems.

and adaptive control for nonlinear systems.



**Renquan LU** was born in 1971. He received his Ph.D. degree in control science and engineering from the Department of Control Science and Engineering, Zhejiang University, Hangzhou, China, in 2004. He is currently a Professor at School of Automation, Guangdong University of Technology, Guangzhou, China. His current research interests include complex systems, networked control systems, and nonlinear systems.