

不确定信息网络专题

基于事件触发和量化的时滞神经网络系统状态估计

刘金良^{①②③}, 汤佳^④, 费树岷^{⑤*}

① 南京财经大学信息工程学院, 南京 210023

② 江苏省现代粮食流通与安全协同创新中心, 南京 210023

③ 江苏省现代物流重点实验室, 南京 210023

④ 南京财经大学应用数学学院, 南京 210023

⑤ 东南大学自动化学院, 南京 210096

* 通信作者. E-mail: smfei@seu.edu.cn

收稿日期: 2016-05-19; 接受日期: 2016-08-02; 网络出版日期: 2016-11-09

国家自然科学基金(批准号: 61403185)、江苏省属高校自然科学研究重大项目(批准号: 15KJA120001)、江苏省“六大人才高峰”资助项目(批准号: 2015-DZXX-021)、江苏省高校“青蓝工程”优秀青年骨干教师培养项目(2014)和江苏高校优势学科建设工程资助项目(PAPD)资助

摘要 本文研究了具有事件触发和量化的时滞神经网络系统状态估计问题。为减轻网络负荷, 本文考虑了基于数据动态传输的事件触发机制以及量化控制, 在此基础上对时滞神经网络系统的状态估计器设计问题进行研究。充分考虑网络对系统性能的影响, 首先建立基于事件触发和量化控制的时滞神经网络系统状态估计数学模型, 然后通过利用 Lyapunov 稳定性理论和线性矩阵不等式技术, 分别给出基于事件触发和量化作用下系统渐近稳定和状态估计器存在的充分条件, 最后通过数值实例验证本文所提方法的有效性。

关键词 神经网络 事件触发 量化 状态估计 网络控制系统

1 引言

作为数学、生物、电子、计算机等多门学科的交叉科学, 神经网络经历了半个多世纪的发展, 已在自动控制、图像识别、数据压缩、组合优化、信息分类、信号处理等学科和方向有了重要的应用。近年来, 由于神经网络在实际生活中的重要性, 越来越多的学者开始投身于各种类型的神经网络系统的研究^[1~5]。文献[3]介绍了模糊神经网络系统中指数稳定及同步问题。文献[4]研究了一类采样数据非线性系统的动态神经网络稳定自适应控制问题。然而在众多应用中, 神经元的状态通常无法全部得到, 因此, 对神经网络的状态进行估计就显得尤为重要^[6,7]。神经网络状态估计旨在通过可得到的测量输出来估计神经元的状态, 使得网络模型的误差系统达到稳定。目前, 神经网络的状态估计问题也已经成为学者们广泛关注的热点之一。Wang 等在文献[8]中首次研究具有时变时滞的神经网络的状

引用格式: 刘金良, 汤佳, 费树岷. 基于事件触发和量化的时滞神经网络系统状态估计. 中国科学: 信息科学, 2016, 46: 1555–1568,
doi: 10.1360/N112016-00131

态估计问题, 并运用线性矩阵不等式方法处理常时滞神经网络的状态估计问题. 在文献 [9] 中, 通过使用 Lyapunov-Krasovskii 函数等方法, 作者详细阐述了带有时变延迟和分布式延迟的 Markov 跳跃神经网络的指数状态估计问题. 文献 [10] 针对混合区间时变时滞神经网络系统, 讨论了该系统的状态估计问题.

另一方面, 伴随着数控时代的到来和网络化控制的兴起, 网络系统设计中考虑量化控制的影响也受到广泛的关注^[11~17]. 有关量化控制问题的研究最早可以追溯到 1956 年, Kalman^[18] 指出量化器的引入可能导致闭环系统出现极限环和混沌现象. 基于此, 许多学者展开了对量化问题的研究, 从最早的理解和克服量化的影响到现在把量化器看作是信息编码器, 提出许多重要的成果^[19, 20]. 考虑量化和网络丢包的影响, Qu 等在文献 [12] 中研究了网络控制系统的事件触发控制问题. Li 等^[13] 针对不确定系统, 在量化反馈控制的基础上, 讨论了离散系统的控制问题. Meng 等^[17] 针对线性多智能体系统, 研究该系统的量化反馈输出和同步问题. 文献 [19] 提出了具有扇形边界的量化反馈控制的方法. 对于神经网络系统, 考虑量化状态估计器设计问题还没有引起太多学者们的关注.

此外, 如何降低网络负荷, 有效地利用带宽也受到国内外学者的高度关注. 为克服周期采样的缺点, 即减少不必要的计算和通信资源的浪费, 在文献 [21, 22] 中都相继采用了一种优于周期采样的方法, 即所谓的事件触发机制, 它是通过一个事件触发判别机制来决定当前采样数据是否被传输, 该方法被认为是一种有效的、优于周期采样的一种方法. 引入事件触发机制可以减少控制任务的执行次数、节约计算成本、提高系统的运行效率. 因此, 近些年来, 很多学者针对事件触发问题展开了讨论^[14, 23~25]. 文献 [14] 基于量化和事件触发机制的考虑, 研究了线性网络系统控制问题. 文献 [24] 讨论了一个离散系统中的事件触发控制问题. 事件触发机制提供了一个有效的方法来决定当前采样数据是否被传输, 这在一定程度上节省了网络宽带, 同时也节省了传感器和执行器的能量消耗. 游和谢^[25] 针对网络控制系统, 介绍了网络控制系统中所采用的网络的特性、在无噪和有噪反馈信道下镇定线性系统所需的最低通信数据率等几种类型问题的研究, 其中事件触发机制和数据的量化处理都有提及, 但是在神经网络系统中, 将两者结合还是很少见的, 因此将事件触发机制应用于神经网络系统模型很有必要.

基于以上热点问题, 本文开展基于事件触发和量化的时滞神经网络系统状态估计器设计问题的研究. 首先引入事件触发机制, 它用来筛选测量输出信号, 其次引入对量化器对传输信号进行量化处理, 进而建立基于事件触发和量化的时滞神经网络系统状态估计模型. 通过使用 Lyapunov 泛函稳定性理论给出状态估计器设计算法, 并给出仿真实例来验证本文给出方法的有效性.

2 系统模型

考虑如下具有时滞的神经网络系统模型^[7]

$$\begin{cases} \dot{x}(t) = -Ax(t) + W_0g(x(t)) + W_1g(x(t - \tau(t))), \\ y(t) = Cx(t), \end{cases} \quad (1)$$

其中 $x(t) = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ 表示状态向量; $A = \text{diag}\{a_1, a_2, \dots, a_n\}$ 是一个正定矩阵, 且 $a_i > 0$; $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \dots, g_n(x_n(t))]^T$ 是神经网络的激励函数; W_0 和 W_1 是激励函数对应项的连接权值矩阵; C 是具有适合维数的常量矩阵; $\tau(t)$ 是系统状态时滞, 并且满足 $\tau(t) \in [\tau_m, \tau_M]$, 其中 τ_m 和 τ_M 分别为 $\tau(t)$ 的下界和上界; $y(t) = [y_1, y_2, \dots, y_r]^T \in \mathbb{R}^r$ 表示系统的测量输出.

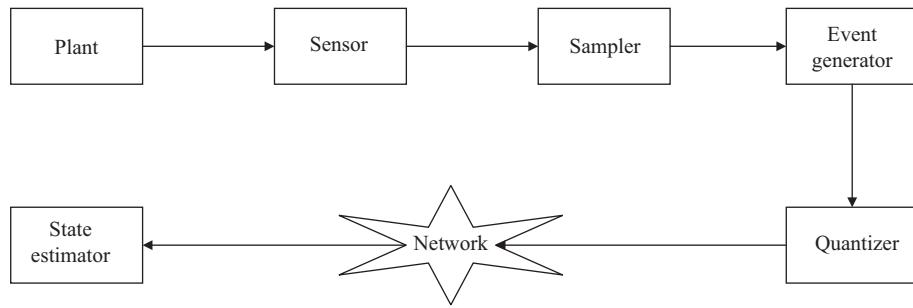


图 1 基于事件触发和量化的时滞神经网络系统状态估计流程图

Figure 1 A typical state estimator for event-triggered delayed neural networks with quantization

假设 1 神经元激励函数满足下列一个条件^[7]:

$$[g(x) - U_1 x]^T [g(x) - U_2 x] \leq 0, \quad (2)$$

其中 U_1, U_2 为实数常量矩阵, 同时满足 $U_2 - U_1 \geq 0$.

注 1 考虑到网络传输资源受限, 为节省网络宽带、减轻网络负担, 本文采用如图 1 所示的事件触发器和量化器. 传感器执行研究对象的信息采集任务, 并将信号传递到采样器, 由采样器以固定的周期采样并传输到事件触发器, 只有当周期采样信号满足一定的条件才被传输到量化器.

本文假设传感器与采样器是时间驱动的, 采样器的采样周期为 h , 采样时刻为 kh ($k = 0, 1, \dots$), 给出类似文献 [21] 中的事件触发条件:

$$[y((k+j)h) - y(kh)]^T \Phi [y((k+j)h) - y(kh)] \leq \sigma y^T((k+j)h) \Phi y((k+j)h), \quad (3)$$

其中 Φ 是合适维数的正定矩阵, $\sigma \in [0, 1]$, $y((k+j)h)$ 表示当前传感器的采样数据, $j = 1, 2, \dots, y(kh)$ 表示传感器最新传出去的数据. 采样状态 $y((k+j)h)$ 满足条件 (3) 将不能传输, 只有采样数据不满足条件 (3) 才会被传输到量化器.

注 2 由事件触发条件式 (3) 可知, 采样数据释放时刻不仅和参数 σ 有关, 而且还和系统的当前采样状态及前一次发送的状态有关.

注 3 若当前的采样数据与最新传输的数据之间的误差满足触发条件 (3), 则该数据不会被传输; 反之, 该数据则被传输到量化器, 进而通过网络传输到状态估计器.

在事件触发机制 (3) 下, 假设触发器释放数据时刻为 $k_0 h, k_1 h, k_2 h, \dots$, $t_0 = 0$ 为初始时刻, 相应的网络诱导时滞为 d_j , 经过事件触发之后系统 (1) 的实际测量输出为

$$\bar{y}(t) = y(k_j h) = C x(k_j h). \quad (4)$$

当采样数据 $y(k_j h)$ 到达量化器, 测量输出经量化后^[11, 12] 为

$$y_1(k_j h) = q(y(k_j h)). \quad (5)$$

量化器 $q(\cdot)$ 满足

$$q(y) = \text{diag}\{q_1, q_2, \dots, q_s\}, \quad (6)$$

$q_i(\cdot)$ 是对称的, 也即 $q_i(-y_i) = -q_i(y_i)$, 并且对数量化器 $q_i(\cdot)$ 定义为

$$q_i(y_i) = \begin{cases} u_l^{(i)}, & \text{if } \frac{1}{1+\delta_{q_i}}u_l^{(i)} < y_i < \frac{1}{1-\delta_{q_i}}u_l^{(i)}, \quad y_i > 0, \\ 0, & \text{if } y_i = 0, \\ -q_i(-y_i), & \text{if } y_i < 0, \end{cases} \quad (7)$$

其中 $\delta_{q_i} = \frac{1-\rho_{q_i}}{1+\rho_{q_i}}$ ($0 < \rho_{q_i} < 1$), ρ_{q_i} 表示量化密度, 是一个固定的常数.
再定义

$$\Delta_q = \text{diag}\{\Delta_{q_1}, \Delta_{q_2}, \dots, \Delta_{q_s}\}, \quad \Delta_{q_i} \in [-\delta_{q_i}, \delta_{q_i}], i = 1, 2, \dots, s. \quad (8)$$

为了方便, 我们假设 $\delta_{q_i} = \delta_q$, δ_q 是一个常数. 类似文献 [12] 中的处理方法, $q(\cdot)$ 可表示为

$$q(y) = (I + \Delta_q)y. \quad (9)$$

结合式 (5) 与 (9), 则状态估计器的实际输入可写为

$$\tilde{y}(t) = q(y(k_j h)) = (I + \Delta_q)y(k_j h), \quad t \in [k_j h + d_j, k_{j+1} h + d_{j+1}). \quad (10)$$

如果 $\Delta_q = 0$, 则 $q(y) = y$, 即无量化作用.

考虑到网络时滞的影响, 本文采用类似于文献 [21] 的方法, 下面将分两种情况来考虑.

情况 A 当 $k_j h + h + \bar{d} \geq k_{j+1} h + d_{j+1}$ 时, 这里 $\bar{d} = \max d_j$, 定义函数

$$d(t) = t - k_j h, \quad t \in [k_j h + d_j, k_{j+1} h + d_{j+1}), \quad (11)$$

因此可以得到 $d_j \leq d(t) \leq (k_{j+1} - k_j)h + d_{j+1} \leq h + \bar{d}$.

情况 B 当 $k_j h + h + \bar{d} < k_{j+1} h + d_{j+1}$ 时, 考虑以下两个区间 $[k_j h + d_j, k_j h + h + \bar{d}]$, $[k_j h + ih + \bar{d}, k_j h + ih + h + \bar{d}]$. 由于 $d_j \leq \bar{d}$, 因此存在一个正整数 $d_M \geq 1$, 使得 $k_j h + d_M h + \bar{d} < k_{j+1} h + d_{j+1} \leq k_j h + d_M h + \bar{d}$. 此外, $y(k_j h)$ 和 $k_j h + ih$ 满足不等式 (3), 这里 $i = 1, 2, \dots, d_M$. 同时

$$\begin{cases} I_1 = [k_j h + d_j, k_j h + h + \bar{d}], \\ I_2^{(i)} = \bigcup_i^{d_M-1} [k_j h + ih + \bar{d}, k_j h + ih + h + \bar{d}], \\ I_3 = [k_j h + d_M h + \bar{d}, k_{j+1} h + d_{j+1}), \end{cases} \quad (12)$$

定义函数

$$d(t) = \begin{cases} t - k_j h, & t \in I_1, \\ t - k_j h - ih, & t \in I_2^{(i)} (i = 1, 2, \dots, d_M - 1), \\ t - k_j h - d_M h, & t \in I_3. \end{cases} \quad (13)$$

综上所述, 根据 $d(t)$ 的定义, 可得

$$\begin{cases} 0 \leq d_j \leq d(t) < h + \bar{d}, & t \in I_1, \\ 0 \leq d_j \leq \bar{d} \leq d(t) < h + \bar{d}, & t \in I_2^{(i)} (i = 1, 2, \dots, d_M - 1), \\ 0 \leq d_j \leq \bar{d} \leq d(t) < h + \bar{d}, & t \in I_3. \end{cases} \quad (14)$$

这里由于 $k_{j+1}h + d_{j+1} \leq k_j + (d_M + 1)h + \bar{d}$, 因此式 (12) 的第 3 行成立. 显然

$$0 \leq d_j \leq d(t) \leq h + \bar{d} \triangleq d_M, \quad t \in [k_jh + d_j, k_{j+1}h + d_{j+1}). \quad (15)$$

在情况 A 中, 由于 $t \in [k_jh + d_j, k_{j+1}h + d_{j+1})$, 定义一个误差向量 $e_k(t) = 0$.

在情况 B 中, 定义

$$e_k(t) = \begin{cases} 0, & t \in I_1, \\ y(k_jh) - y(k_jh + ih), & t \in I_2^{(i)} (i = 1, 2, \dots, d_M - 1), \\ y(k_jh) - y(k_jh + d_Mh), & t \in I_3. \end{cases} \quad (16)$$

由 $e_k(t)$ 的定义和事件触发条件 (3), 可得

$$e_k^T(t)\Phi e_k(t) \leq \sigma y^T(t - d(t))\Phi y(t - d(t)). \quad (17)$$

注 4 通过 $d(t)$ 的定义可得, $d(t)$ 不仅依赖于采样数据释放时刻, 而且还依赖于采样周期 h , 需要指出的是 $d(t)$ 与神经网络的状态时滞 $\tau(t)$ 是不同的.

根据实际测量输出, 本文选择如下状态估计系统:

$$\begin{cases} \dot{\hat{x}}(t) = -A\hat{x}(t) + K(\hat{y}(t) - \tilde{y}(t)), \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \quad (18)$$

其中 $\hat{x}(t)$ 为估计状态向量, K 为待求解的状态估计器增益.

定义误差项为 $e(t) = x(t) - \hat{x}(t)$, 由式 (1), (10), (16) 和 (18) 可得误差系统:

$$\begin{aligned} \dot{e}(t) = & -(A + KC)e(t) + KCx(t) - K(I + \Delta_q)Cx(t - d(t)) \\ & - K(I + \Delta_q)e_k(t) + W_0g(x(t)) + W_1g(x(t - \tau(t))). \end{aligned} \quad (19)$$

令 $\bar{x}(t) = [x^T(t) \quad e^T(t)]^T$, 通过结合式 (1) 和 (19) 得到以下增广系统:

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}\bar{x}(t - d(t)) + \bar{W}_0g(H\bar{x}(t)) + \bar{W}_1g(H\bar{x}(t - \tau(t))) + \bar{C}e_k(t), \quad (20)$$

其中

$$\begin{aligned} \bar{A} &= \begin{bmatrix} -A & 0 \\ KC & -(A + KC) \end{bmatrix}, \quad \bar{W}_0 = \begin{bmatrix} W_0 \\ W_0 \end{bmatrix}, \quad \bar{W}_1 = \begin{bmatrix} W_1 \\ W_1 \end{bmatrix}, \quad H^T = \begin{bmatrix} I \\ 0 \end{bmatrix}, \\ \bar{B} &= \begin{bmatrix} 0 & 0 \\ -K(I + \Delta_q)C & 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 \\ -K(I + \Delta_q) \end{bmatrix}. \end{aligned}$$

接下来, 为方便后面的理论分析, 给出以下几个重要引理.

引理 1 ([26]) 对任意的向量 $x, y \in \mathbb{R}^n$ 和正定对称矩阵 $Q \in \mathbb{R}^{n \times n}$, 下面不等式成立:

$$2x^T y \leq x^T Q x + y^T Q^{-1} y. \quad (21)$$

引理 2 ([27]) 对给定的常数 τ_1 与矩阵 $R > 0$, 有下面不等式成立:

$$-\tau_1 \int_{t-\tau_1}^t \dot{x}^T(s) R \dot{x}(s) \leq \begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix}. \quad (22)$$

引理 3 ([28]) 假设 $\tau(t) \in [\tau_m, \tau_M]$, $d(t) \in [0, d_M]$, $Q_i (i = 1, 2, 3, 4, 5)$ 是具有合适维数的已知矩阵, 不等式 $Q_1 + (\tau_M - \tau(t))Q_2 + (\tau(t) - \tau_m)Q_3 + (d_M - d(t))Q_4 + d(t)Q_5 < 0$ 成立, 当且仅当下面不等式成立:

$$\begin{cases} Q_1 + (\tau_M - \tau_m)Q_2 + d_M Q_4 < 0, \\ Q_1 + (\tau_M - \tau_m)Q_3 + d_M Q_4 < 0, \\ Q_1 + (\tau_M - \tau_m)Q_2 + d_M Q_5 < 0, \\ Q_1 + (\tau_M - \tau_m)Q_3 + d_M Q_5 < 0. \end{cases} \quad (23)$$

引理 4 ([29]) 对于矩阵 $R > 0$, X 和任意的实数 η , 下面的不等式成立:

$$-XR^{-1}X \leq \eta^2 X - 2\eta R. \quad (24)$$

引理 5 ([30]) A, D, E, F 是具有适当维数的实矩阵并且满足 $\|F\| \leq 1$, 则对任意的变量 $\varepsilon > 0$ 有下面的不等式成立:

$$DFE + E^T F^T D^T \leq \varepsilon^{-1} DD^T + \varepsilon E^T E. \quad (25)$$

3 主要结论

在定理 1 中, 假设 A, C, W_0, W_1 和状态估计器增益 K 是已知的, 利用 Lyapunov 泛函和线性矩阵不等式技巧, 在考虑事件触发机制和量化的前提下, 对系统 (20) 的稳定性进行分析.

定理 1 对于给定的时滞信息 τ_m, τ_M, d_M , 事件触发机制参数 σ 和反馈增益矩阵 K , 如果存在正定矩阵 $P > 0, Q_i > 0, R_i > 0 (i = 1, 2, 3), \Phi > 0$, 合适维数的矩阵 M, N, S, Z , 以及参数 $\lambda_1 > 0, \lambda_2 > 0$, 使得下面矩阵不等式成立:

$$\Sigma = \begin{bmatrix} \Omega_{11} + \Upsilon + \Upsilon^T & * & * \\ \Omega_{21} & \Omega_{22} & * \\ \Omega_{31}(p) & 0 & \Omega_{33} \end{bmatrix} < 0 \quad (p = 1, 2, 3, 4), \quad (26)$$

其中

$$\Omega_{11} = \begin{bmatrix} \Delta_1 & * \\ \Delta_2 & \Delta_3 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} \Gamma_1 & * & * & * \\ R_2 & -Q_1 - R_2 & * & * \\ 0 & 0 & -\lambda_2 \bar{U}_1 & * \\ 0 & 0 & 0 & -Q_2 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} \bar{B}^T P & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{W}_0^T P - \lambda_1 \bar{U}_2^T & 0 & 0 & 0 \\ \bar{W}_1^T P & 0 & -\lambda_2 \bar{U}_2^T & 0 \\ \bar{C}^T P & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}\Delta_3 &= \text{diag}\{\sigma\Omega, -Q_3, -\lambda_1 I, -\lambda_2 I, -\Phi\}, \quad \Gamma_1 = P\bar{A} + \bar{A}^T P + Q_1 + Q_2 + Q_3 - R_2 - \lambda_1 \bar{U}_1, \\ \Upsilon &= \begin{bmatrix} S & M & N-M & -N & Z-S & -Z & 0 & 0 & 0 \end{bmatrix}, \quad M^T = \begin{bmatrix} 0 & M_2^T & M_3^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ N^T &= \begin{bmatrix} 0 & 0 & N_3^T & N_4^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad Z^T = \begin{bmatrix} 0 & 0 & 0 & Z_5^T & Z_6^T & 0 & 0 & 0 \end{bmatrix}, \quad S^T = \begin{bmatrix} S_1^T & 0 & 0 & 0 & S_5^T & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Omega_{21} &= \begin{bmatrix} \tau_{21}R_1\bar{A} & 0 & 0 & 0 & \tau_{21}R_1\bar{B} & 0 & \tau_{21}R_1\bar{W}_0 & \tau_{21}R_1\bar{W}_1 & \tau_{21}R_1\bar{C} \\ \tau_mR_2\bar{A} & 0 & 0 & 0 & \tau_mR_2\bar{B} & 0 & \tau_mR_2\bar{W}_0 & \tau_mR_2\bar{W}_1 & \tau_mR_2\bar{C} \\ \sqrt{d_M}R_3\bar{A} & 0 & 0 & 0 & \sqrt{d_M}R_3\bar{B} & 0 & \sqrt{d_M}R_3\bar{W}_0 & \sqrt{d_M}R_3\bar{W}_1 & \sqrt{d_M}R_3\bar{C} \end{bmatrix}, \quad \Omega = \begin{bmatrix} C^T\Phi C & 0 \\ 0 & 0 \end{bmatrix}, \\ \Omega_{31}(1) &= \begin{bmatrix} \tau_{21}M^T \\ \sqrt{d_M}S^T \end{bmatrix}, \quad \Omega_{31}(2) = \begin{bmatrix} \tau_{21}M^T \\ \sqrt{d_M}Z^T \end{bmatrix}, \quad \Omega_{31}(3) = \begin{bmatrix} \tau_{21}N^T \\ \sqrt{d_M}S^T \end{bmatrix}, \quad \Omega_{31}(4) = \begin{bmatrix} \tau_{21}N^T \\ \sqrt{d_M}Z^T \end{bmatrix}, \\ \Omega_{22} &= \text{diag}\{-R_1, -R_2, -R_3\}, \quad \Omega_{33} = \text{diag}\{-R_1, -R_3\}, \quad \tau_{21} = \sqrt{\tau_M - \tau_m}, \\ \bar{U}_1 &= \frac{H^T U_1^T U_2 H + H^T U_2^T U_1 H}{2}, \quad \bar{U}_2 = -\frac{H^T U_1^T + H^T U_2^T}{2}, \end{aligned}$$

则系统 (20) 漐近稳定.

证明 构造如下形式的 Lyapunov 泛函:

$$V(\bar{x}_t) = V_1(\bar{x}_t) + V_2(\bar{x}_t) + V_3(\bar{x}_t), \quad (27)$$

其中

$$\begin{aligned}V_1(\bar{x}_t) &= \bar{x}^T(t)P\bar{x}(t), \\ V_2(\bar{x}_t) &= \int_{t-\tau_m}^t \bar{x}^T(s)Q_1\bar{x}(s)ds + \int_{t-\tau_M}^t \bar{x}^T(s)Q_2\bar{x}(s)ds + \int_{t-d_M}^t \bar{x}^T(s)Q_3\bar{x}(s)ds, \\ V_3(\bar{x}_t) &= \int_{t-\tau_M}^{t-\tau_m} \int_s^t \dot{\bar{x}}^T(v)R_1\dot{\bar{x}}(v)dvds + \tau_m \int_{t-\tau_m}^t \int_s^t \dot{\bar{x}}^T(v)R_2\dot{\bar{x}}(v)dvds + \int_{t-d_M}^t \int_s^t \dot{\bar{x}}^T(v)R_3\dot{\bar{x}}(v)dvds.\end{aligned}$$

对 $V_1(\bar{x}_t), V_2(\bar{x}_t), V_3(\bar{x}_t)$ 沿系统 (20) 关于 t 进行求导得

$$\begin{aligned}\dot{V}(\bar{x}_t) &= \dot{V}_1(\bar{x}_t) + \dot{V}_2(\bar{x}_t) + \dot{V}_3(\bar{x}_t) \\ &= 2\bar{x}^T(t)P\dot{\bar{x}}(t) + \bar{x}^T(t)(Q_1 + Q_2 + Q_3)\bar{x}(t) - \bar{x}^T(t - \tau_m)Q_1\bar{x}(t - \tau_m) \\ &\quad - \bar{x}^T(t - \tau_M)Q_2\bar{x}(t - \tau_M) - \bar{x}^T(t - d_M)Q_3\bar{x}(t - d_M) + (\tau_M - \tau_m)\dot{\bar{x}}^T(t)R_1\dot{\bar{x}}(t) \\ &\quad - \int_{t-\tau_M}^{t-\tau_m} \dot{\bar{x}}^T(s)R_1\dot{\bar{x}}(s)ds + \tau_m^2 \dot{\bar{x}}^T(t)R_2\dot{\bar{x}}(t) - \tau_m \int_{t-\tau_m}^t \dot{\bar{x}}^T(s)R_2\dot{\bar{x}}(s)ds \\ &\quad + d_M \dot{\bar{x}}^T(t)R_3\dot{\bar{x}}(t) - \int_{t-d_M}^t \dot{\bar{x}}^T(s)R_3\dot{\bar{x}}(s)ds. \quad (28)\end{aligned}$$

应用自由权矩阵方法^[31] 得

$$\begin{cases} 2\xi^T(t)M[\bar{x}(t - \tau_m) - \bar{x}(t - \tau(t)) - \int_{t-\tau(t)}^{t-\tau_m} \dot{\bar{x}}(s)ds] = 0, \\ 2\xi^T(t)N[\bar{x}(t - \tau(t)) - \bar{x}(t - \tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{\bar{x}}(s)ds] = 0, \\ 2\xi^T(t)S[\bar{x}(t) - \bar{x}(t - d(t)) - \int_{t-d(t)}^t \dot{\bar{x}}(s)ds] = 0, \\ 2\xi^T(t)Z[\bar{x}(t - d(t)) - \bar{x}(t - d_M) - \int_{t-d_M}^{t-d(t)} \dot{\bar{x}}(s)ds] = 0, \end{cases} \quad (29)$$

其中 M, N, S 和 Z 是具有合适维数的矩阵, 且 $\xi^T(t) = [\bar{x}^T(t), \bar{x}^T(t - \tau_m), \bar{x}^T(t - \tau(t)), \bar{x}^T(t - \tau_M), \bar{x}^T(t - d(t)), \bar{x}^T(t - d_M), g^T(H\bar{x}(t)), g^T(H\bar{x}(t - \tau(t))), e_k^T(t)]^T$.

由引理 1, 可得下面不等式成立:

$$\left\{ \begin{array}{l} -2\xi^T(t)M \int_{t-\tau(t)}^{t-\tau_m} \dot{\bar{x}}(s)ds \leq (\tau(t) - \tau_m)\xi^T(t)MR_1^{-1}MR_1^{-1}M^T\xi(t) + \int_{t-\tau(t)}^{t-\tau_m} \dot{\bar{x}}(s)^T R_1 \dot{\bar{x}}(s)ds, \\ -2\xi^T(t)N \int_{t-\tau_M}^{t-\tau(t)} \dot{\bar{x}}(s)ds \leq (\tau_M - \tau(t))\xi^T(t)NR_1^{-1}N^T\xi(t) + \int_{t-\tau_M}^{t-\tau(t)} \dot{\bar{x}}(s)^T R_1 \dot{\bar{x}}(s)ds, \\ -2\xi^T(t)S \int_{t-d(t)}^t \dot{\bar{x}}(s)ds \leq d(t)\xi^T(t)SR_3^{-1}S^T\xi(t) + \int_{t-d(t)}^t \dot{\bar{x}}(s)^T R_3 \dot{\bar{x}}(s)ds, \\ -2\xi^T(t)Z \int_{t-d_M}^{t-d(t)} \dot{\bar{x}}(s)ds \leq (d_M - d(t))\xi^T(t)ZR_3^{-1}Z^T\xi(t) + \int_{t-d_M}^{t-d(t)} \dot{\bar{x}}(s)^T R_3 \dot{\bar{x}}(s)ds. \end{array} \right. \quad (30)$$

根据引理 2, 有

$$-\tau_m \int_{t-\tau_m}^t \dot{\bar{x}}^T(s)R_2 \dot{\bar{x}}(s)ds \leq \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t - \tau_m) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t - \tau_m) \end{bmatrix}. \quad (31)$$

利用假设 1 可得

$$\begin{bmatrix} \bar{x}(t) \\ \bar{g}(H(\bar{x}(t))) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2^T & I \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{g}(H(\bar{x}(t))) \end{bmatrix} \leq 0. \quad (32)$$

由式 (32), 对于 $\lambda_1 > 0, \lambda_2 > 0$, 有如下不等式成立:

$$-\lambda_1 \begin{bmatrix} \bar{x}(t) \\ \bar{g}(H(\bar{x}(t))) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2^T & I \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{g}(H(\bar{x}(t))) \end{bmatrix} \geq 0, \quad (33)$$

$$-\lambda_2 \begin{bmatrix} \bar{x}(t - \tau(t)) \\ \bar{g}(H(\bar{x}(t - \tau(t)))) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2^T & I \end{bmatrix} \begin{bmatrix} \bar{x}(t - \tau(t)) \\ \bar{g}(H(\bar{x}(t - \tau(t)))) \end{bmatrix} \geq 0. \quad (34)$$

结合式 (17) 和 (27)~(34), 可得

$$\begin{aligned} \dot{V}(\bar{x}_t) &\leq 2\bar{x}^T(t)P\bar{x}(t) + \bar{x}^T(Q_1 + Q_2 + Q_3)\bar{x}(t) - \bar{x}^T(t - \tau_m)Q_1\bar{x}(t - \tau_m) \\ &\quad - \bar{x}^T(t - \tau_M)Q_2\bar{x}(t - \tau_M) - \bar{x}^T(t - d_M)Q_3\bar{x}(t - d_M) \\ &\quad + \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t - \tau_m) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t - \tau_m) \end{bmatrix} \\ &\quad + 2\xi^T(t)M[\bar{x}(t - \tau_m) - \bar{x}(t - \tau(t))] + 2\xi^T(t)N[\bar{x}(t - \tau(t)) - \bar{x}(t - \tau_M)] \\ &\quad + 2\xi^T(t)S[\bar{x}(t) - \bar{x}(t - d(t))] + 2\xi^T(t)Z[\bar{x}(t - d(t)) - \bar{x}(t - d_M)] \\ &\quad + (\tau(t) - \tau_m)\xi^T(t)MR_1^{-1}M^T\xi(t) + (\tau_M - \tau(t))\xi^T(t)NR_1^{-1}N^T\xi(t) \\ &\quad + d(t)\xi^T(t)SR_3^{-1}S^T\xi(t) + (d_M - d(t))\xi^T(t)ZR_3^{-1}Z^T\xi(t) \\ &\quad - \lambda_1 \begin{bmatrix} \bar{x}(t) \\ \bar{g}(H(\bar{x}(t))) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2^T & I \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{g}(H(\bar{x}(t))) \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
& -\lambda_2 \left[\begin{array}{c} \bar{x}(t-\tau(t)) \\ \bar{g}(H(\bar{x}(t-\tau(t)))) \end{array} \right]^T \left[\begin{array}{cc} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2^T & I \end{array} \right] \left[\begin{array}{c} \bar{x}(t-\tau(t)) \\ \bar{g}(H(\bar{x}(t-\tau(t)))) \end{array} \right] \\
& + \sigma \bar{x}^T(t-d(t)) \Omega \bar{x}(t-d(t)) - e_k^T(t) \Phi e_k(t) \\
& + (\tau_M - \tau_m) \dot{\bar{x}}^T(t) R_1 \dot{\bar{x}}(t) + \tau_m^2 \dot{\bar{x}}^T(t) R_2 \dot{\bar{x}}(t) + d_M \dot{\bar{x}}^T(t) R_3 \dot{\bar{x}}(t).
\end{aligned} \tag{35}$$

通过 Schur 补, 由式 (35) 可得

$$\begin{aligned}
\dot{V}(\bar{x}_t) & \leq \xi^T(t)(\Omega_{11} + \Upsilon + \Upsilon^T)\xi(t) + (\tau(t) - \tau_m)\xi^T(t)MR_1^{-1}M^T\xi(t) \\
& + (\tau_M - \tau(t))\xi^T(t)NR_1^{-1}N^T\xi(t) + d(t)\xi^T(t)SR_3^{-1}S^T\xi(t) + (d_M - d(t))\xi^T(t)ZR_3^{-1}Z^T\xi(t) \\
& + (\tau_M - \tau_m)\dot{\bar{x}}^T(t)R_1 \dot{\bar{x}}(t) + \tau_m^2 \dot{\bar{x}}^T(t)R_2 \dot{\bar{x}}(t) + d_M \dot{\bar{x}}^T(t)R_3 \dot{\bar{x}}(t).
\end{aligned} \tag{36}$$

结合式 (26) 和 (36), 由引理 4 及 Schur 补可知 $\dot{V}(\bar{x}_t) < 0$ 成立. 通过 Lyapunov 稳定性理论可得系统 (20) 漐近稳定.

根据定理 1 所得的稳定性条件, 在定理 2 中将给出在事件触发机制和量化的作用下神经网络系统 (20) 的状态估计增益矩阵 K 的设计算法.

定理 2 对于给定的时滞信息 τ_m, τ_M, d_M , 事件触发机制参数 σ , 以及参数 $\delta_q, \varepsilon_p > 0$ ($p = 1, 2, 3, 4$), 如果存在正定矩阵 $P_1 > 0, P_2 > 0, Q_i > 0, R_i > 0$ ($i = 1, 2, 3$), $\Phi > 0$, 具有合适维数的矩阵 M, N, S, Z 以及参数 $\lambda_1 > 0, \lambda_2 > 0$, 使得下面线性矩阵不等式成立:

$$\Pi = \begin{bmatrix} \tilde{\Sigma}(p) & * & * \\ \varepsilon_4 \bar{\mathcal{L}}_1 & -\varepsilon_4 I & * \\ \delta_q \bar{\mathcal{L}}_2 & 0 & -\varepsilon_4 I \end{bmatrix} < 0, \quad (p = 1, 2, 3, 4), \tag{37}$$

其中

$$\begin{aligned}
\tilde{\Sigma}(p) &= \begin{bmatrix} \Pi_{11} + \Upsilon + \Upsilon^T & * & * \\ \Pi_{21} & \tilde{\Omega}_{22} & * \\ \Omega_{31}(p) & 0 & \Omega_{33} \end{bmatrix}, \quad A_1 = \begin{bmatrix} -P_1 A & 0 \\ YC & -P_2 A - YC \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 \\ -YC & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 \\ -Y \end{bmatrix}, \\
\Pi_{11} &= \begin{bmatrix} \Delta_4 & * \\ \Delta_5 & \Delta_3 \end{bmatrix}, \quad \Delta_4 = \begin{bmatrix} \hat{T}_1 & * & * & * \\ R_2 & -Q_1 - R_2 & * & * \\ 0 & 0 & -\lambda_2 \bar{U}_1 & * \\ 0 & 0 & 0 & -Q_2 \end{bmatrix}, \quad \Delta_5 = \begin{bmatrix} A_2^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{W}_0^T P - \lambda_1 \bar{U}_2^T & 0 & 0 & 0 \\ \bar{W}_1^T P & 0 & -\lambda_2 \bar{U}_2^T & 0 \end{bmatrix}, \\
\Pi_{21} &= \begin{bmatrix} \tau_{21} A_1 & 0 & 0 & 0 & \tau_{21} A_2 & 0 & \tau_{21} P \bar{W}_0 & \tau_{21} P \bar{W}_1 & \tau_{21} A_3 \\ \tau_m A_1 & 0 & 0 & 0 & \tau_m A_2 & 0 & \tau_m P \bar{W}_0 & \tau_m P \bar{W}_1 & \tau_m A_3 \\ \sqrt{d_M} A_1 & 0 & 0 & 0 & \sqrt{d_M} A_2 & 0 & \sqrt{d_M} P \bar{W}_0 & \sqrt{d_M} P \bar{W}_1 & \sqrt{d_M} A_3 \end{bmatrix}, \\
\tilde{\Omega}_{22} &= \text{diag}\{-2\varepsilon_1 P + \varepsilon_1^2 R_1, -2\varepsilon_2 P + \varepsilon_2^2 R_2, -2\varepsilon_3 P + \varepsilon_3^2 R_3\}, P = \text{diag}\{P_1, P_2\}, \\
\bar{\mathcal{L}}_1 &= \begin{bmatrix} A_3^T & 0_{n \times 8n} & \tau_{21} A_3^T & \tau_m A_3^T & \sqrt{d_M} A_3^T & 0 & 0 \end{bmatrix}, \quad \bar{\mathcal{L}}_2 = \begin{bmatrix} 0_{n \times 4n} & \check{C}_1 & 0_{n \times 3n} & I & 0_{n \times 5n} \end{bmatrix},
\end{aligned}$$

$$\hat{\Gamma}_1 = A_1 + A_1^T + Q_1 + Q_2 + Q_3 - R_2 - \lambda_1 \bar{U}_1, \check{C}_1 = [C \ 0].$$

其他参数的定义同定理 1, 则系统 (20) 漐近稳定, 且状态估计增益矩阵 $K = P_2^{-1}Y$.

证明 定义 $P = \text{diag}\{P_1, P_2\}$, $P_2K = Y$, $\bar{B} = \hat{B} + \bar{K}\bar{C}_1$, $\bar{C} = \bar{K} + \bar{K}\bar{C}_2$, 其中,

$$\hat{B} = \begin{bmatrix} 0 & 0 \\ -KC & 0 \end{bmatrix}, \quad \bar{C}_1 = [\Delta_q C \ 0], \quad \bar{K} = \begin{bmatrix} 0 \\ -K \end{bmatrix}, \quad \bar{C}_2 = \Delta_q.$$

在式 (26) 左右两边分别乘以 $\text{diag} = \{I, PR_1^{-1}, PR_2^{-1}, PR_3^{-1}, I\}$ 和其转置, 并结合引理 4 可得

$$\hat{\Sigma} = \begin{bmatrix} \Omega_{11} + \Upsilon + \Upsilon^T & * & * \\ \hat{\Omega}_{21} & \tilde{\Omega}_{22} & * \\ \Omega_{31}(p) & 0 & \Omega_{33} \end{bmatrix} < 0, \quad (p = 1, 2, 3, 4), \quad (38)$$

其中

$$\hat{\Omega}_{21} = \begin{bmatrix} \tau_{21}P\bar{A} & 0 & 0 & 0 & \tau_{21}P\bar{B} & 0 & \tau_{21}P\bar{W}_0 & \tau_{21}P\bar{W}_1 & \tau_{21}P\bar{C} \\ \tau_mP\bar{A} & 0 & 0 & 0 & \tau_mP\bar{B} & 0 & \tau_mP\bar{W}_0 & \tau_mP\bar{W}_1 & \tau_mP\bar{C} \\ \sqrt{d_M}P\bar{A} & 0 & 0 & 0 & \sqrt{d_M}P\bar{B} & 0 & \sqrt{d_M}P\bar{W}_0 & \sqrt{d_M}P\bar{W}_1 & \sqrt{d_M}P\bar{C} \end{bmatrix}.$$

而式 (38) 又可重写为

$$\hat{\Sigma} = \Sigma_{11} + \mathcal{L}_1^T \mathcal{L}_2 + \mathcal{L}_2^T \mathcal{L}_1 < 0, \quad (39)$$

其中

$$\tilde{\Omega}_{11} = \begin{bmatrix} \Delta_1 & * \\ \Delta_6 & \Delta_3 \end{bmatrix}, \quad \Delta_6 = \begin{bmatrix} \hat{B}^T P & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{W}_0^T P - \lambda_1 \bar{U}_2^T & 0 & 0 & 0 \\ \bar{W}_1^T P & 0 & -\lambda_2 \bar{U}_2^T & 0 \\ \bar{K}^T P & 0 & 0 & 0 \end{bmatrix}, \quad \Sigma_{11} = \begin{bmatrix} \tilde{\Omega}_{11} + \Upsilon + \Upsilon^T & * & * \\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & * \\ \Omega_{31}(p) & 0 & \Omega_{33} \end{bmatrix},$$

$$\tilde{\Omega}_{21} = \begin{bmatrix} \tau_{21}P\bar{A} & 0_{n \times 3n} & \tau_{21}P\hat{B} & 0 & \tau_{21}P\bar{W}_0 & \tau_{21}P\bar{W}_1 & \tau_{21}P\bar{K} \\ \tau_mP\bar{A} & 0_{n \times 3n} & \tau_mP\hat{B} & 0 & \tau_mP\bar{W}_0 & \tau_mP\bar{W}_1 & \tau_mP\bar{K} \\ \sqrt{d_M}P\bar{A} & 0_{n \times 3n} & \sqrt{d_M}P\hat{B} & 0 & \sqrt{d_M}P\bar{W}_0 & \sqrt{d_M}P\bar{W}_1 & \sqrt{d_M}P\bar{K} \end{bmatrix},$$

$$\mathcal{L}_1 = [\bar{K}^T P \ 0_{n \times 8n} \ \tau_{21}\bar{K}^T P \ \tau_m\bar{K}^T P \ \sqrt{d_M}\bar{K}^T P \ 0_{n \times 2n}], \quad \mathcal{L}_2 = [0_{n \times 4n} \ \bar{C}_1 \ 0_{n \times 3n} \ \bar{C}_2 \ 0_{n \times 5n}].$$

由式 (39), 应用引理 5 可知, 存在 ε_4 使得

$$\hat{\Sigma} \leq \Sigma_{11} + \varepsilon_4 \mathcal{L}_1^T \mathcal{L}_1 + \varepsilon_4^{-1} \delta_q^2 \bar{\mathcal{L}}_2^T \bar{\mathcal{L}}_2. \quad (40)$$

由式 (26), (37) 和 (40), 利用 Schur 补, 根据定理 1 可得系统 (20) 漐近稳定. 由 $P_2K = Y$ 可知状态估计器增益为 $K = P_2^{-1}Y$, 定理得证.

注 5 本文使用的事件触发机制和量化机制能够有效地减少网络传输负担, 延长状态估计器等的使用寿命. 同时还可以看出, 所设计的状态估计器不仅依赖于网络通讯时滞和事件触发机制参数, 而且还受量化器的量化参数 δ_q 的影响.

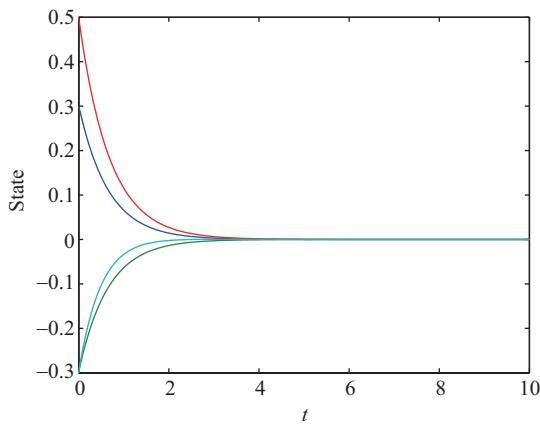


图 2 (网络版彩图) 系统状态 $x(t)$ 与估计状态 $\hat{x}(t)$ 的轨迹

Figure 2 (Color online) The state responses of $x(t)$ and its estimation $\hat{x}(t)$

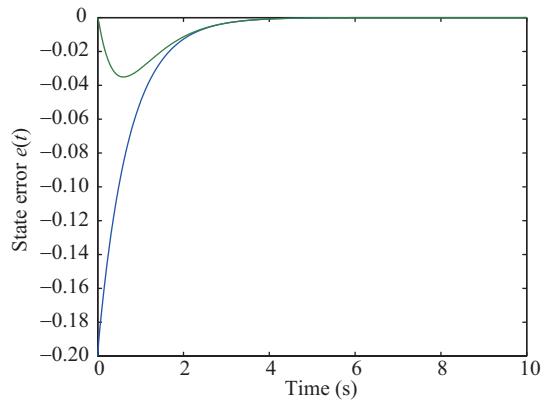


图 3 (网络版彩图) 系统状态误差图

Figure 3 (Color online) The estimation error $e(t)$

4 数值例子

为验证本文给出方法的有效性, 下面将给出具体的数值算例.

这里直接假设神经网络系统 (1) 具有如下参数:

$$\begin{aligned} A &= \begin{bmatrix} 1.5 & 0 \\ 0 & 2 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.2 & -0.3 \\ -0.3 & 0.2 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix}, \quad C = \begin{bmatrix} -0.9 & -0.8 \\ -0.7 & -0.5 \end{bmatrix}, \\ U_1 &= \begin{bmatrix} 0.3 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0 & 0.95 \end{bmatrix}, \quad g(t) = \begin{bmatrix} 0.5x_1(t) - \tanh(2x_1(t)) + 0.2x_2(t) \\ 0.95x_2(t) - \tanh(0.75x_1(t)) \end{bmatrix}. \end{aligned}$$

下面将选择合适的参数来说明事件触发器和量化器对系统的影响.

取 $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.1$, 时滞 $\tau_m = 0.01$, $\tau_M = 0.05$, $d_M = 0.1$, 对应的触发参数 $\sigma = 0.05$ 和量化密度 $\rho = 0.5$, 这里 $\delta_q = \frac{1-\rho}{1+\rho}$, 通过使用 Matlab 的 LMI 工具箱求解 (37), 可以得到下面的矩阵:

$$\begin{aligned} P_1 &= \begin{bmatrix} 153.5560 & -23.7902 \\ -23.7902 & 132.8509 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 145.1459 & -25.4209 \\ -25.4209 & 109.5115 \end{bmatrix}, \quad Y = \begin{bmatrix} 3.2061 & 5.4329 \\ 11.6651 & 5.8164 \end{bmatrix}, \\ K &= \begin{bmatrix} 0.0425 & 0.0487 \\ 0.1164 & 0.0644 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 101.9030 & -5.6446 \\ -5.6446 & 105.9252 \end{bmatrix}, \quad \lambda_1 = 125.2997, \quad \lambda_2 = 210.3698. \end{aligned}$$

选定系统的初始条件为 $x(0) = [0.3 \ -0.3]^T$, 估计器的初始状态为 $\hat{x}(0) = [0.5 \ -0.3]^T$. 基于上述参数, 考虑事件触发器和量化器的作用, 可得图 2~5 的仿真曲线图. 图 2 给出系统状态 $x(t)$ 与估计状态 $\hat{x}(t)$ 的轨迹变化图; 图 3 描述系统状态误差 $e(t)$ 的动态变化; 图 4 表示经过量化器后系统的实际输出 $\tilde{y}(t)$; 图 5 显示的是传输信号释放时刻和释放区间变化图.

从图 3 的仿真曲线图可以看出, 误差状态在很短的时间内便能趋于稳定, 这表明所设计状态估计器的有效性. 通过图 5 描述的事件触发机制中信号的释放时刻与释放间隔图可以看出, 系统在趋于稳

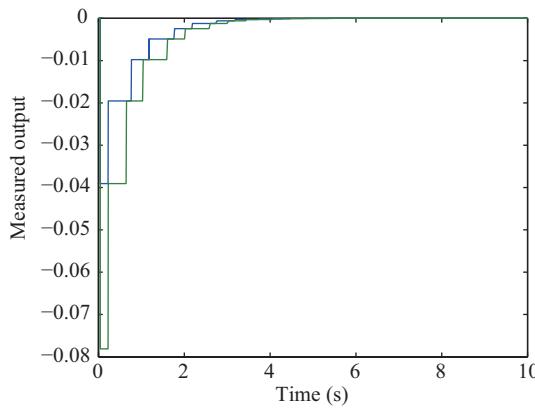
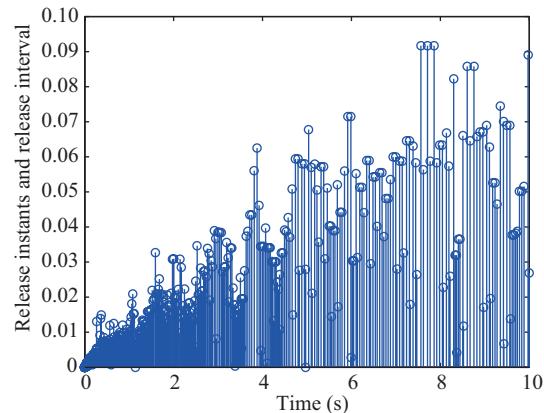
图 4 (网络版彩图) 量化输出 $\bar{y}(t)$ **Figure 4** (Color online) The quantized output $\bar{y}(t)$ 

图 5 (网络版彩图) 释放时刻与释放间隔图

Figure 5 (Color online) The release instants and intervals

定之前, 采样比较密集, 系统渐渐趋于稳定之后, 采样变得稀疏, 节约了传输资源, 这说明本文所采用事件触发机制的有效性.

5 结论

本文研究了一类基于事件触发和量化的时滞神经网络系统状态估计器设计问题. 为减少网络传输压力、节省网络带宽, 本文引入了基于数据动态传输筛选的事件触发机制, 并对测量输出进行量化处理, 建立基于事件触发和量化的时滞神经网络系统状态估计模型. 在此基础上通过使用 Lyapunov 泛函稳定性理论和线性矩阵不等式技术研究时滞神经网络系统的状态估计问题, 并得到该问题存在可行解的充分条件, 最后通过数值算例验证本文所提方法的有效性.

致谢 作者感谢南京邮电大学岳东教授在本文事件触发仿真方面给予的指导.

参考文献

- 1 Jiang B, Shen Q, Shi P. Neural-networked adaptive tracking control for switched nonlinear pure-feedback systems under arbitrary switching. *Automatica*, 2015, 61: 119–125
- 2 Chen W S, Hua S Y, Ge S S. Consensus-based distributed cooperative learning control for a group of discrete-time nonlinear multi-agent systems using neural networks. *Automatica*, 2014, 50: 2254–2268
- 3 Yang S J, Li C D, Huang T W. Exponential stabilization and synchronization for fuzzy model of memristive neural networks by periodically intermittent control. *Neural Networks*, 2016, 75: 162–172
- 4 Sun F C, Sun Z Y, Zhang R J. Stable adaptive control for sampled-data nonlinear systems using neural networks. *Acta Autom Sin*, 1996, 22: 648–656 [孙富春, 孙增圻, 张戎军. 采样非线性系统的神经网络稳定自适应控制. 自动化学报, 1996, 22: 648–656]
- 5 Liu G Q, Zhou S M. Robust stability for a class of uncertain Hopfield neural networks of neutral-type with time-varying delays. *Acta Autom Sin*, 2013, 39: 1421–1430 [刘国权, 周书民. 一类含有时变时滞的不确定中立型 Hopfield 神经网络的鲁棒稳定性判据. 自动化学报, 2013, 39: 1421–1430]
- 6 Liu R, Wang Z D, Liu X H. Design of exponential state estimators for neural networks with mixed time delays. *Phys Lett A*, 2007, 364: 401–412

- 7 Li N, Hu J, Li L. Exponential state estimation for delayed recurrent neural networks with sampled-data. *Nonlinear Dynam*, 2012, 69: 555–564
- 8 Wang Z D, Ho C W D, Liu H X. State estimation for delayed neural networks. *IEEE Trans Neur Net*, 2005, 16: 279–284
- 9 Zhang D, Yu L. Exponential state estimation for Markovian jumping neural networks with time-varying discrete and distributed delays. *Neural Networks*, 2012, 35: 103–111
- 10 Wang H W, Song Q K. State estimation for neural networks with mixed interval time-varying delays. *Neurocomputing*, 2010, 73: 1281–1288
- 11 Shi P, Wang H J, Lim C C. Network-based event-triggered control for singular systems with quantizations. *IEEE Trans Ind Electron*, 2016, 63: 1230–1238
- 12 Qu F L, Guan Z H, He D X, et al. Event-triggered control for networked control systems with quantization and packet losses. *J Franklin Inst*, 2015, 351: 974–986
- 13 Li Z M, Chang X H, Du X K, et al. H_∞ control of discrete-time uncertain linear systems with quantized feedback. *Neurocomputing*, 2016, 174: 790–794
- 14 Hu S L, Yue D. Event-triggered control design of linear networked systems with quantizations. *ISA Trans*, 2012, 51: 153–162
- 15 Liu S, Li T, Xie L, et al. Continuous-time and sampled-data based average consensus with logarithmic quantizers. *Automatica*, 2013, 49: 3329–3336
- 16 Li T, Xie L. Distributed coordination of multi-agent systems with quantized-observer based encoding-decoding. *IEEE Trans Automat Contr*, 2012, 57: 3023–3037
- 17 Meng Y, Li T, Zhang J F. Output feedback quantized observer-based synchronization of linear multi-agent systems over jointly connected topologies. *Int J Robust Nonlin Contr*, 2015
- 18 Kalman R E. Nonlinear aspects of sampled-data control systems. In: Proceedings of the Symposium on Nonlinear Circuit Theory, Brooklyn, 1956. 6: 273–313
- 19 Fu M, Xie L. The sector bound approach to quantized feedback control. *IEEE Trans Automat Contr*, 2005, 50: 1698–1711
- 20 Liu S, Li T, Xie L. Continuous-time and sampled-data based average consensus with logarithmic quantizers. *Automatica*, 2013, 49: 3329–3336
- 21 Yue D, Tian E G, Han Q L. A delay system method for designing event-triggered controllers of networked control systems. *IEEE Trans Automat Contr*, 2013, 58: 475–481
- 22 Zhang J, Peng C. An adaptive event-triggered communication scheme for networked control systems with randomly occurring non-linearities and uncertainties. *Neurocomputing*, 2011, 174: 475–482
- 23 Sahoo A, Xu H, Jaganathan S. Adaptive neural network-based event-triggered control of single-input single-output nonlinear discrete-time systems. *IEEE Trans Neural Netw Learn Syst*, 2016, 27: 151–164
- 24 Eqtami A, Dimarogonas D V, Kyriakopoulos K J. Event-triggered control for discrete-time systems. In: Proceedings of the American Control Conference, Baltimore, 2010. 58: 4719–4724
- 25 You K Y, Xie L H. Survey of recent progress in network control system. *Acta Autom Sin*, 2013, 39: 101–118 [游科友, 谢立华. 网络控制系统的最新研究综述. *自动化学报*, 2013, 39: 101–118]
- 26 Chen W, Liu X, Chu X. Robust control of a class of uncertain nonlinear systems. *Syst Contr Lett*, 1997, 19: 139–149
- 27 Lis J G, Perlmutter D D. Stability of Time-Delay Systems. Boston: Birkhauser, 2003. 951–953
- 28 Tian E G, Yue D, Zhang Y. Delay-dependent robust H_∞ control for T-S fuzzy system with interval time-varying delay. *Fuzzy Sets Syst*, 2009, 160: 1708–1719
- 29 Xiong J, Lam J. Stabilization of networked control systems with a logic ZOH. *IEEE Trans Automat Contr*, 2009, 54: 358–363
- 30 Cao Y, Sun Y, Lam J. Delay-dependent robust H_∞ control for uncertain systems with time varying delays. *IEEE Proc Contr Theory Appl*, 1998, 143: 338–344
- 31 He Y, Wu M, She J H, et al. Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties. *IEEE Trans Automat Contr*, 2004, 49: 828–832

Event-based state estimation for delayed neural network systems with quantization

Jinliang LIU^{1,2,3}, Jia TANG⁴ & Shumin FEI^{5*}

1 College of Information Engineering, Nanjing University of Finance and Economics, Nanjing 210023, China;

2 Collaborative Innovation Center for Modern Grain Circulation and Safety, Nanjing 210023, China;

3 Jiangsu Key Laboratory of Modern Logistics, Nanjing 210023, China;

4 Department of Applied Mathematics, Nanjing University of Finance and Economics, Nanjing 210023, China;

5 School of Automation, Southeast University, Nanjing 210096, China

*E-mail: smfei@seu.edu.cn

Abstract This paper presents an investigation of the state estimation problem for a class of delayed neural network systems with event-triggered communication and quantization. The network bandwidth burden is reduced by using both an event-triggered communication scheme and quantization with which the state estimator design for delayed neural network systems is concerned. Considering the influence of the communication network, an event-based state estimator error dynamic model for delayed neural network systems is firstly constructed by taking the effect of the event-triggered scheme and quantization into consideration. Then by employing the Lyapunov functional approach and the linear matrix inequality technique, some sufficient conditions are obtained under which the state estimator exists and the estimator error dynamics is asymptotically stable. Finally, a numerical example is provided to demonstrate the usefulness of the proposed approach.

Keywords neural networks, event-triggered scheme, quantization, state estimation, networked control systems



Jinliang LIU was born in Shandong Province, China, in 1980. He received his Ph.D. degree from Donghua University in 2011. From 2013 to 2016, he worked as a postdoctoral researcher in the School of Automation, Southeast University. Currently, he is a visiting research associate in the Department of Mechanical Engineering, the University of Hong Kong. He has been with the College of Information Engineering,

Nanjing University of Finance and Economics since 2011. His research interests include networked control systems, complex dynamical networks, and T-S fuzzy systems.



Jia TANG was born in Henan Province, China, in 1992. She received her B.S. degree from Anyang Normal University in 2014. Currently, she is pursuing an M.S. degree in the Department of Applied Mathematics, Nanjing University of Finance and Economics. Her research interests include networked control systems, T-S fuzzy systems, and time-delay systems.



Shumin FEI received his Ph.D. degree from Beijing University of Aeronautics and Astronautics in 1995. From 1995 to 1997, he was a postdoctoral research fellow at Southeast University. Presently, he is a professor and doctoral advisor at the School of Automation, Southeast University. His research interests include nonlinear systems, stability theory of delayed systems, and complex systems, amongst others.